Experimental Investigations of 3D Fatigue Crack Propagation

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ABSTRACT. *In the present paper, some interesting questions concerning 3D fatigue crack propagation are discussed. The main focus is on experimental investigations of 3D fatigue crack propagation under Mode I loading conditions. Transparent specimens of PMMA are used, in order to be able to observe and to document accurate sequences of the real 3D crack front evolution profiles via in situ photographic measurement. Such well documented experimentally observed sequences of real crack front profiles are necessary to identify an appropriate crack propagation criterion via comparing the experimental observations with corresponding numerical simulations. The aim of this paper is to present a new 3D crack propagation criterion, based on simple 2D fatigue tests on CT specimens, which allows to predict the correct distribution of the incremental crack extension also for complicated loading conditions under which only parts of the crack front propagate. Another focus of the experimental investigations concerns the influence of the additional corner singularities in the case of surface breaking cracks. Results of both, numerical simulations of 3D fatigue crack propagation an corresponding experimental investigations, are presented.*

INTRODUCTION

For a reliable life time estimation of cyclic loaded pre-cracked components, an effective numerical tool is essential, which enables a realistic simulation of real 3D fatigue crack propagation problems. Due to the non-linearity of the problem, an incremental and iterative simulation procedure is required. Within each incremental loop, the following three steps have to be performed. Firstly, the whole boundary value problem has to be solved, including an accurate and effective evaluation of the relevant fracture mechanics parameters along the crack front. Secondly, based on a suitable fatigue crack propagation criterion, the new crack front has to be determined and thirdly, the numerical model for the next incremental loop has to be updated automatically. To ensure that the crack propagation criterion is fulfilled at the end of the incremental step too, an iterative preditor-/corrector concept can be taken into account.

The crucial points in such a numerical simulation are the still open questions, (1) which crack propagation criterion is relevant for the determination of the kink angle (for mixed mode problems) and the correct distribution of the incremental crack extension along the crack front and (2) which influences of the additional corner singularities of surface braking cracks have to be taken into account. To answer these questions, precise experimental investigations are necessary. Only a careful comparison between experimental observations, e.g. well documented sequences of incremental crack front shapes, and corresponding numerical simulations enable an identification of the relevant crack propagation criterion. In order to be able to observe and store the crack front propagation by photographic in situ measurements, specimens of the transparent material PMMA were used.

TECHNIQUE OF SIMULATION

Fracture mechanic in 3D can be described as a sequence of 2D fracture mechanic problems on planes perpendicular to the smooth crack front. Thereby, the parameters of the 2D fracture mechanic, e.g. the crack length a or the stress intensity factor K_i, are used in dependence of their position P along the crack front. These 3D parameters now are $a(P)$ and $K_i(P)$ and additional parameters like the curvature of the crack front $r(P)$ and the angle g between crack front and the normal of the surface (for surface breaking cracks) have to be defined (see Fig. 1).

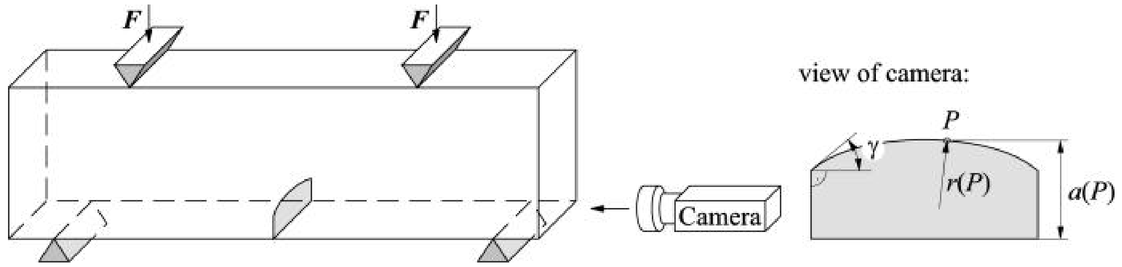

Figure 1. Four-point-bending specimen and the parameters to describe a 3D crack front.

To simulate crack propagation, the numerical simulation has to be done incrementally, because the stress state changes during crack growth. After solving the boundary value problem to obtain the state of stress for the actual crack front, the stress-intensity-factors $K_i(P)$ for every point P along the crack front are calculated and afterwards, due to a suitable crack propagation criterion (e.g. depending on these SIFs), the crack has to be enlarged in direction and magnitude [1]. Finally, the numerical model for the next incremental loop has to be updated. To check whether the created new crack front fulfils the crack propagation criterion, one or more iterative corrector steps are required. In this paper only Mode I problems are considered ($K_I \neq 0$ and $K_{II}=K_{III}=0$), thus the kink angle is zero.

To enlarge the crack by the user-defined incremental crack growth length Δa_0, there are a few possibilities to distribute Δa_0 along the crack front, e.g. depending on the K-factor- or energy-release-rate-criterion [2]:

$$\Delta a(P) = \Delta a_0 \frac{G(P)}{G(P)_{\max}} \text{ with } G(P) = \frac{1-\nu^2}{E}\left[K_I^2(P) + K_{II}^2(P)\right] + \frac{1+\nu}{E} K_{III}^2(P) \tag{1}$$

or on the crack propagation rate [3]:

$$\Delta a(P) = \frac{da}{dN}(\Delta K) \cdot N \tag{2}$$

known from 2D experiments, where N are the number of load cycles. In the energy-release-rate-criterion (Eq. 1) it is assumed, that the crack always propagates at all points P along the crack front, because it is always $G(P) > 0$. This is in contradiction to experimental observations, as documented in Fig. 2. In these pictures of a rectangular four-point-bending specimen (the load acts in vertical direction) it can be seen, that the crack only propagates at the bottom to overcome the asymmetric crack front and become $K(P) = const$ for all P, because for this uniform K-distribution the crack front is energetically most favourable. Obviously, there exists a lower threshold (ΔK_{th}) for fatigue crack propagation, that has to be considered in the criterion. This "local crack propagation" can mostly observed either around machined notches, precipitates and cavities in metal alloys or if the loading conditions change dramatically.

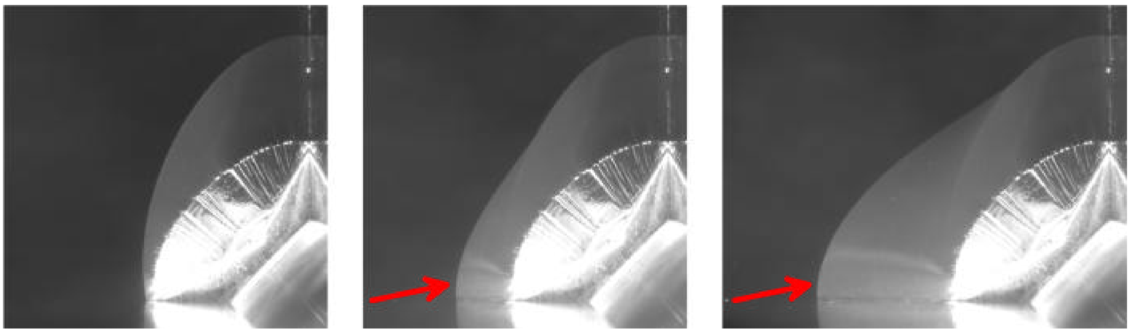

Figure 2. Local crack propagation at the lower side of the specimen.

A crack propagation criterion based on the crack propagation rate (Eq. 2) includes this lower threshold ΔK_{th} for non crack propagation, but it must be checked, whether this 2D criterion can be applied in 3D cases (see section EXPERIMENTS).

Another question in simulating crack propagation is the influence of additional corner singularity onto the crack front shape. Corner singularities occur at non-smooth parts of a crack or in the vicinity where a crack front intersects the free surface. In this paper, only the intersection between crack front and surface is considered. The corner singularity can be obtained by solving an eigenvalue problem, detailed described in [4, 5]. The result of this eigenvalue problem is the exponent α_i, depending on the geometry (the angle γ between crack front and normal of the surface) and the material (Poisson's ratio n). In the framework of fatigue crack propagation, three solutions can be indicated: for a_i=0.5 the generalized stress-intensity-factors K_i^* correspond with the classical stress-intensity-factors K_i. The appropriate angle g is denoted as g_r. In case of $a_i > 0.5$, K_i becomes zero ($K_i \to 0$) and for $a_i < 0.5$, K_i tends to infinite ($K_i \to \infty$). For PMMA, with a Poisson's ratio of $n = 0.365$, the exponent a_i becomes 0.5 for the angel $g_{r_{PMMA}} \approx 14°$.

Now, the first interesting question is, if a manufactured crack front (e.g. a machined notch) that intersects the free surface under the angle $g_m \neq g_r$ changes its angle (when fatigue propagation starts) to become the characteristic angle g_r, thus a_i becomes 0.5 and the classical stress-intensity-factor K_i exists, and furthermore how this crack propagation will occur (abruptly or continuously).

The second question deals with the problem, if the angle g_r can be influenced by changing other parameter, e.g. the geometry of the free surface.

To answer these questions, experimental investigations on specimens with different cross sections and suddenly changing boundary conditions (creating a new free intersecting surface) are necessary. By conducting this experiments, the behaviour of the angle g must be observed. If the resultant angle g is always equal g_r, the crack front ever bends in this way to become the real square root singularity (r^{a_i} with $\alpha_i = 0.5$) of a smooth crack.

EXPERIMENTS

Development of a New Crack Propagation Criterion

To develop a realistic crack propagation criterion, where crack arrest on some parts of the crack front can be assumed, a comparison between experimental observations and numerical calculations was made. The experiments on four-point-bending specimens with square cross section (50×50mm) and part circular corner crack (Figs 2 and 3 left) yield the real crack growth along the crack front. From the calculation of the corresponding state of stress, the SIFs $\Delta K_i(P)$ are gained.

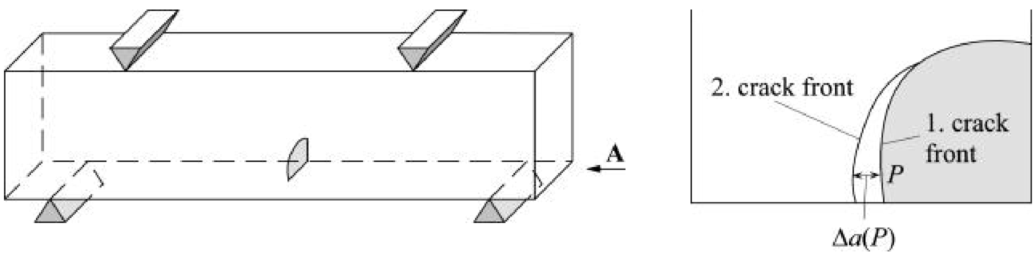

Figure 3. Four-point-pending specimen with quarter rounded initial crack and the detection of the real crack growth length along the crack front.

To observe the crack propagation along the crack front, photographic pictures of the crack front were taken and the crack front was measured in situ during the fatigue test. The difference between the positions of the crack fronts of two successive pictures is the real crack growth length $\Delta a(P)$ (see Fig. 3 right). To ensure that the real crack growth length has the same dimension as in an incremental simulation (The simulations are more accurate for small crack growth length, but more incremental steps are required – and vice versa!), two pictures with similar crack growth length were chosen.

The simulation of the crack front of the first picture yields the SIFs $K_i(P)$ along the crack front. A corresponding order of the crack growth length per load cycle da/dN in dependence of the ΔK-factor is shown in Fig. 4.

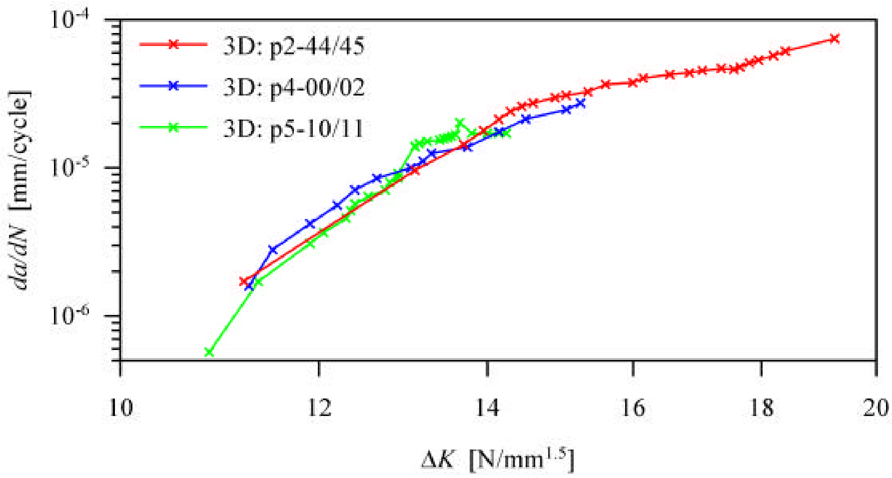

Figure 4. Crack growth length per load cycle in dependence of the *K*-factor.

The gradients in Fig. 4 correlates with the first two regions of the crack propagation rate – known from 2D – as shown in Fig. 5.

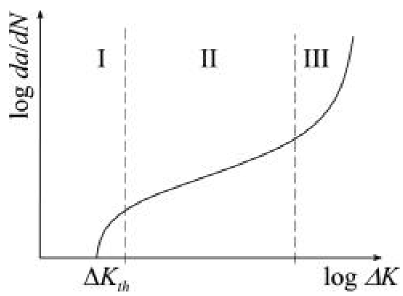

Figure 5. Crack propagation rate known from 2D test.

The first region describes a very slow crack growth velocity with its lower threshold ΔK_{th} of fatigue crack growth. The second region can be described with the Paris-Erdogan-Law:

$$\frac{da}{dN} = C \cdot \Delta K^m, \tag{3}$$

with the material parameters C and m. The last region is characterised by a very high crack growth velocity and the failure of the specimen at its right end.

To obtain the crack propagation rate for 2D, tests with four-point-bending specimens (rectangular cross section with a thickness of 12.5mm and 50mm height) of the same semi-finished product were made (see Fig. 6), because specimens of different semi-finished products (given in common literature) show very different material properties in a order of magnitude of up to 25%. The reason why four-point-bending specimen are used for these 2D test – rather than CT or CTS specimen – is, that the friction at the constraints are the same as in the 3D four-point-bending test with the specimens of square cross section and has to be considered in the simulations.

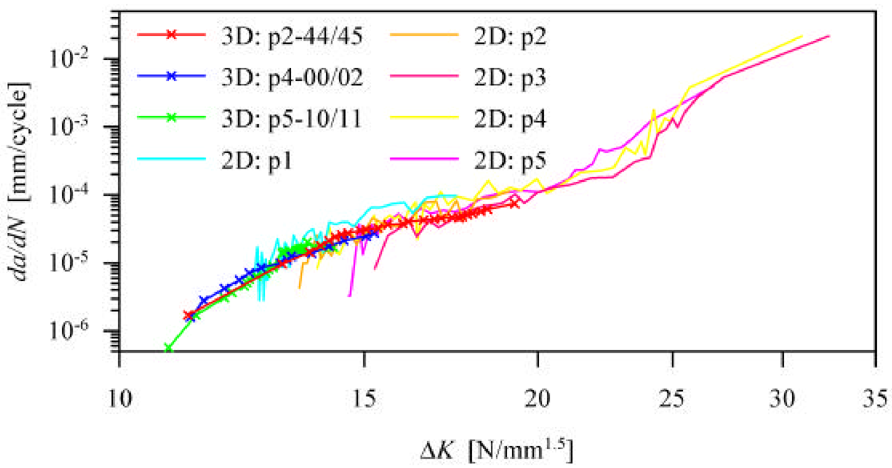

Figure 6. Crack propagation rates for 2D and 3D tests.

Based on the course of the gradients in Fig. 6, it can be concluded, that the known crack propagation rates of 2D can be taken as crack propagation criterion for 3D crack growth simulations.

Influences of Additional Corner Singularities on the Crack Front Shape

The influences of the additional corner singularity on the crack front shape can be observed by changing the free surface of the specimen. For this purpose, a test series with four-point-bending specimens of different cross sections has been carried out.

Firstly, test on specimens with rectangular cross section and six different sizes (5 specimens per size) were made (the cross sections are shown in Fig. 7 left), to obtain the angle g. An Example of one of these taken pictures of crack fronts is shown in Fig. 7 right. As a result, the value of the angle g between crack front and normal of the surface was found to be g $14^{\circ \pm 4}$, independently of the size of the specimen.

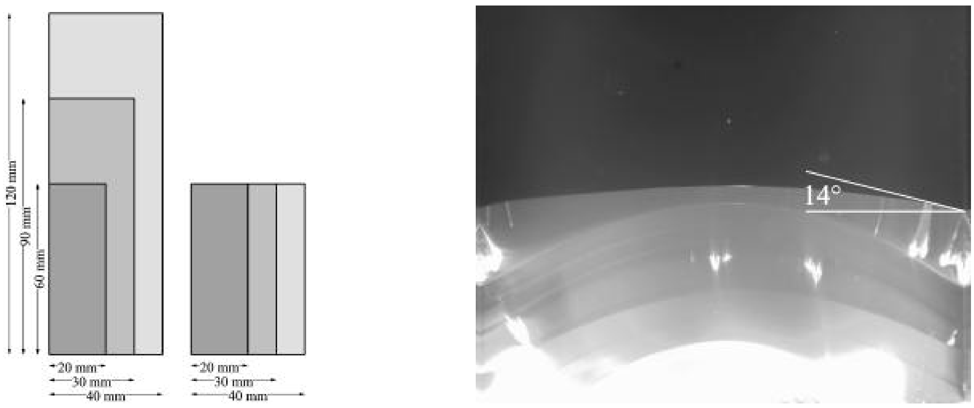

Figure 7. Specimens with rectangular cross sections and angle $\gamma \approx 14°$.

Secondly, specimens with different trapezoidal cross section are investigated. But it has to be distinguished if the crack growths into the smaller or wider part of the specimen (see Fig. 8a). Five specimens per trapezoidal angle (these angles are $2\beta = \pm 30°/60°/90°$) are investigated. Figure 8b shows a crack of a specimen with $2\beta = -90°$, whereas Figure 8c shows a specimen with $2\beta = 60°$. The interesting fact of these test is, that the angle γ doesn't have any dependence of the angle β, thus γ is always about 14°.

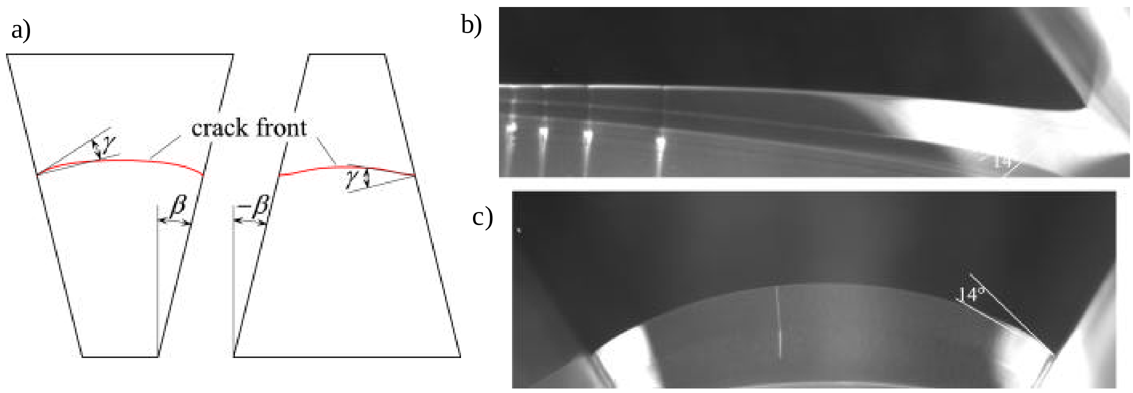

Figure 8. Specimens with different trapezoidal cross sections and angles $\gamma \approx 14°$.

The only difference of the positive and negative β-specimens is, that the crack fronts bend only in one direction for $\beta > 0°$ (convex) along the whole crack front, and for the negative values of β, the crack bends concave in the vicinity of the free surface and convex in the middle of the specimen.

Based on these investigations of the trapezoidal specimens it could be shown, that the crack front always bends to a shape to become the real $r^{-0.5}$ singularity at the vicinity of a free surface as it is in the middle of the specimen.

The last question is, how the fatigue crack propagation occurs (abruptly or continuously). Therefore, the specimen with the trapezoidal were notched lengthwise, to create an new free surface in the middle of the existing crack front. The following fatigue crack propagation shows a continuous crack growth rate (see the maker loads in Fig. 9) with the characteristic angle g 14° at the new surface.

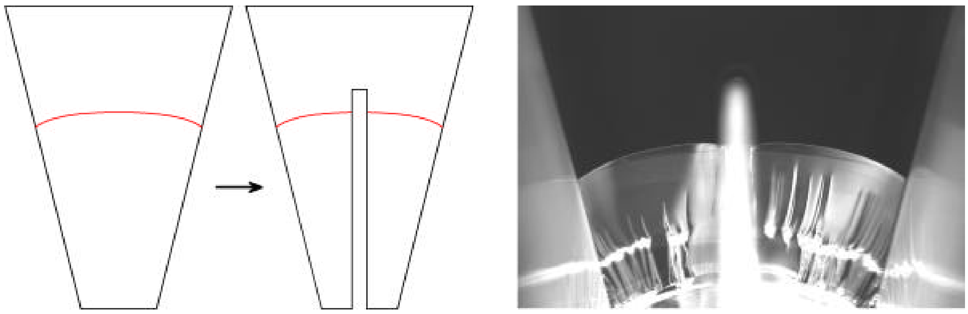

Figure 9. Specimens with trapezoidal cross sections and additional lengthwise notch.

CONCLUSION

For the simulation of fatigue crack propagation in 3D, a crack propagation criterion and the influence of the additional corner singularity of surface braking cracks were gained from experimental investigations on four-point-bending specimens under Mode I. In a next step, the crack propagation criterion has to be extended to arbitrary mixed-mode problems, mainly the kink angle has to be examined.

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