

Numerical and Experimental Analysis of Fatigue Crack Growth in Elastic-Plastic Materials

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***Abstract** In this paper, the crack propagation in the compact-tension-shear (CT) specimens of elastic-plastic materials, 7005 aluminum alloy under mixed-mode loading conditions is studied numerically and experimentally. The elastic numerical method which uses the maximum circumferential stress $\mathbf{s}_{qq_{max}}$ and elastic-plastic method, $J-M^p$ based criterion developed by the authors recently [4], are used in order to estimate the growth direction of a crack in elastic-plastic materials under cyclic loading and monotonic loading. A series of static and fatigue experiments are carried out. A mixed-mode-loading device developed by Richard [8] and the CT specimens are used in these experiments. The photos of crack growth paths subjected to different mixed-mode loading conditions are presented in this work. The experimental results show that the plastic zone has important effect on the direction of crack growth path, especially in the case of monotonic loading. Furthermore, the use of $J-M^p$ based criterion is discussed by comparing the numerical simulations and the experimental observations.*

INTRODUCTION

Fatigue crack growth behavior is always an important research subject. The majority of fatigue crack growth studies is concentrated on single-mode-loading and usually is performed under mode-I loading condition in elastic material where different methods and criteria have been proposed since 1960s. However, fatigue crack growth in elastic-plastic material has not been studied thoroughly, especially in the case of mixed-mode loading.

For a crack in elastic material, several different criteria can be used to determine the crack growth path under mixed mode loading, such as the maximum circumferential stress $\mathbf{s}_{qq_{max}}$ criterion (Erdogan and Sih [1]), the maximum energy release rate criterion (Palaniswamy and Knauss [2]), the crack tip opening displacement (or angle) criterion (Sutton et al. [3]) and so on. Recently we have developed the $J-M_p$ based criteria (Li et al. [4]) to assess the propagation of a crack in elastic-plastic material under mixed mode loading.

In this paper, the numerical and experimental studies of the growth path of a crack in elastic-plastic materials under mixed-loading condition have been done. The growth paths of a crack in a specimen issued from Aoki et al.'s experimental work [5][8],

which is called the compact-tension-shear specimen, are simulated numerically by using two different methods. The first one is the elastic method, in which the $S_{qq_{max}}$ criterion is used to calculate the crack growth angle. The second is elastic-plastic method based on the $J-M^p$ criteria. These two methods simulate the crack growth in monotonic loading condition. The experimental studies of the fatigue crack growth under mixed-mode loading conditions are performed by using the Compact Tension Specimen (CT) with the mixed-mode loading devices. The experimental crack growth paths of different mixed loads are compared with the two numerical predictions.

NUMERICAL STUDIES

Elastic method for predicting the crack growth paths under monotonic loading

In the case of a crack in elastic material, the $S_{qq_{max}}$ criterion [1] is more often used. According to this criterion, the crack always propagates in the direction of the maximum circumferential stress. The circumferential stress S_{qq} is expressed as follow:

$$s_{qq} = \frac{1}{4\sqrt{2pr}} \left[K_I (\cos \frac{q}{2} + \cos \frac{3q}{2}) - 3K_{II} (\sin \frac{q}{2} + \sin \frac{3q}{2}) \right] \quad (1)$$

Where r and q are the polar coordinates from the crack tip.

When s_{qq} is maximum ($\frac{\partial S_{qq}}{\partial q} = 0$), one can obtain:

$$K_I \sin q + K_{II} (3 \cos q - 1) = 0 \quad (2)$$

Then the bifurcation angle q_0 can be determined:

$$\text{tg} \left(\frac{q_0}{2} \right) = \frac{1}{4} \left(\frac{K_I}{K_{II}} \right) \pm \frac{1}{4} \sqrt{\left(\frac{K_I}{K_{II}} \right)^2 + 8} \quad (3)$$

The result of the crack growth direction calculated by this criterion will be showed in this work.

Elastic-plastic method for predicting the crack growth paths under monotonic loading

When a crack exists in an elastic-plastic material, the angle of crack growth depends on the competition between cleavage tensile fracture, essentially relates to the void growth and coalescence near the crack tip, and ductile shearing fracture, essentially, depends on the plasticity progression. Recently, we have developed the $J-M^p$ based criteria [4] in order to determine this crack growth angle. The main idea of the $J-M^p$ based criterion is as follows:

In the case of a crack in an elastic-plastic material under mixed mode loading, Shih (1981) [6] showed that the stress, strain and displacement fields near the crack tip are dominated by the HRR singularity, and can be characterized by two parameters, the J -integral and the mixity parameter M^p , which varies from zero to one. When $M^p = 0$, it is the case of pure mode II and when $M^p = 1$, it is the case of pure mode I. A numerical

method has been developed (Li, 1998) [7] to determine these two parameters for a crack under mixed mode loading.

Experimental studies show that, for an elastic-plastic material, there exists a transition from tensile type fracture to shear type fracture. This transition is controlled by the critical value of the mixity parameter M_c^p which can be determined by means of experiments according to the critical fracture toughness J_{IC} and J_{IIC} (J_{IC} is obtained from a pure mode I tensile test and J_{IIC} from a pure mode II shear test). If the mixity parameter M^p for a given loading case is greater than M_c^p , the crack will propagate by tensile type fracture. It means that it will propagate in the direction of the maximum circumferential stress $S_{\theta\theta}^{max}$. The $S_{\theta\theta}^{max}$ criterion can be used to determine the crack growth angle. On the other hand, if M^p is smaller than M_c^p , the crack will propagate by shear type fracture along one of slip bands. Then the crack growth angle can be determined according to the slip band criterion [4].

By this criterion, the propagation of a crack in compact-tension-shear specimen under different loading cases is simulated.

EXPERIMENTAL STUDIES

The mixed-mode fatigue crack growth experiments are conducted on a Compact Tension specimens (CT) with a loading device as shown in figure 1. The location of loading holes in the figure 1(b) provides a range of loads which result in a full spectrum of mode mixities [5][8]. The material used in the investigation was 7005 aluminum alloy plate, Young's modulus $E=72\text{GPa}$ and yielding stress $S_0=280\text{MPa}$. The CT specimen's dimensions are about 90mm in width and 10mm in thickness, which is thick enough to satisfy the plane strain condition. From an initial notch performed, a fatigue pre-crack was introduced up to $a/w \approx 0.5$.

The fatigue tests were conducted on the MTS-810 material test system at room temperature. The CT specimens were tested under three loading angles, 0° , 30° , and 60° respectively. Two or three specimens were used for each loading condition. During the tests, the load ratio for all loading angles was kept constant as 0.5, also, maximum and minimum load values were constants for all the loading angles, and loads were applied sinusoidally at a frequency of 25Hz.

In the second part of the test, the crack growth direction in elastic-plastic material under mixed-mode loading was investigated.

RESULTS AND DISCUSSION

Numerical results

The numerical calculations are carried out on the compact-tension-shear specimen, it is supposed that the hardening coefficient of this material is about $n = 7$. For this material,

the two values of critical fracture toughness are about: $J_{IC} \approx 14$ N/mm and $J_{IIC} \approx 46$ N/mm. According to critical values J_{IC} and J_{IIC} , we deduce the critical mixity parameter M_c^p which is about 0.75.

The loading angles α are selected as 60° , 30° and 0° . For each loading angle, two simulations are carried out in order to determine the crack growth path. The first one is the elastic simulation, in which the $s_{qq_{max}}$ criterion is used to calculate the crack growth angle. In the second simulation, the direction of crack propagation is predicted by $J-M^p$ based criterion. The mixity parameter M^p is calculated in each step of crack growth.

The numerical results of the initial growth direction are listed in table 1, s_0 is the crack growth angle in the first step. Figures 2 to 4 give the different crack growth paths obtained by these two simulations.

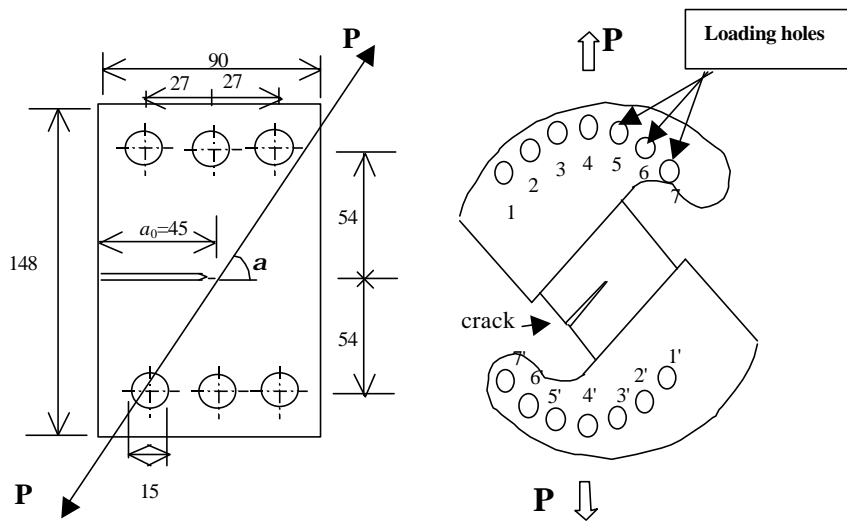


Fig.1 (a) Loads applied on the specimen (b) Device for mixed mode loading

When the loading angle α is 60° , it can be found in table 1 that the crack growth angle is equal to -28° in the first step when following the $s_{qq_{max}}$ criterion. It changes only a little in the following steps (figure2 (a)). In the case of elastic-plastic simulation, we get $M^p = 0.85$ in the first step (table 1), it is greater than the critical value M_c^p which is about 0.75. According to the $J-M^p$ based criterion it is a tensile type fracture. The angle of crack growth is predicted as -32° . In the following steps, the tensile type fracture and the shear type fracture take place alternately. Tensile type fracture is observed when M^p is greater than M_c^p , and shear type fracture will occur when M^p is smaller than M_c^p , therefore, the crack grows obviously in zigzag way. Figure 2 shows these two different crack growth paths.

The same numerical studies are performed in the case of 30° loading. From tables 1, it can be noted that when the loading condition approaches pure mode II, the directions of crack growth predicted by these two models are quite different. In the first step, the crack will grow in the direction of -51° when $s_{qq_{max}}$ criterion is used (table 1), and then,

it almost follows the same direction (figure3 (a)). However, when the $J-M^p$ criterion is used, the crack will grow in the opposite direction (48°) in the first step, then the growth angle changes a little, and it is always the shear type fracture as M^p is always smaller than 0.75. These crack growth paths in 30° loading can be observed in figure 3.

When the crack is subjected to pure mode II loading, the theoretical growth angle is $\pm 72^\circ$ from the crack axis. In this investigation, the predicting bifurcation angle of $S_{\sigma\sigma_{max}}$ criterion is -70° , which is close to the theoretical value, then the crack grows almost following the same direction (figure 4(a)). However, the prediction of the $J-M^p$ criterion is 0° in the first step, and the crack grows along the direction of 0° until the fracture. The crack growth paths of pure mode II obtained from these two criteria are given in figure 4. Figure 5 and 6 show the finite element mesh during the crack growth under different loading conditions by using the two criteria.

Table 1 numerical prediction of initial bifurcation angle

elastic criterion				plastic criterion			
loading angle	60°	30°	0°	loading angle	60°	30°	0°
$K_I (MPa \sqrt{mm})$	325.6	187.8	10.8	M^p	0.85	0.63	0.0006
$K_{II} (MPa \sqrt{mm})$	91.8	161.3	187.4				
q_0	-28°	-51°	-70°	q_0	-32°	48°	0°

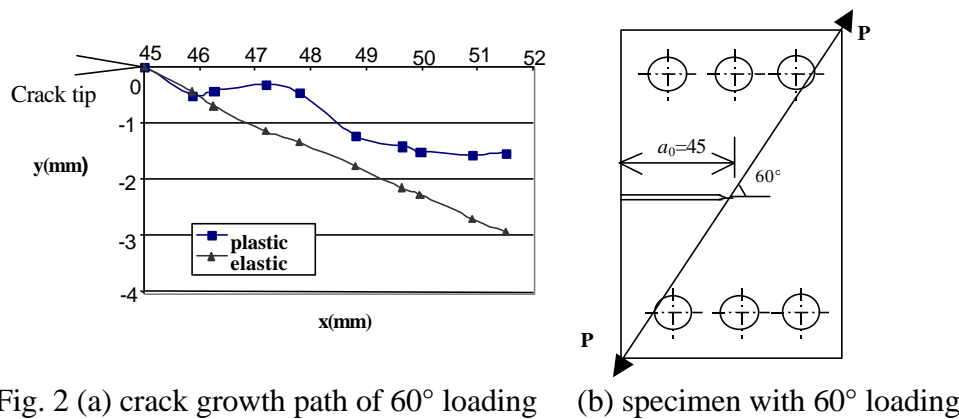


Fig. 2 (a) crack growth path of 60° loading

(b) specimen with 60° loading

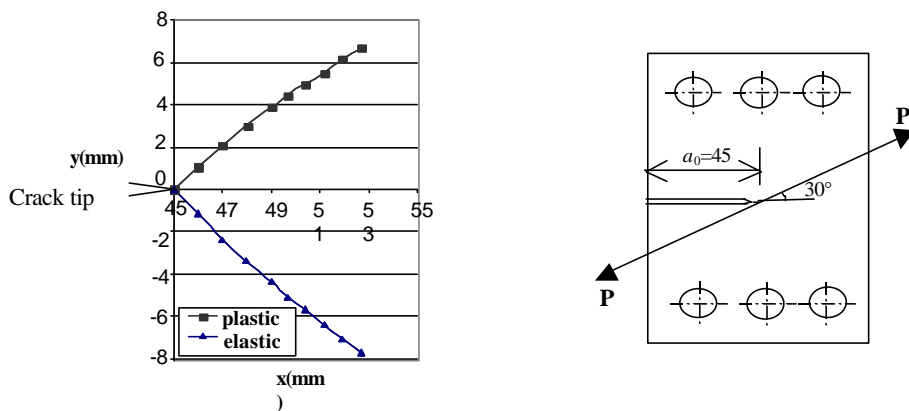


Fig. 3 (a) crack growth path of 30° loading

(b) specimen with 30° loading

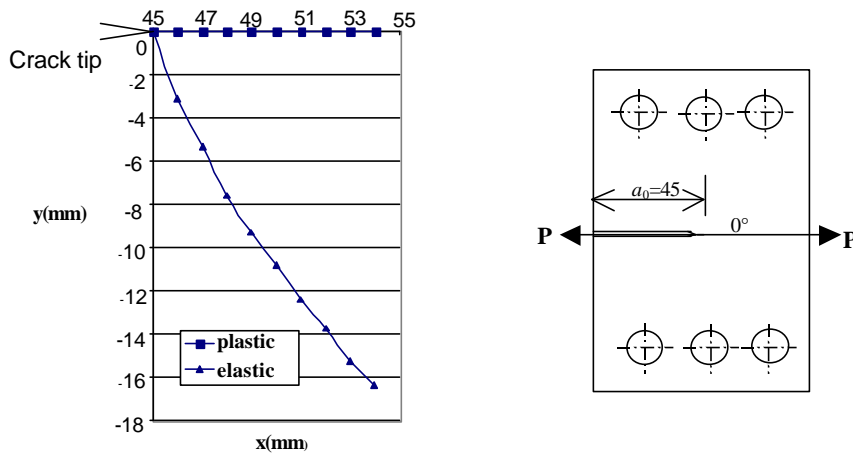
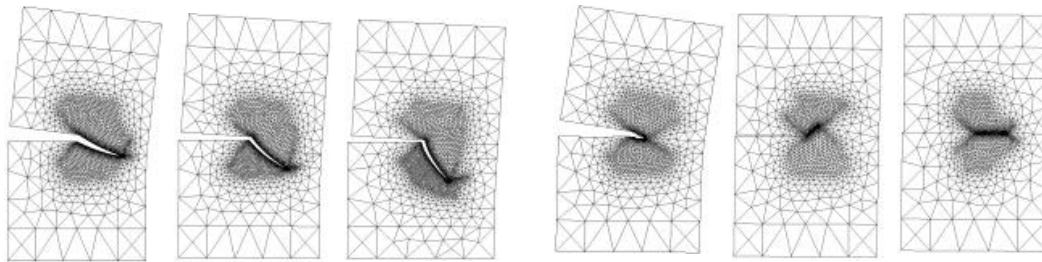


Fig 4 (a) crack growth path of 0° loading (b) specimen with 0° loading



(a)60° loading (b)30° loading (c)0° loading (a)60° loading (b)30° loading (c)0° loading
 Fig. 5 elastic simulations of crack growth path Fig. 6 plastic simulations of crack growth path

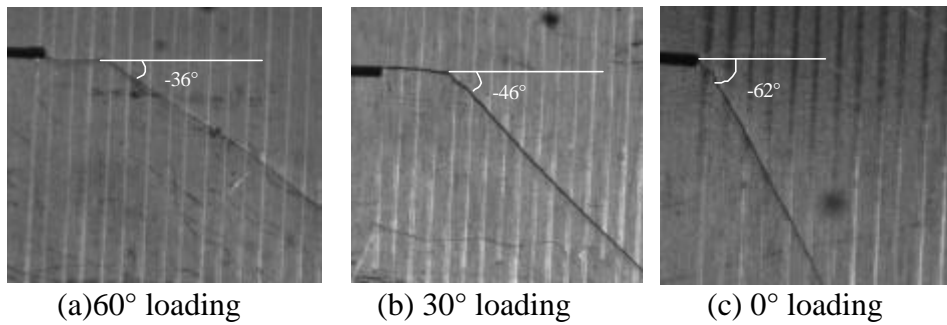
Experimental results

Figures 7 and 8 are the photos of the crack growth path, which were taken in the fatigue test and the monotonic test. The pre-crack was prepared under pure mode I loading, and the pre-crack length is 4 mm, i.e., the cracks grow from 45mm under mixed-mode load. (The interval of two stripes is 1mm.)

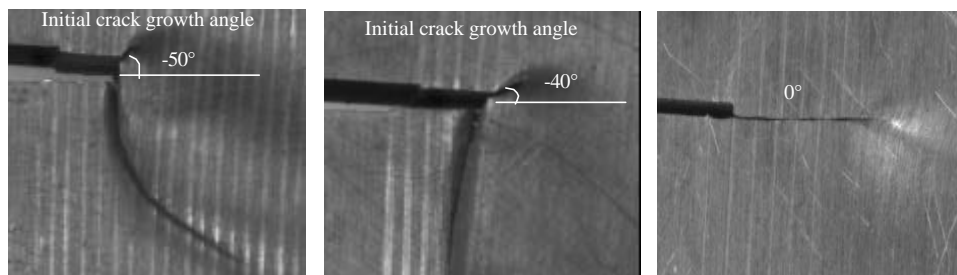
Comparing these photos with the figure 2 to 6, we find that the growth paths of the fatigue crack almost approaches the numerical simulations of $S_{q_{max}}$ criterion. So, in the case of small scale yielding, crack growth path under cyclic loading can be predicted by elastic method, even though in elastic-plastic material. Nevertheless, because of the effect of plastic zone, the values of bifurcation angles obtained from the testing are not exactly equal to that of the numerical predictions which are estimated by elastic method.

However, when a crack in elastic-plastic material is subjected to monotonic mixed-mode loading (figure 8), the growth paths are quite different from the elastic prediction.

From figure 8, the following points can be noted:



(a)60° loading (b) 30° loading (c) 0° loading
Fig. 7 photos of crack growth path under cyclic loading



(a)60° loading (b) 30° loading (c) 0° loading
Fig. 8 photos of crack growth path under monotonic loading

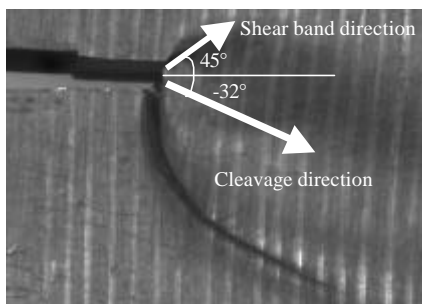


Fig.9 bifurcation under 60° loading

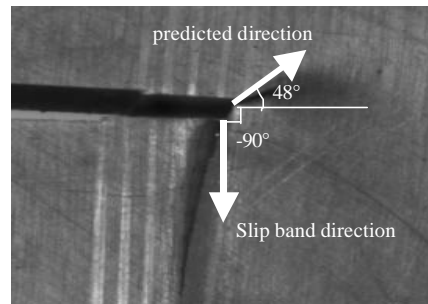


Fig.10 bifurcation under 30° loading

(1) When the loading angle is 0°, the experimental path is similar to the prediction of $J-M_c^p$ criterion.

(2) When the loading angle is 60° (figure 9), the expected initial crack growth angle corresponding to the tensile-type is about -32° . The experimental observation shows an initial crack growth angle of about 45° , this is in disagreement with the expected one. Nevertheless, the initial crack growth angle seems to follow the shear band (shear type fracture[4]) and not the cleavage (tensile type) direction. Later it can be noted obviously that the crack grows along the border of the plastic zone associated to a necking effect. In this condition, the plane strain state is totally not satisfied.

(3) When the loading angle is 30° (figure 10), the initial growth angle in the monotonic test approaches the numerical results of $J-M_c^p$ criterion in the first step. Later, the crack seems to follow another slip band (-90°)[4]. It is to be noted that for this case, the crack in the one hand, is in the situation of tensile–shear type transition; in the

other hand, the effect of plastic zone on crack growth is significant, the plane strain condition is difficult to satisfy because of great deformation.

CONCLUSION

In this work, the growth bifurcation angle of crack in CT specimens of 7005 aluminum alloy under mixed-mode loading conditions was studied numerically and experimentally. The numerical simulations and the experimental observations indicate:

1. In the case of small scale yielding, the fatigue crack growth path in elastic-plastic material (7005 aluminum alloy) can almost be assessed by elastic criterion. The crack grows always in the direction of the initial bifurcation angle.

2. When the crack is in elastic-plastic material under monotonic loading, the elastic method is unable to estimate the direction of the crack bifurcation. The crack grows almost along the border of the plastic zone. Nevertheless, the $J-M^p$ based criterion can provide an excellent prediction for pure mode II loading.

3. Experimental observations show that the thickness effect of the specimen has to be taken into account. The $J-M^p$ based criteria can only be used in the case of plane strain condition. Other criteria have to be developed later for plane stress condition

Reference

- [1] Erdogan F. and Sih, G.C., On the crack extension in plates under plane loading and transverse shear. *Transaction of the ASME, J. Basic Eng*, **85**, 519-527, 1963
- [2] Palaniswamy K. and Knauss, W.G., On the problem of crack extension in brittle solids under general loading. In: Memat-Nasser S. (Ed.), *Mechanics Today*, Vol. **4**, Pergamon Press, 87-148, 1978
- [3] Sutton M.C., Deng X., Ma F., Newman Jr. J.C. and James M., Development and application of a crack tip opening displacement-based mixed mode fracture criterion. *Int. J. Struct. Solids*, **37**, 3591-3618, 2000
- [4] Li J., Zhang X.B. and Recho, $J-M^p$ Based Criteria for Bifurcation Assesment of a Crack in Elastic-Plastic Materials under Mixed Mode I-II Loading. *Engineering Fracture Mechanics*. (Accepted)
- [5] Aoki S., Kishimoto K. Yoshida T. Sakata M. and Richard A., Elastic-Plastic fracture behavior of an aluminum alloy under mixed mode loading. *J. Mech. Phys. Solids*, **38**, No. 195-213, 1990
- [6] Shih C.F., Small-scale yielding analysis of mixed mode plane-strain crack problems. *ASTM STP*, **560**, 187-210, 1974
- [7] Li J, Estimation of the mixity parameter of a plane strain elastic-plastic crack by using the associated J_integral. *Engineering Fracture Mechanics*, No. 61, pp.355-368, 1998
- [8] Richard H. A. and Benitz K., *Int. J. Fractures*, **22**, R55, 1982