Numerical Simulation of Delamination Growth under Fatigue Loading

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ABSTRACT. *This paper presents an implicit algorithm to simulate delamination growth under fatigue loading when the growth is governed by the Paris law. It is based on the weak form of the law, the resulting non linear problem being solved by the Newton method. This algorithm was first compared to the explicit Euler and improved Euler schemes on a DCB specimen loaded in mode I and then applied to the delamination growth simulation in a rectangular plate submitted to a compressive loading. Accurate results were obtained with large values of the number of cycles increment.*

INTRODUCTION

Layered composite materials are now widely used in aircraft structural components. If they allow weight saving, these materials are sensitive to low velocity impacts which produce delaminations (debonding of two adjacent layers) [1]. These delaminations can grow under in-service fatigue loading, weakening the structure. As a consequence, this kind of damage must be taken into account at the design stage. It is then of prime importance to have numerical tools able to predict the residual strength of damaged structural elements.

The present work extends to the case of fatigue loading an algorithm developed previously to simulate delamination growth under monotonic loading [2]. It consists in writing the growth law, here the Paris law [3], as a non linear variational problem and then, in solving it by the Newton method. This algorithm is an implicit one. It is first compared to the explicit Euler and improved Euler schemes on a DCB specimen for which the solution is known analytically. Then, it is applied to a delaminated plate submitted to a compressive loading.

THE PARIS LAW

The straight crack case

The Classical Paris Law

Let us consider a two dimensional domain submitted to a cyclic loading and having a propagating crack. It is assumed that the crack growth is governed by the Paris law which is written as

$$\begin{cases} \dfrac{da}{dN} = C\big(G(a)\big)^m \\ a(0) = a_0 \end{cases} \tag{1}$$

where a is the crack length, N is the number of cycles, G is the crack driving force, C and m are material parameters identified experimentally, and a_0 is the initial crack length. Assuming the crack is straight and remains straight during growth, the crack tip location is given by the integration of the Paris law. It is a non linear problem solved numerically using the Euler scheme in most cases [4], [5].

The Paris Law Revisited

We recall that, in the case of monotonic loading, the developed algorithm is based on the assumption that, for a given level of loading and in the case of a stable growth, the front at arrest minimises the total energy E of the structure, sum of the potential energy J and of the fracture energy D. The idea is to define a fracture energy D in such a way that the characterisation of the minimum gives the Paris law.

For an increment DN of the number of cycles, let D be defined by :

$$D = \frac{m\,C\Delta N}{1+m}\left(\frac{\Delta a}{C\Delta N}\right)^{1+1/m} \tag{2}$$

Then, the characterisation of the minimum of E is given by the following equation

$$E^{(1)} = J^{(1)} + D^{(1)} = 0$$

where the superscript (1) indicates the first derivative with respect to a crack tip displacement. After differentiation, one obtains the following relation

$$G - \left(\frac{\Delta a}{C\Delta N}\right)^{1/m} = 0 \tag{3}$$

which is similar to the Griffith criterion in which $\left(\frac{\Delta a}{C\Delta N}\right)^{1/m}$ can be viewed as a variable critical energy release rate G_C .

Extension to Delamination

Taking one's inspiration from the monotonic loading case, one can think to define a fracture energy as a function of the growth rate of the delaminated area. Unfortunately, this choice gave bad results. It seems that, for delamination, the Paris law cannot be associated to an energy balance. In order to obtain an implicit algorithm, the Paris law is first expressed in a weak form :

$$\int_{g_f}\left[G-\left(\frac{\Delta a.\boldsymbol{n}}{C\Delta N}\right)^{1/m}\right]\boldsymbol{q}.\boldsymbol{n} \;=\; 0 \quad \forall\boldsymbol{q} \tag{4}$$

where g_f is the delamination front, $\boldsymbol{n}$ is the unit normal to the front outward to the delaminated area, and $\boldsymbol{q}$ is any admissible front displacement. Then, Eq. (4) is solved with the Newton method. To this end, the first derivative of Eq. (4) with respect to a front displacement must be computed. The first derivative of G (the opposite of the second derivative of the mechanical energy J) is given in [6] whereas

$$\begin{aligned}\frac{\partial}{\partial g_f}\left(\int_{g_f}\left(\frac{\Delta a.\boldsymbol{n}}{C\Delta N}\right)^{1/m}\boldsymbol{q}.\boldsymbol{n}\right).\boldsymbol{q} \;&=\; \frac{1}{mC\Delta N}\int_{g_f}\left(\frac{\Delta a.\boldsymbol{n}}{C\Delta N}\right)^{-1+1/m}(\boldsymbol{q}.\boldsymbol{n})^2\\ &+\left(1+\frac{1}{m}\right)\int_{g_f}\left(\frac{\Delta a.\boldsymbol{n}}{C\Delta N}\right)^{1/m}(\nabla.\boldsymbol{q})(\boldsymbol{q}.\boldsymbol{n}) - \int_{g_f}\left(\frac{\Delta a.\boldsymbol{n}}{C\Delta N}\right)^{1/m}\boldsymbol{n}.\nabla\boldsymbol{q}.\boldsymbol{q}\\ &-\frac{1}{mC\Delta N}\int_{g_f}\left(\frac{\Delta a.\boldsymbol{n}}{C\Delta N}\right)^{-1+1/m}(\boldsymbol{n}.\nabla\boldsymbol{q}.\Delta a)\boldsymbol{q}.\boldsymbol{n}\\ &-\frac{1}{m}\int_{g_f}\frac{\boldsymbol{t}.\nabla\boldsymbol{q}.\boldsymbol{t}}{\|\boldsymbol{t}\|^2}\left(\frac{\Delta a.\boldsymbol{n}}{C\Delta N}\right)^{1/m}\boldsymbol{q}.\boldsymbol{n}\end{aligned} \tag{5}$$

where $\boldsymbol{t}$ is the tangent to the front such that $(\boldsymbol{t},\boldsymbol{n})$ is direct.

It is recalled that the approximation of the second derivative of the potential energy is a fully populated symmetric matrix noted $\left[J^{(2)}\right]$ whereas the approximation of Eq. (5)

is a symmetric matrix noted $\lfloor D^{(2)} \rfloor$. The front displacement is then obtained solving the linear system

$$\lfloor E^{(2)} \rfloor \cdot \{q\} = \left(\lfloor J^{(2)} \rfloor + \lfloor D^{(2)} \rfloor\right) \cdot \{q\} = -\{E^{(1)}\}$$

where $-\{E^{(1)}\}$ is the approximation of the left hand side of Eq. (4) and $\{q\}$is the front displacement vector.

NUMERICAL EXAMPLES

DCB Specimen

A beam made of an isotropic material of Young modulus E is first studied. The beam has a crack of length a along its mid-axis. It is assumed that the beam is submitted to a mode I cyclic loading at the cracked arms tips (Fig. 1). Let h be the thickness of the cracked arms and $\boldsymbol{d}$ be their normal displacement taken as the control variable. The crack driving force G has the following expression :

$$G = \frac{\boldsymbol{a}}{a^4} \; ; \; \boldsymbol{a} = \frac{3Eh^3\boldsymbol{d}^2}{4}$$

The integration of the Paris law is straightforward, giving :

$$a = \left[(4m+1)C\boldsymbol{a}N + \boldsymbol{b}\right]^{1/4m+1} \; ; \; \boldsymbol{b} = \frac{1}{4m+1} a_0^{4m+1}$$

In order to apply the implicit algorithm, the two first derivatives of the fracture energy D (Eq. (4)) are given :

$$D^{(1)} = \left(\frac{\Delta a}{C\Delta N}\right)^{1/m}$$

$$D^{(2)} = \frac{1}{mC\Delta N}\left(\frac{\Delta a}{C\Delta N}\right)^{-1+1/m}$$

The numerical computations were made with E = 150000 MPa, h = 1,5 mm, a_0 = 30 mm, C = 5,154, m = 3.74 and $\boldsymbol{d}$ = 1 mm. The crack extension a – a_0 is reported in Fig. 2 as a function of the number of cycles N for a constant increment ΔN = 4000. Both the implicit and the improved Euler schemes give results close to the analytical solution. A computation made with ΔN = 20000 (Fig. 3) shows the efficiency of the implicit method.

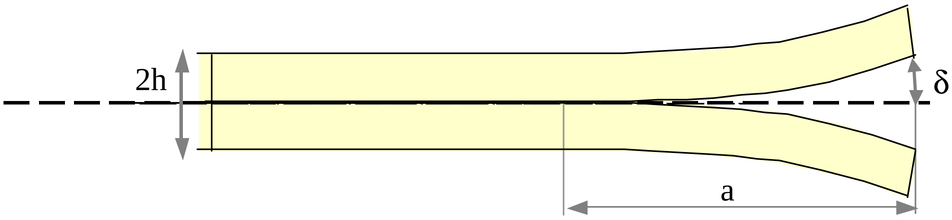

Figure 1. DCB specimen.

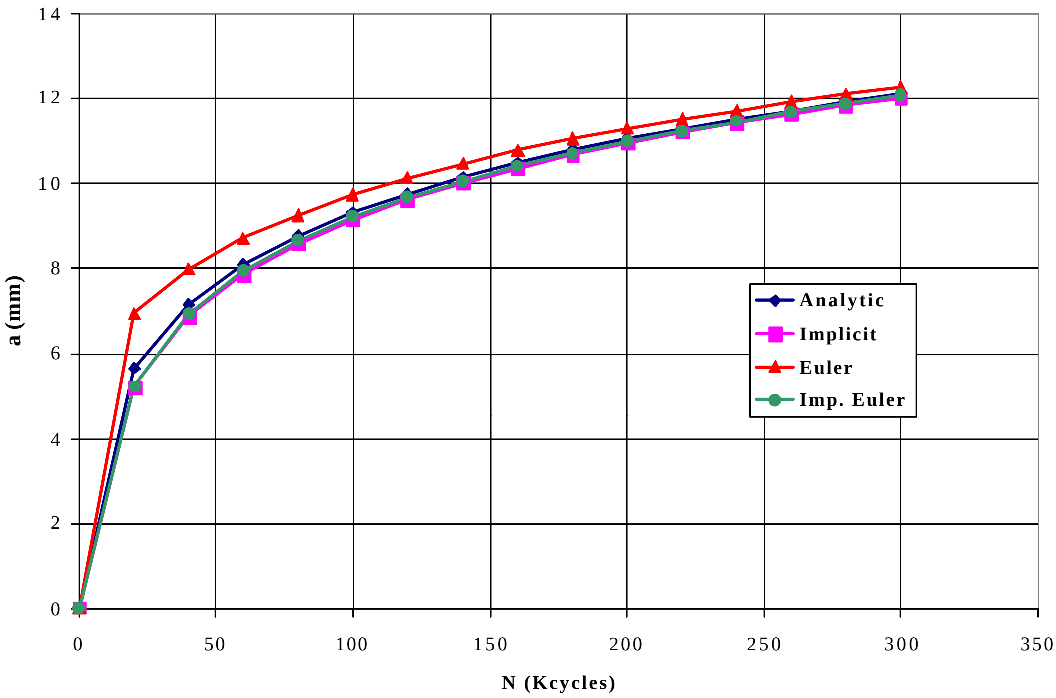

Figure 2. Crack extension as a function of the number of cycles for DN = 4000.

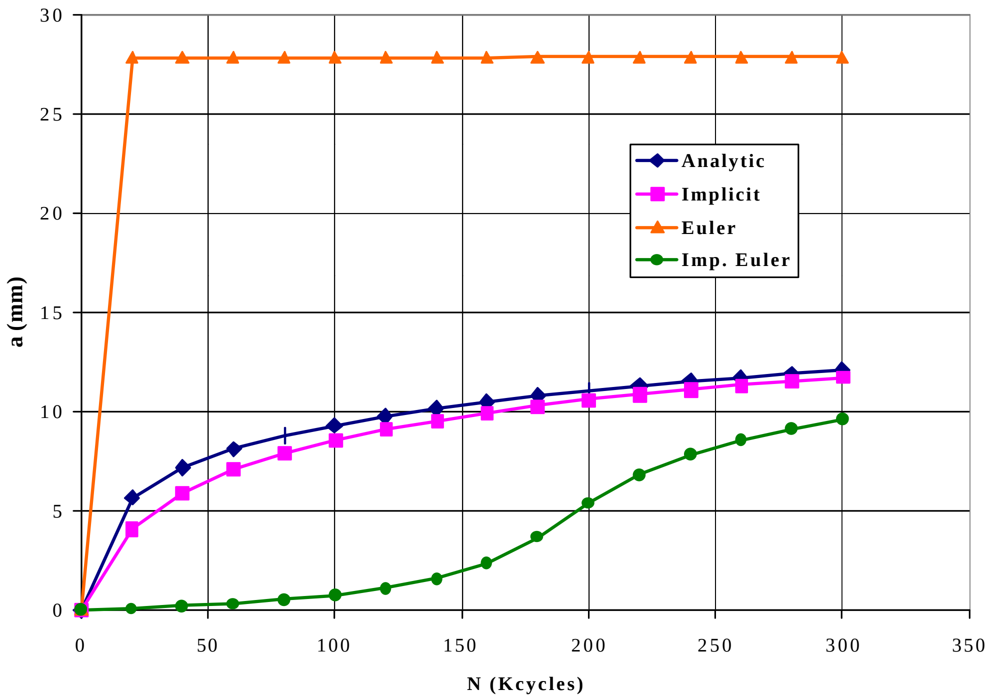

Figure 3. Crack extension as a function of the number of cycles for DN = 20000

Rectangular Plate loaded in Compression

The second numerical example concerned a 16 plies 55 mm x 40 mm rectangular plate loaded in compression and previously studied by Krüger *et al.* [7]. The stacking sequence was [$\pm$5 // +45 / $\pm$5 / -45 / 0 / $\pm$85 / 0 / -45 / $\mp$5 / +45 / $\mp$5], each ply being .125 mm thick. The plate had a 10 mm diameter centred circular artificial delamination located at the second interface. It was loaded in compression, the maximal pressure level being of 220 MPa.

Computations were conducted applying a simply supported boundary condition along the plate edges. The finite element mesh is depicted in Fig. 4. The following parameters values identified from the experimental results were used in the computations :

$$C = 5.174 \ ; \ m = 3.74 \ ; \ DN = 20000$$

As the delamination buckling occurred at a load level of 185 MPa, the arc-length method of Crisfield [8] was used to follow the post-buckled solution, so it was not possible to stop the computation at the maximum value of the applied load exactly.

The computed growth pattern was different from the one reported in [7], but similar to the one described in the same reference for an initial delamination of 20 mm diameter. The delamination first grows slowly in the longitudinal direction (the loading direction) and then, at N = 100000 cycles, it grows quickly in the transverse direction until the front reaches the lateral plate edges. After, both the two resulting fronts move slowly in a stable manner (the crack driving force is a decreasing function of the crack length). Fig. 5 depicts such a pattern.

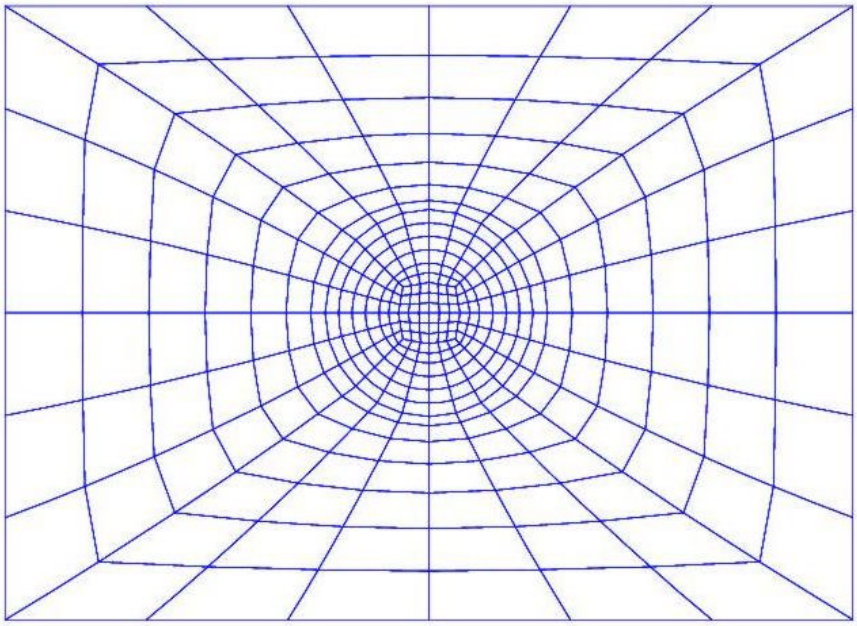

Figure 4. Delaminated plate : finite elements mesh.

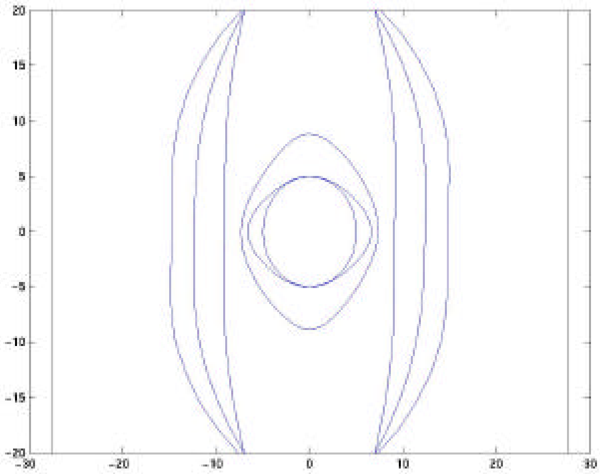

Figure 5. Delaminated plate : front locations during growth.

CONCLUSION

An implicit algorithm was developed to simulate delamination growth in layered composite structures submitted to fatigue loading. It is based on a variational formulation of the Paris law. It was first applied to a DCB specimen, and then to a plate submitted to compression. The first results are promising showing a relative insensitivity to the cycles increment values, but a complete evaluation of its performances (accuracy, robustness) requires more test cases. The main difficulty lies in the necessity of remeshing at each iteration, especially as in the plate case, when the initial closed front is divided into two open curves moving separately.

REFERENCES

1. Joshi, S.P., Sun, C.T. (1987) *J. Compos. Mater.* **9**, 40-46.
2. Ousset, Y. (1999) *Eur. J. Mech. A/Solids* **18**, 291-312.
3. Paris, P., Erdogan, F. (1963) *J. Basic Engng.* **85**, 528-534.
4. Shuter, D.M., Geary, W. (1995) *Fatigue Fract. Engng. Mater. Struct.* **18**, 653-659.
5. Nykänen, T.J. (1996) *Fatigue Fract. Engng. Mater. Struct.* **19**, 99-109.
6. Münch, A., Ousset, Y. (2002) *Comput. Methods Appl. Mech. Engrg.* **191**, 2045-2067.
7. Krüger, R., Hänsel, C., König, M. (1996) Report of the Institute of Static and Dynamic of Aerospatial Structures, University of Stuttgart, Germany.
8. Crisfield, M.A. (1983) *Computers & Structures* **13**, 55-62.