

# Cyclic Crack Propagation at Geometric Discontinuities

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**ABSTRACT.** *Cracks frequently change their shape at geometric discontinuities such as corners or holes. Using the MTU automatic fatigue crack propagation software CRACKTRACER this phenomenon is investigated for a simple specimen with square cross section and drilling hole. The analysis shows that the change in crack shape takes place as soon the discontinuity is reached and can be interpreted as being triggered by the resulting change of intersection angle with the free boundary. This leads to a violation of the previously existing equilibrium conditions and a sharp increase of the local stress intensity factors ( $K$ -factors). Thereupon, the crack rapidly converges towards a new equilibrium configuration.*

## INTRODUCTION

Nearly all structures exhibit some kind of geometric discontinuities such as corners. If a crack initiates in the neighborhood of such a discontinuity the question naturally arises how this geometric feature will influence the crack propagation. Previous investigations in a notched plane specimen [1] have revealed that the stress intensity factor and the propagation speed locally increase until a new equilibrium configuration is reached. The purpose of the present analysis is to have a closer look at this phenomenon by investigating crack propagation in a specimen having both corner discontinuities and a drilling hole and to establish a link between the crack front intersection angle and the  $K$ -increase. To this end the MTU automatic cyclic crack propagation software CRACKTRACER is used.

## NUMERICAL CRACK PROPAGATION PROCEDURE

Starting in the mid-nineties a numerical crack propagation tool has been developed at MTU with the target to cope with any initial crack in any component in a fully automatic way. The software and several aero engine applications have been discussed in the literature (see e.g. [1] and [2]). Here, only a brief overview of the underlying procedure will be given.

Cyclic crack propagation calculations with CRACKTRACER start from a 20-node brick mesh of the uncracked structure, a description of the initial crack geometry and

crack propagation data. The basic loop consists of a preprocessing step, a finite element calculation and a postprocessing step (Fig. 1).

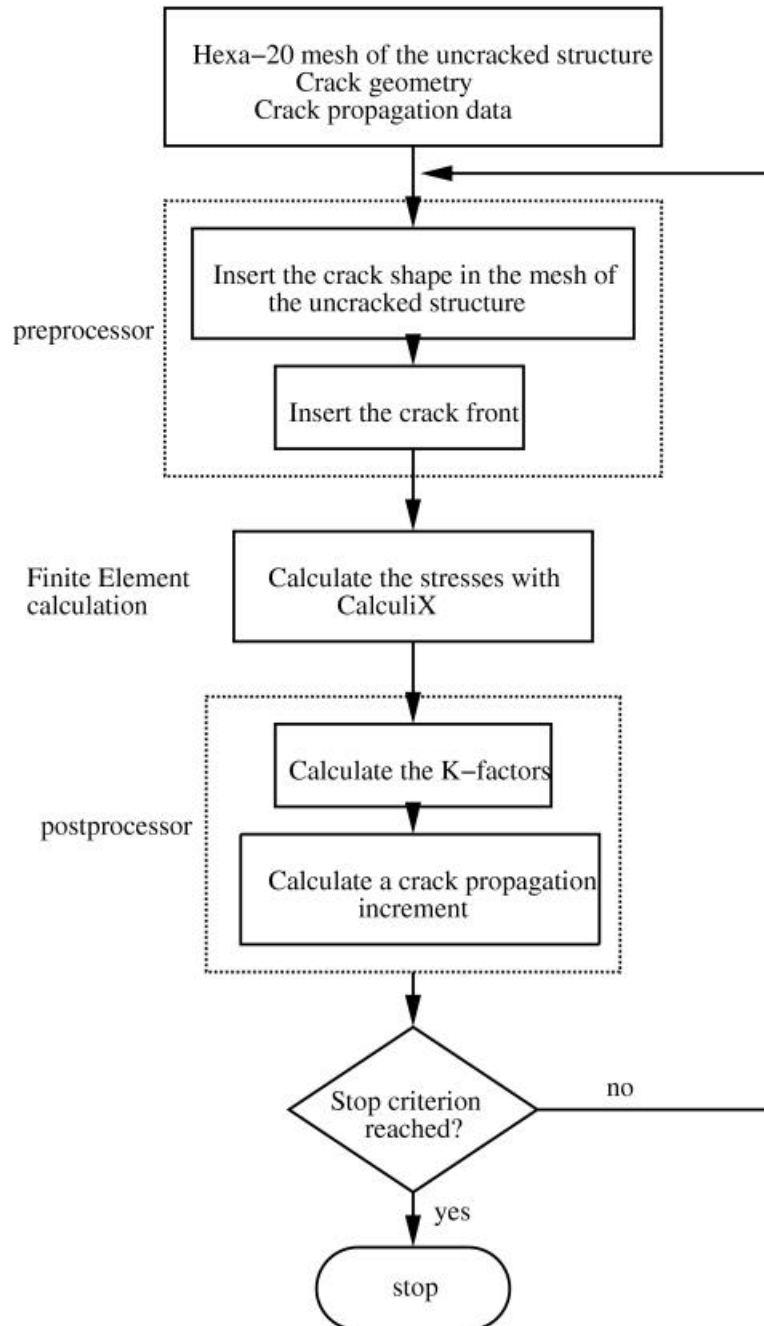


Figure 1. Flow diagram of CRACKTRACER.

The first operation in the preprocessing step is the insertion of the crack shape through local mesh modifications. Starting point is always the mesh of the uncracked

structure. Notice that the crack shape is not necessarily plane and can take an arbitrary two-dimensional shape in three-dimensional space. Then, the crack front is inserted into the crack shape and the mesh is once again appropriately modified. The automatic modification of the mesh is the most tricky part of the algorithm. It results in a pure 20-node brick mesh for the cracked structure. At the crack front collapsed quarter-point elements are taken [1]. An example of the mesh modification is shown in Fig. 2. It is characterized by a spider-web like element configuration at the crack front.

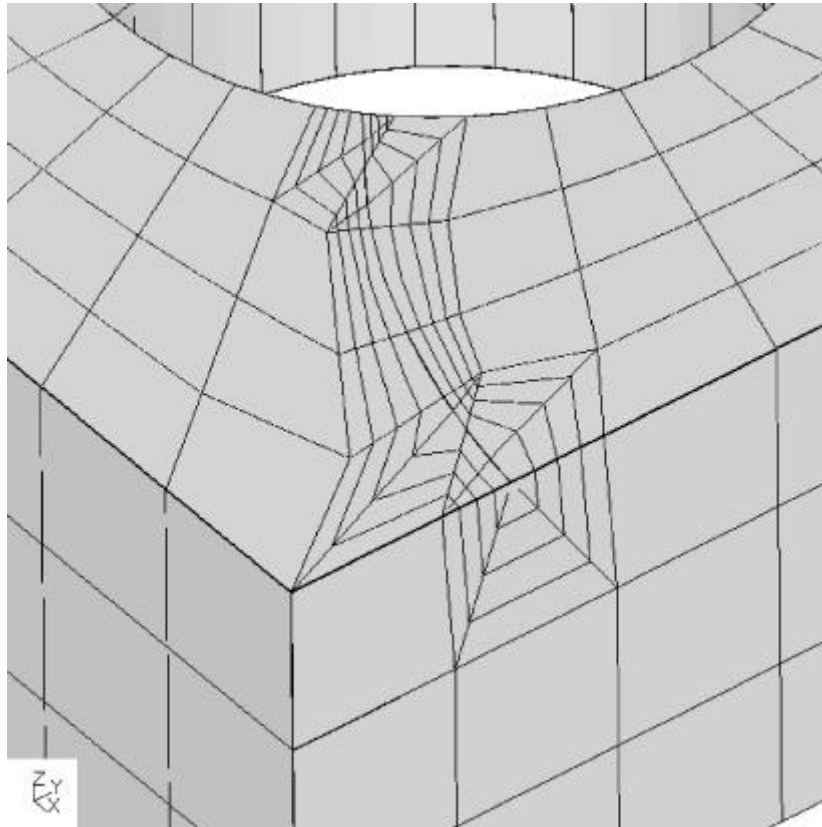


Figure 2. Mesh at the crack front

After application of the appropriate mechanical and thermal loading the stresses can be calculated using any generic finite element package. At MTU this is usually done with the free software package CalculiX [3].

In the postprocessing step the K-factors are calculated based on the stresses in the reduced integration points in front of the crack tip. Although other procedures exist (J-integral, virtual crack closure integral...) diligent use of the stresses has proven to be very straightforward and flexible [4]. Application of the crack propagation law (such as Paris, Forman..) to each node along the crack front leads to a crack propagation increment and a new crack front. Usually a maximum crack propagation increment per iteration is prescribed by the user and the number of cycles needed to reach this increment is determined (in the assumption that K is constant during the iteration). This

leads to equally spaced crack fronts and a much more stable procedure than by specifying a constant number of cycles between the loops. Then, the loop is reiterated with the new crack front, until some stopping criterion such as crack length or maximum K-value is attained.

The procedure has been applied to widely different engine parts such as blades, vane clusters and disks and has proven to be very valuable and to yield accurate results ([1], [2]) in the sense that the maximum deviation is usually less than a factor of two in life. The main advantages of the procedure are its automatism and the lack of geometric assumptions during crack propagation. The user does not prescribe how the crack fronts have to look like: the crack freely converges towards a state which is in equilibrium with the actual stress field. This also applies to the intersection of the crack front with the free boundary and exactly this characteristic will yield valuable information in the present investigation.

## CASE STUDY

### *Geometry*

In order to have plenty of opportunity to analyze the crack behavior at geometric discontinuities a specimen with square cross section and drilling hole was selected (Fig.3). The specimen is subject to tensile forces at both ends and a small semi-circular initial crack was inserted in the symmetry plane in the middle of the right outer face (Fig. 4). Poisson's coefficient  $\nu = 0.3$  was taken throughout.

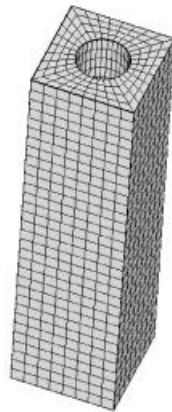


Figure 3. Specimen geometry and finite element mesh.

### *Crack Propagation*

Due to cyclic tensile loading the crack propagates as illustrated in Fig. 4. Each line represents a new crack front and corresponds to a finite element calculation. A new calculation was performed as soon as the maximum crack propagation increment along the crack front exceeded 50  $\mu\text{m}$ . The number of cycles between two crack fronts decreases as the crack propagates since the K-values increase. This, however, was not

the object of the investigation and for convenience a Paris-type law without critical value was used. The value of the maximum crack propagation increment was chosen such that the effect of geometric discontinuities on the crack propagation could be analyzed in detail.

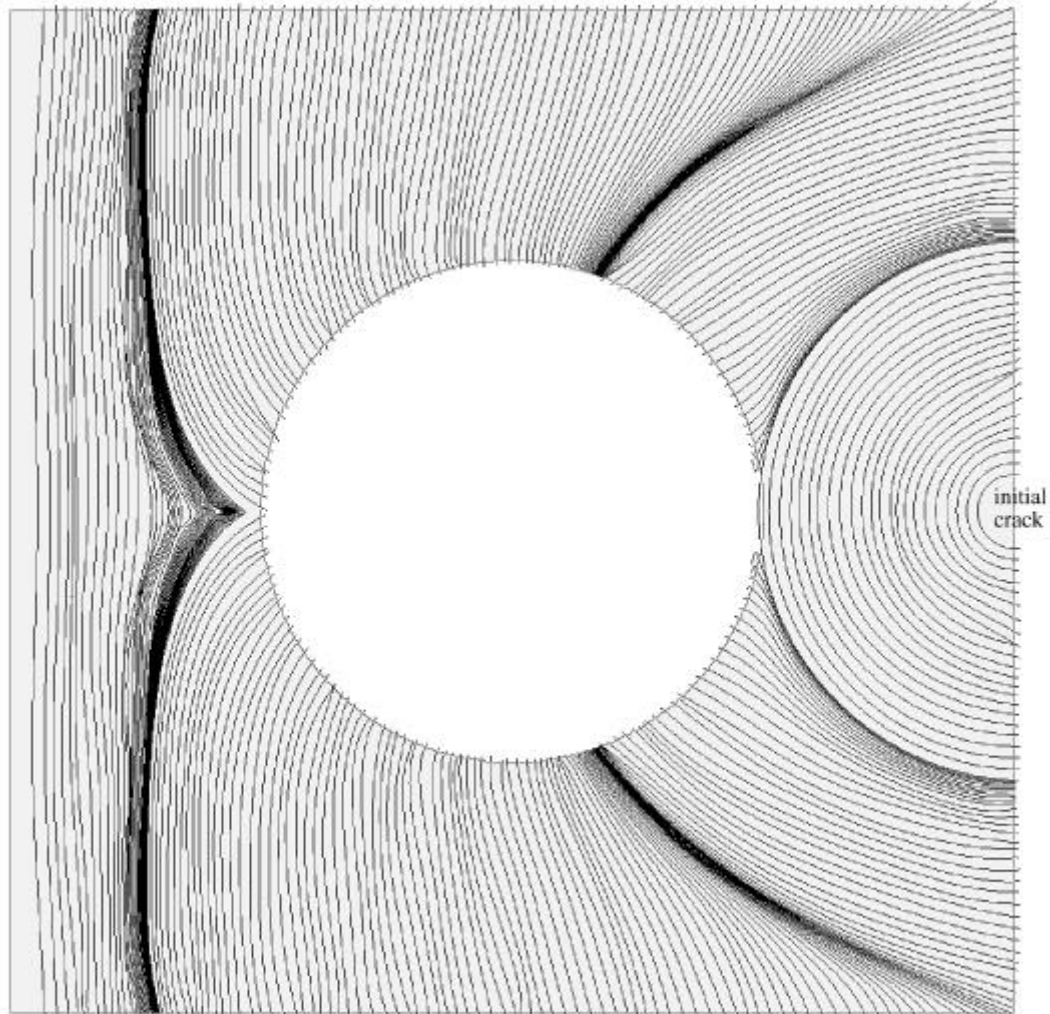


Figure 4. Crack propagation fronts

The initial half circular crack propagates at first in a self-similar way until the hole is reached. Then, the crack front abruptly splits into two parts which rapidly evolve into slightly curved through cracks. This is obtained by an acceleration of the crack propagation at the hole surface. Subsequently, two of the specimen edges are crossed leading again to a local propagation acceleration, this time at the outer surface of the specimen. From now on, crack propagation is smooth until the hole is crossed. Then, the two crack fronts are united leading to a kink in the new crack front. This kink is smoothed through local crack acceleration until a smooth relatively straight crack

ensues. Notice that a local acceleration takes place at those locations where a geometric discontinuity arises and that smooth crack propagation along the complete crack front is characterized by intersections with the free boundary which are close to  $90^\circ$ . This is particularly obvious at the hole surface: just before crossing the complete hole the crack locally curves in an extreme way to ensure a more or less straight angle at the hole boundary.

### *Stress intensity values*

It is very informative to investigate the  $K$ -values for the present analysis. The stress intensity factors at the free boundary are plotted in Figs 5 and 6. For crack fronts which do not cross the hole the midpoint on the front was taken instead. For the sake of clarity, iterations where geometric discontinuities occur are extra marked.

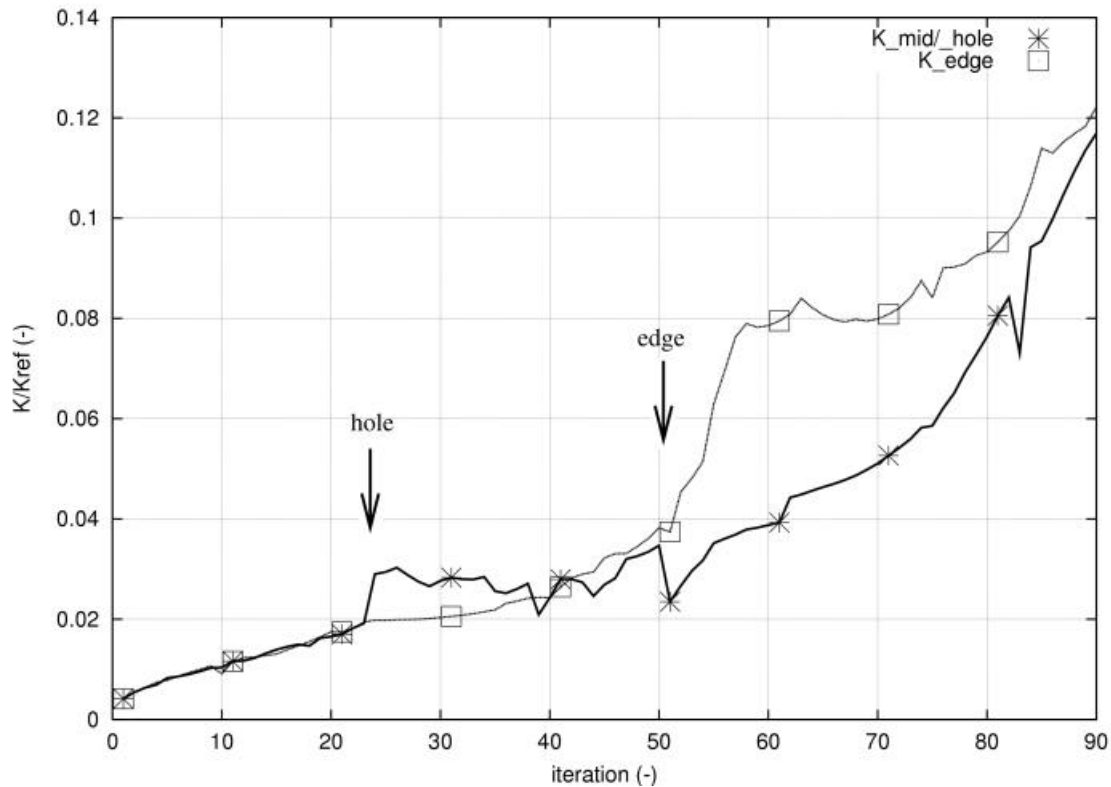


Figure 5. Stress intensity factors at the free boundary

At the first discontinuity (the hole) the  $K$ -factors along the hole significantly increase, leading to accelerated crack propagation. The  $K$ -factors at the outer boundary exhibit a small decrease in slope and crack propagation slightly decreases. A similar scenario occurs at the edge and after the hole is crossed. Notice that changes in  $K$  only occur after the geometric discontinuity was crossed: there is no anticipation on behalf of the crack (except maybe after crossing the hole in Fig. 6). This is also plainly visible in Fig. 4: geometric changes of the crack front take place *after* crossing the discontinuities.

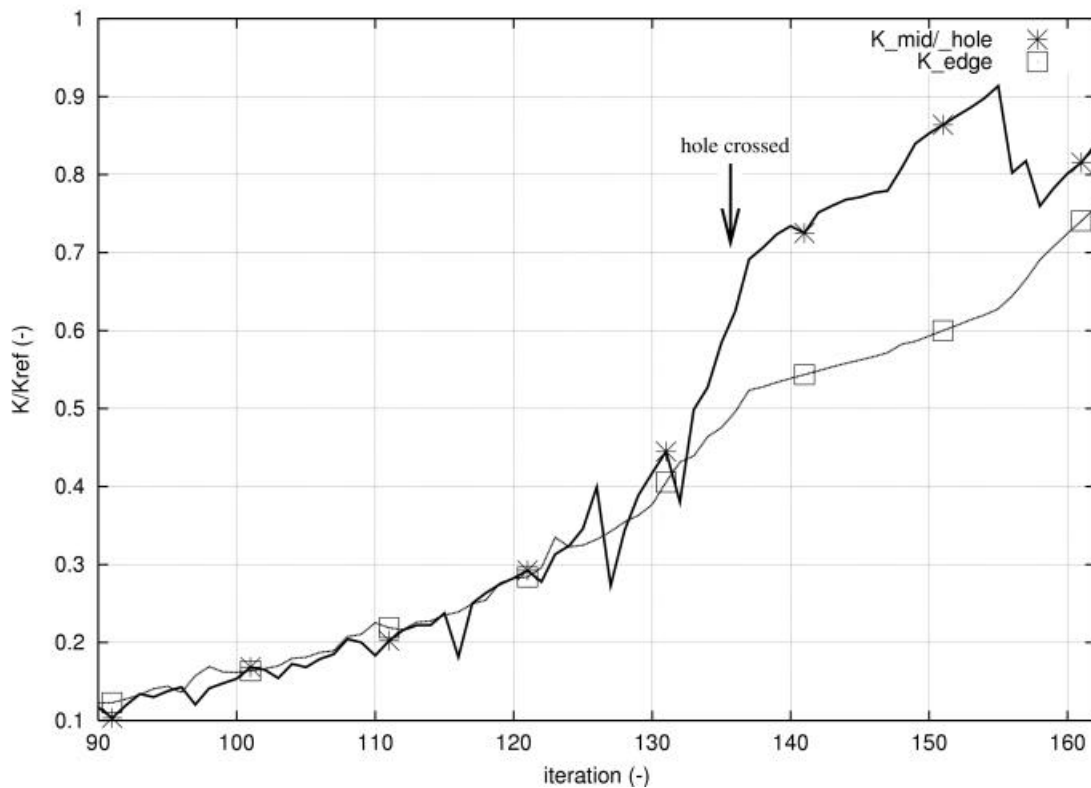


Figure 6. Stress intensity factors at the free boundary

***Intersection angle at the free boundary***

From the previous observations it seems that the main driving force is the abrupt change in intersection angle due to the geometric discontinuity. Figure 7 shows the angle between the tangent at the free surface and a tangent to the crack front. A half circular crack in an infinite domain has its largest K-values at the free boundary [5]. Thus, the crack propagates faster at the boundary and an angle of smaller than 90° is expected. This is confirmed by the values of about 80° at the start of the calculation (Fig. 7). From iteration 15 on there is a trend at the outer boundary towards an angle of 100°. This value is important since it represents the equilibrium value for a through crack under mode I and  $\nu = 0.3$  [6] (recall that a through crack has its lowest K-values at the free boundary [5]). This tendency is at first unaltered by the crossing of the hole boundary. Then, the edge is crossed and the angle is massively disturbed, but it converges again towards the through crack value. At the hole, on the other hand, there is a constant change due to the curvature of the boundary. After the crossing of the hole the angle tends towards about 70°, except for a small range between iteration 80 and 110 where values of 100° are reached. This is the range where both the outer boundary and the hole boundary are largely parallel and the crack behaves as a through crack.

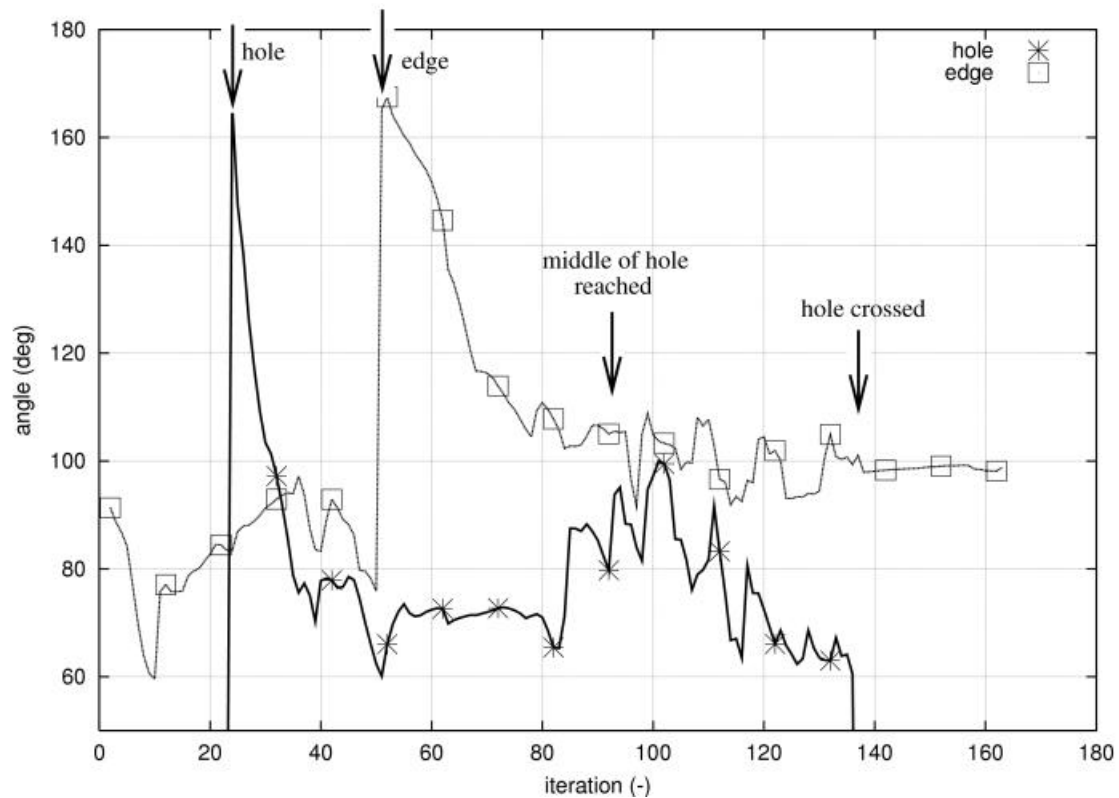


Figure 7. Intersection angle at the free boundary

## CONCLUSIONS

Crack propagation calculations were performed for a specimen with several geometric discontinuities. As soon as the crack crosses a discontinuity it results in a abrupt change of intersection angle with the free boundary. This leads to locally increasing K-values and accelerated crack propagation which tends to restore an equilibrium intersection angle.

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