Numerical Determination of Crack Paths in Three-Dimensional Structures with the Program System ADAPCRACK3D

M. Fulland[1] and H.A.Richard[1]

[1] Institute of Applied Mechanics, University of Paderborn, Pohlweg 47-49, 33098 Paderborn, Germany, e-mail: fulland@fam.upb.de

ABSTRACT. *This paper deals with the Finite-Element based three-dimensional crack simulation program ADAPCRACK3D, which has been developed at Institute of Applied Mechanics at University of Paderborn, and its special abilities concerning the prediction of crack paths. Therefore in a first section "fracture mechanical requirements" especially concerning crack deflection angles and crack growth increment for the crack path determination in ADAPCRACK3D are presented. In a second section the numerical aspects of realizing crack paths within a simulation are discussed. It is shown in which way a fully automatic manipulation of an FE-mesh can be carried out in order to adjust the FE-mesh step by step to the growing crack. A simulation example of the crack propagation in a shutter ring of a hydraulic press proves the significance of the presented algorithms as well as of the whole program system.*

INTRODUCTION

In many cases the lifetime of technical structures and components depends on the behaviour of cracks. Hence it is of enormous importance to determine the lifetime of a crack in a structure as well as the path it takes. For the purpose of predicting the crack growth currently quite a lot programs are available, that are able to analyse a two-dimensional structure. Those programs generally yield good results if both the (originally three-dimensional) structure can be approximated in two dimensions quite exactly and only „in-plane loadings" are applied to it. Whenever those preconditions are not fulfilled for a component, a fully three-dimensional simulation is inevitable. When passing over from 2D to 3D, it becomes apparent that unfortunately only very few codes do exist, which would be able to handle this task at least rudimentarily. This comes on the one hand from the enormously increasing numerical problems, that arise from the necessity to automatically adjusting the underlying FE- (or also BE-) mesh from simulation step to simulation step. Therefore some codes e.g. simplify this task by defining special "crack blocks" respectively "crack elements", which of course restricts the generality of the particular program. On the other hand the second major problem is the issue of a reliable fracture criterion describing the crack propagation in a three-dimensional structure under consideration of all three crack opening modes. The Finite-Element-based crack simulation program ADAPCRACK3D, which has been developed at Institute of Applied Mechanics (FAM) at University of Paderborn, however is able to accomplish the demands of mesh adaptation in a very general manner and uses a new and very promising fracture criterion (also recently proposed at FAM). The criterion and thus the fracture mechanical determination of crack paths as well as its numerical realisation will be discussed in the following.

DETERMINATION OF CRACK PATHS WITH ADAPCRACK3D

In a three-dimensional structure a crack path is given as a surface within the cracked object. When using an incremental simulation approach as it is e.g. realised in ADAPCRACK3D, an additional crack growth area has to be determined in any simulation step (Fig. 1, left-hand side). In the related FE-model the crack front consequently transfers to a contiguous set of piecewise linear edges connecting a number of crack front nodes, while the crack (growth) surfaces are depicted by a number of FE-faces (Fig. 1, right-hand side).

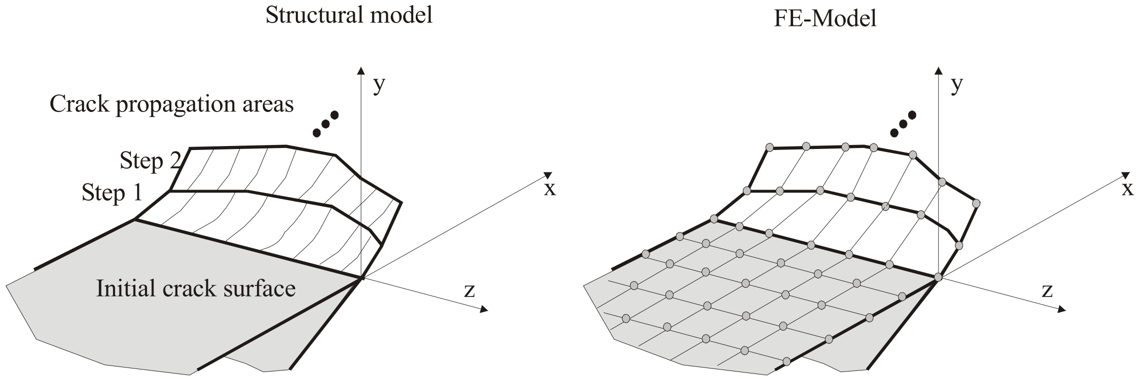

Figure 1. Crack propagation areas for a 3D-simulation in an incremental approach

The description of the crack propagation area relies on the knowledge of the local *propagation direction* as well as on the local *crack growth increment* at every point of the actual crack front of the structural model (respectively at every node of the FE-model).

Local Propagation direction

In order to determine a local propagation direction at any of the crack front nodes it is necessary to know the stress intensity factors for all three crack opening modes K_I, K_{II} and K_{III} at that particular node. In ADAPCRACK3D those stress intensity factors are calculated by using the MVCCI-method [1,2]. The full description of the crack growth direction in a three-dimensional structure requires two propagation angles as can be seen in Fig. 2, where φ_0 denotes the local kinking of the crack front and ψ_0 the local twisting.

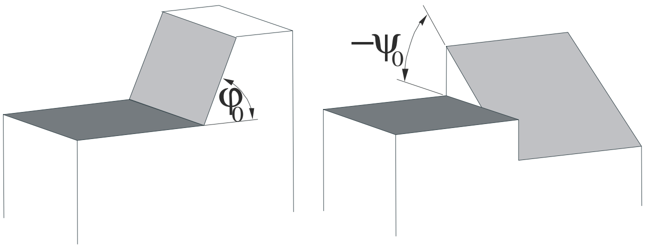

Figure 2. Descritption of local crack growth direction by two angles

The calculation of the angles is based on the σ_1'-criterion by Schöllmann et al. [3]. In this criterion both angles depend on the stress intensity factors K_I, K_{II} and K_{III} for all three crack opening modes. They are given by:

$$\begin{aligned}
&-6K_I \tan\left(\frac{\varphi_0}{2}\right) - K_{II}\left(6 - 12\tan^2\left(\frac{\varphi_0}{2}\right)\right) + \left\{\left[4K_I - 12K_{II}\tan\left(\frac{\varphi_0}{2}\right)\right]\cdot\right.\\
&\cdot\left[-6K_I \tan\left(\frac{\varphi_0}{2}\right) - K_{II}\left(6 - 12\tan^2\left(\frac{\varphi_0}{2}\right)\right)\right] - 32K_{III}^2 \tan\left(\frac{\varphi_0}{2}\right)\cdot\\
&\left.\cdot\left(1 + \tan^2\left(\frac{\varphi_0}{2}\right)\right)^2\right\}\cdot\left\{\left[4K_I - 12K_{II}\tan\left(\frac{\varphi_0}{2}\right)\right]^2 + 64K_{III}^2\left(1 + \tan^2\left(\frac{\varphi_0}{2}\right)\right)^2\right\}^{-1/2} = 0
\end{aligned} \tag{1}$$

$$\psi_0 = \frac{1}{2}\arctan\left(\frac{8K_{III}\cos\frac{\varphi_0}{2}}{K_I\left(3\cos\frac{\varphi_0}{2} + \cos\frac{3\varphi_0}{2}\right) - K_{II}\left(3\sin\frac{\varphi_0}{2} + 3\sin\frac{3\varphi_0}{2}\right)}\right) \tag{2}$$

It is noticeable that Equation 1 is an implicit function of φ_0, which has to be solved numerically.

Determination of the local crack growth increment

Reasonably the magnitude of the local loading at any point of the crack front can be described by calculating a comparative stress intensity factor K_v. By application of the σ_1'-criterion, for instance, this K_v is given by

$$\begin{aligned}
K_v = \frac{1}{2}\cos\left(\frac{\varphi_0}{2}\right)&\left\{K_I\cos^2\left(\frac{\varphi_0}{2}\right) - \frac{3}{2}K_{II}\sin(\varphi_0)\right.\\
&\left.+\sqrt{\left[K_I\cos^2\left(\frac{\varphi_0}{2}\right) - \frac{3}{2}K_{II}\sin(\varphi_0)\right]^2 + 4K_{III}^2}\right\}.
\end{aligned} \tag{3}$$

The simulation sequence of ADAPCRACK3D is controlled by the crack growth increment. This means that in every simulation step the particular node, which is loaded by the biggest comparative stress intensity, is propagated by the (user-defined) maximum crack growth increment (Δa_{max}) per step [1]. Any other node along the crack front gets a smaller increment according to the following calculation procedure. With the underlying crack growth rate relation $da/dN=f(K_v, R)$, where f() is either the law of Erdogan/Ratwani [4] or the Forman/Mettu-Equation [5] and $R=\sigma_{min}/\sigma_{max}$ is the stress ratio, the number of necessary loading cycles can be approximated by $N_i=\Delta a_{max}/f(K_{v,max},R)$. The application of this N_i to the local crack growth rate at any other node of the crack front results in smaller increments $\Delta a_i=N_i * f(K_{v,i},R)$ at those locations (see Fig. 3).

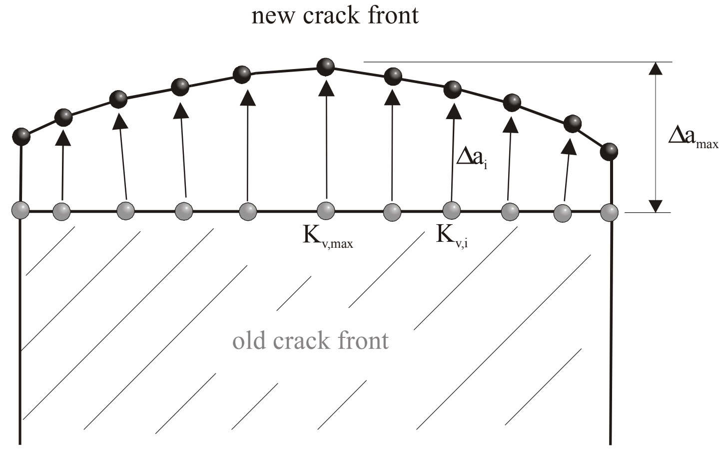

Figure 3. Crack growth increment along a 3D crack front

Combining of the calculated increment and the crack deflection angles finally results in a “propagated node location” for each node of the crack existing crack front. By connecting these nodes by piecewise linear lines (as part of FE-faces) a new crack front can be obtained as can be seen in Fig. 3.

NUMERICAL REALISATION OF CRACK GROWTH BY ADAPCRACK3D

In ADAPCRACK3D the description of the initial crack (in the first simulation step) as well as the additional crack extension areas (in all following steps) is given by a set of triangular FE-faces (Fig. 4). In order to realise the crack in the global mesh of the structure an adaptive re-meshing technique in combination with a local de-bonding is applied. In this approach the FE faces/edges/nodes of the given crack (resp. crack extension) description are created by an eligible mesh adaptation algorithm in a first step, and then in a second step those FE-object can “easily” be de-bonded in order to propagate the crack within the object.

[1] Therefore the number of loading cycles applied in each simulation step is variable!

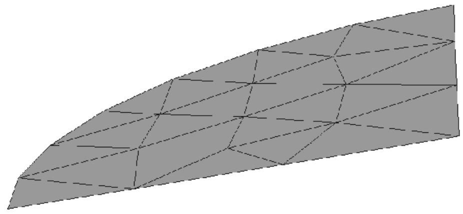

Figure 4. FE-description of a quarter elliptic initial crack by triangular faces

Taking a closer look at Fig. 4, it becomes obvious, that the mesh adaptation algorithm has to be able to insert just one single triangular element into the global mesh, as the whole crack simply can be gathered by iteration of this procedure on all faces. This inserting algorithm consists of three sub-processes:

1. Insertion of the three nodes of a face
2. Realisation of the interconnecting edges
3. Realisation of the whole face (in contrast to 2D, the face in 3D does not automatically exist, if the three edges do!)

Even the sub-processes 2 and 3 can be realised by insertion of additional nodes at appropriate locations within the mesh, so the requirement of mesh manipulating algorithms reduces "on the programming baseline level" to just one algorithm capable of inserting nodes into an existing mesh. The node insertion itself is performed by using a modified *Delaunay algorithm*, which was specially adapted for the context of crack simulations. In doing so the sub-processes 2 and 3 are realised by following node insertion procedures:

- Interconnecting edges are created by additional nodes in the middle of the missing edges.
- The realisation of a face is based on an adaptation of the *Bisection algorithm* by Rivara [6]. The missing face is (recursively) subdivided into two smaller faces by inserting a bisecting edge from the middle of the longest edge to the opposite node.

The algorithm described above is able to realise the necessary manipulation of the mesh (and thus the geometry) in every step of a crack growth simulation. Generally it is necessary to take a lot of effort in the field of *mesh improvement algorithms* [2,7] in order to keep a sufficiently well shaped mesh during the whole simulation.

Submodeling technique

The proposed algorithm for mesh manipulation delivers a geometrically correct mesh with respect to the crack growth in any simulation step. However, this mesh is far from being well-posed for fracture mechanical evaluations, as it generally shows neither any geometrical nor numerical regularity. Therefore the FE-submodeling technique is applied additionally in ADAPCRACK3D, which provides a sort of very regular mesh around the crack front, that advances in the simulation with the growing crack (Fig. 5).

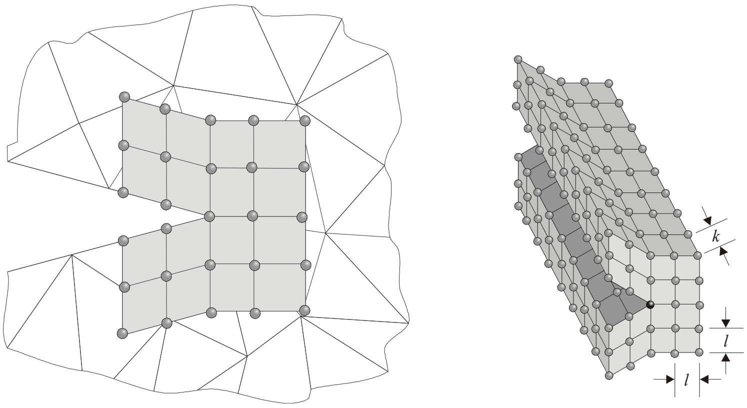

Figure 5. FE-submodel for a straight through-crack (right hand side) and cross-section of submodel with underlying global mesh (left hand side)

This submodel is –due to its regular structure– ideally suited for fracture mechanical evaluations especially with the favoured MVCCI-method. It is of a special importance that the submodel does not need any physical connection to the underlying global model, which would be a very complicated task. It is rather sufficient to define all nodes at the surface of the submodel as so-called *driven nodes*, whose kinematic boundary conditions then automatically are interpolated from the solution of the global model.

SIMULATION EXAMPLE

Figure 6. Shutter ring of a hydraulic press with detailed view on the crack propagation

The practical relevance of the presented program ADAPCRACK3D shall now be demonstrated by an industrial example of the crack growth in a shutter ring of a hydraulic press. During the trial operation of this press after a short time (~80.000 loading cycles) a multitude of fast growing cracks could be observed, which forced an untimely substitution of the ring (Fig. 6). Due to the symmetry of the shutter ring, it is sufficient to calculate a model according to Fig. 7, which consists of a 30°-segment. The model is loaded by area loads at the lower side of the teeth by an overall load of 3.33MN per tooth.

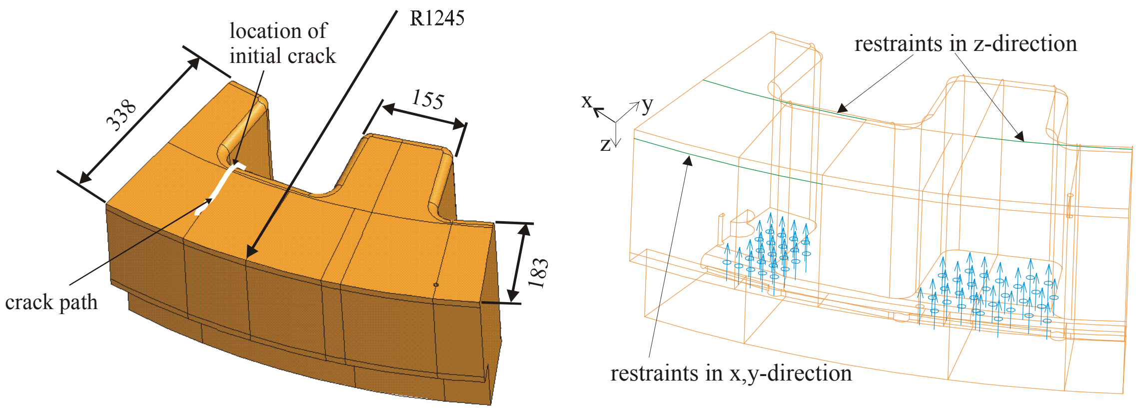

Figure 7. Schematic overview on measurements,
bearings and loadings of the shutter ring

In the simulation it becomes obvious that, for the regarded crack initiation location at the inner side of the shutter ring, a semi-circular shaped initial crack of just 0.4mm is sufficiently large in order to start crack growth from the fracture mechanical point of view, as even for this crack size the Threshold-value for the material (221.4 N/mm$^{3/2}$, R=0) is exceeded. The crack path, which is calculated during the following 36 simulation steps, can be gathered from Fig. 8.

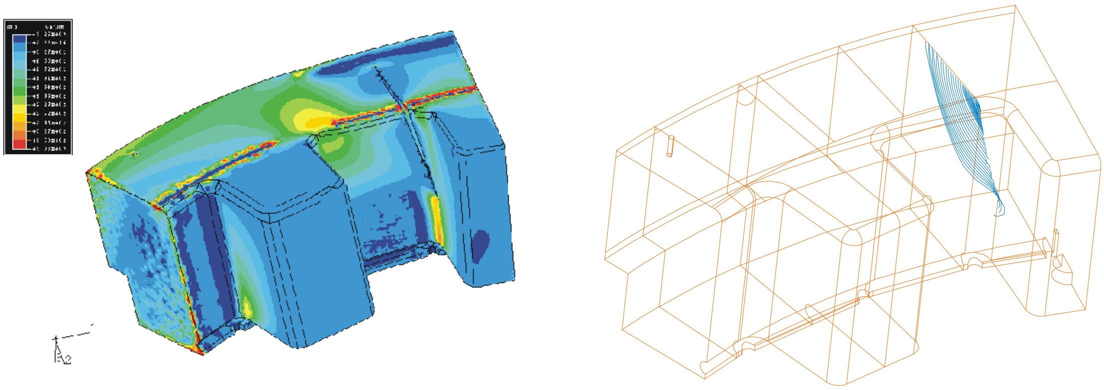

Figure 8. Simulated crack propagation

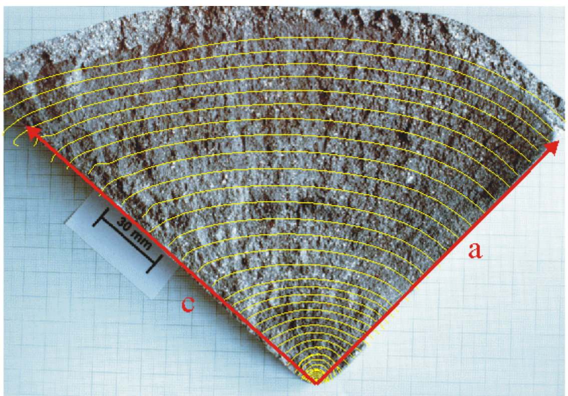

Figure 9. Top view crack surface with simulated crack fronts

Apparently the calculated crack path is in excellent agreement with the real path as found in right-hand side of Fig. 6. This also is approved in the top view on the surface with the calculated crack fronts (Fig. 9). In this figure *a* denotes the crack length at the upper side of the shutter ring and *c* the crack length at the inner side along the tooth.

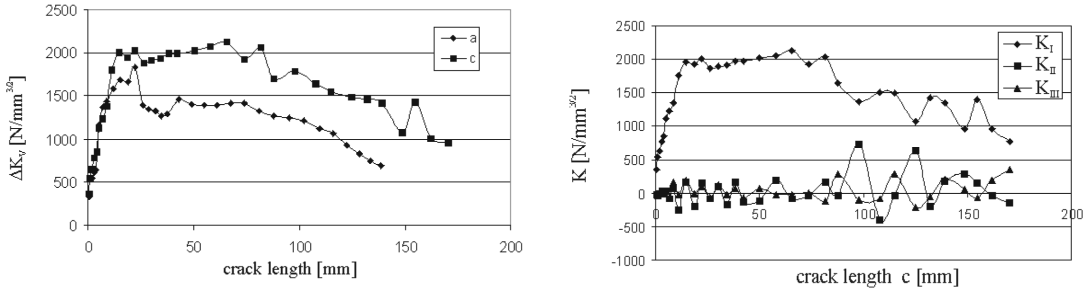

Figure 10. Development of stress intensities along the crack length

In the development of the stress intensities for both *a* and *c* for small crack lengths at first an increase of the ΔK_V-value to nearly the fracture toughness (K_{Ic}=2400N/mm$^{3/2}$) can be observed, whereas in the following a „plateau area” develops. Beginning from about 90mm crack length even a slow decrease of the stress intensity can be found until the end of the simulation. The right-hand side of Fig. 10 presents the stress intensities for the three modes for crack length *c*. At the beginning the crack is clearly dominated by Mode I, while up from 90mm crack length an increasing influence of Mode II and Mode III can be observed, which cause the crack to kink towards the neighbouring tooth (see Figs 6 and 8, each right-hand side).

To conclude this, it can be pointed out, that ADAPCRACK3D is a powerful tool for the simulation of crack growth processes in three dimensional structures, which yields excellent agreement with the real crack case.

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