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The Effect of Weld Metal Strength Mismatch on *J* and CTOD

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ABSTRACT Welded structures may need to withstand severe plastic deformation without fracture in the presence of a defect. To predict whether this is possible it is necessary to characterise the crack deformation in terms of J or δ as a function of applied strain. This is a difficult task since most elastic–plastic fracture design methods are based on applied stress. The complexity of the problem is further increased by any strength mismatch which exists between the weld and parent plate. This paper describes the results of elastic–plastic finite element analyses of double 'V' welds with strength mismatch of ±30 percent compared to the parent plate. It is shown that the strength mismatch can significantly alter the value of J at a given remote applied strain.

Notation

a	Crack depth
C	Constant used to link non-dimensional J to non-dimensional strain
e	Strain
Δe	Increment in strain
e_y	Yield strain
J	Computed value of J integral in finite element analysis or estimated value from limit load formulae
ΔJ	Increment in J value for an increment of strain
L	Length of specimen or structure
m	Constant linking J with δ
W	Specimen width in the plane of the crack
Y	Crack geometry factor in expression for linear elastic stress intensity factor

Greek symbols

δ	CTOD
σ	Stress
σ_y	Yield stress

Abbreviations

CTOD	Crack tip opening displacement

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Introduction

It is common engineering policy to use weld metals that are stronger than the plate which they join. The rationale for this is that the weld metal is less tough and more likely to contain defects than the plate. It is hoped that overmatching weld metal yield stress will offset these potential problems. Toughness decreases with increasing yield stress, so that, when trying to overmatch high strength plating it may be difficult to find a weld metal that is tough enough, even allowing for the supposed benefits of overmatching. Residual stress and fabrication cracking problems also worsen as weld yield strength increases. It is thus difficult to identify the optimum combination of weld yield strength and toughness for a given application.

This report describes an attempt to quantify the effect of weld mismatch on defect tolerance using elastic–plastic finite element analysis. The geometry analysed is illustrated in Fig. 1. It consists of a two-dimensional plane strain idealisation of a shallow edge crack in a double 'V' weld in a long strip subjected to pure bending or pure tension at the ends. Weld yield was varied between 30 percent undermatched and 30 percent overmatched compared to the surrounding plate.

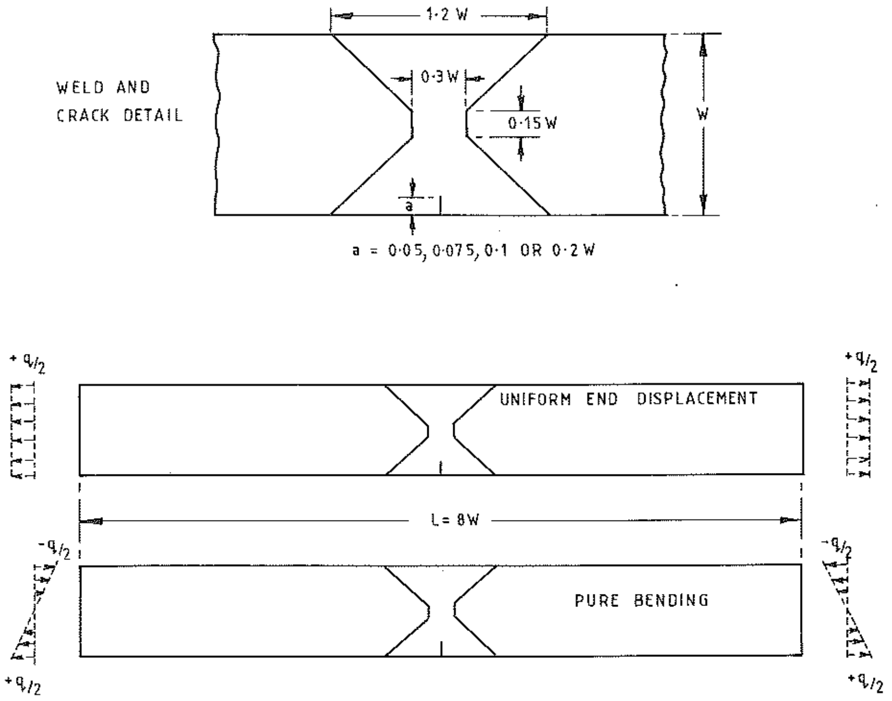


Fig 1 Geometry analysed

Computational details

The finite element mesh used for the numerical study of weld mismatch effects is shown in Fig. 2. All analyses were performed at Swansea University using an in-house program based on eight-noded, parabolic, isoparametric elements. The mesh in Fig. 2 contains 198 such elements.

Previous work at Swansea University on this problem **(1)** has shown that path dependence of the J integral results when integration contours cross the boundary of a significantly mismatched weld. In the present study J was evaluated as the average of contours contained wholly within the weld zone. CTOD was defined by a straight line extrapolation to the crack tip from the two crack flank nodes nearest the mouth of the crack.

The analysis was performed using bilinear stress strain behaviour with low-work hardening chosen to be characteristic of high strength steel and weld metal

$$\frac{\sigma}{\sigma_y} = 1 + 0.0022\left(\frac{e}{e_y} - 1\right) \tag{1}$$

where σ and e are applied stress and strain, and σ_y and e_y are the respective yield values. Undermatching and overmatching were simulated by holding the yield stress of the plate constant and increasing or decreasing σ_y for the weld zone between 30 percent undermatched and 30 percent overmatched. The linear slope of the work hardening curve was kept the same for both plate and weld.

Both tension and bending analyses were performed by imposing end displacements in fifty equal increments up to a maximum applied strain of approximately $2e/e_y$ uniform end strain in tension and $4e/e_y$ outer fibre strain in bending. Most analyses were performed with a crack depth to specimen thickness $a/W = 0.1$, but some computations were also carried out at $a/W = 0.05$, 0.075, and 0.2.

J results are presented in non-dimensionalised form

$$\frac{JE}{\sigma_y^2 a} \text{ versus } \frac{e}{e_y}$$

where E is Young's modulus and a is crack depth. The definition of strain is shown in Fig. 3.

Finite element results

Tension

Figures 4(a) and 4(b) show plots of non-dimensionalised J integral versus remote strain for various degrees of weld mismatch at $a/W = 0.1$ in tension. σ_y and e_y are the yield stress and strain of the plate in all cases.

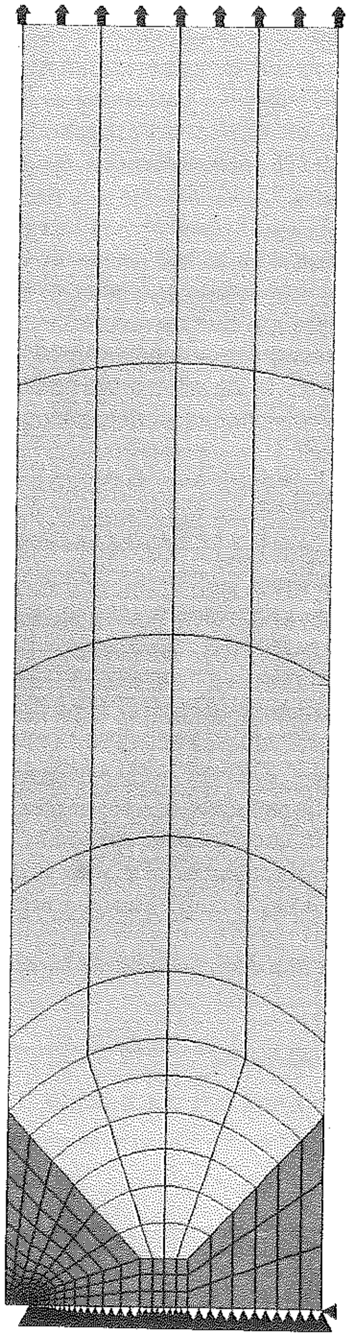


Fig 2 Typical finite element mesh (198 eight noded elements)

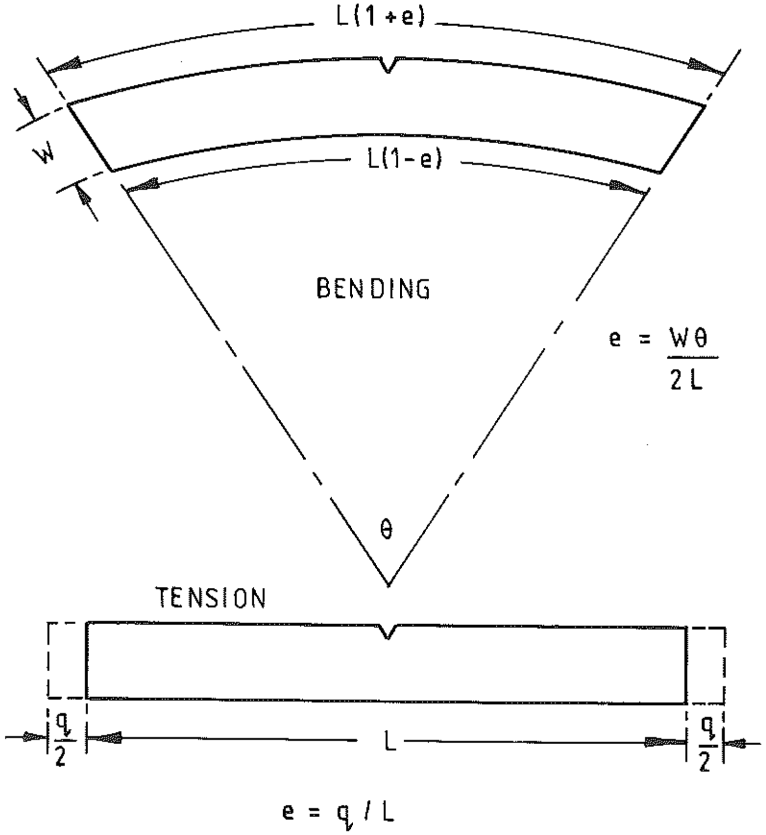


Fig 3 Definitions of remote strain used in bending and tension

The overmatched welds (Fig. 4(a)) show the expected trend that increasing the degree of weld overmatch provides increasing protection from crack tip deformation. Figure 4(b) shows that J is also reduced when the weld is undermatched. At first sight this seems impossible, but it can be understood by examining the relative plastic flow fields illustrated in Fig. 5.

(i) The matched weld shows a net section yield pattern characteristic of a deep edge crack in a finite width plate. For a high work hardening material a change to gross yield (yield on sections remote from the crack) might be expected for a crack as shallow as $a/W = 0.1$. However, for the very mild work-hardening behaviour used here net section yield is maintained.

(ii) With the overmatched weld, the whole plate outside the weld has gone into gross yield. A net section yield pattern exists between the crack tip and the gross yield region, but the total effect is that the crack tip now sees a much smaller proportion of the axially applied displacement.

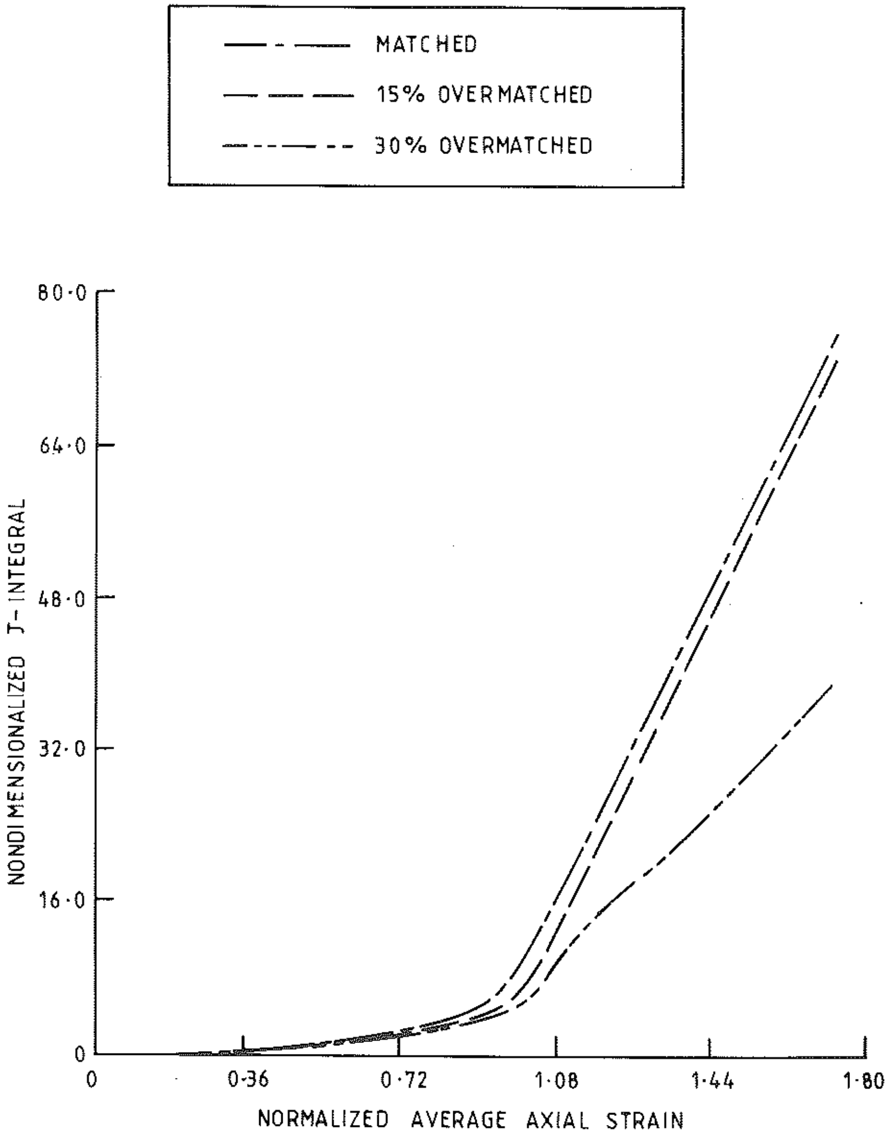


Fig 4a Non-dimensional J ($JE/\sigma_y^2 a$) versus non-dimensional strain (e/e_y). Overmatched double 'V' welds in tension. Edge crack, $a/W = 0.1$

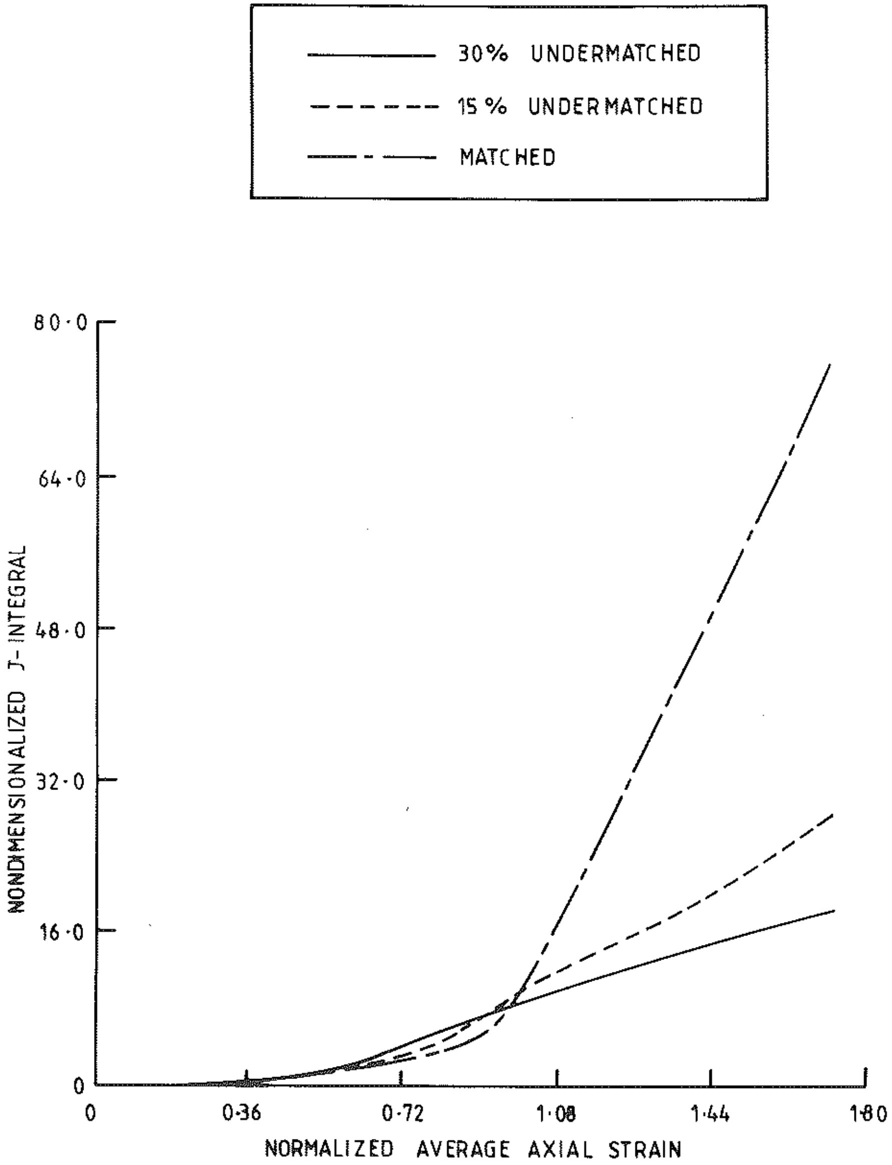


Fig 4b Non-dimensional J ($JE/\sigma_y^2 a$) versus non-dimensional strain (e/e_y). Undermatched welds in tension. Edge crack, $a/W = 0.1$

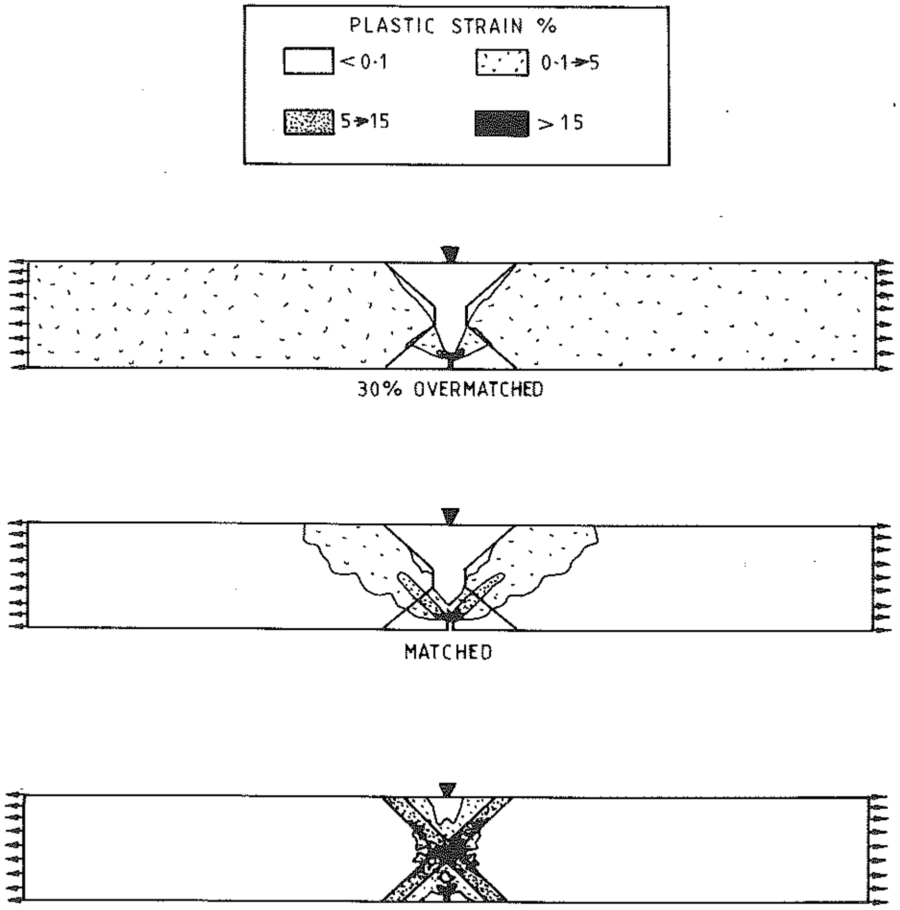


Fig 5 **Plastic zone development in tension. Edge crack $a/W = 0.1$**

(iii) In the undermatched case all remotely applied displacement is concentrated in the weld. However, the crack tip does not see this displacement since it is absorbed by intense bands of strain which follow the weld profile and concentrate axial strain in the weld root.

A check was made to examine the effect of an angled crack following the fusion line in the undermatched case. As the algorithm for J calculation did not cover angled cracks, the comparison was made in terms of δ. The computation, which was carried out for a 15 percent undermatched weld, showed less crack tip opening for the angled crack than for the weld centre line crack, although additional mode II deformations were now evident. Further work is planned to investigate the behaviour of a buried crack at the weld root.

Bending

Figure 6 shows J versus imposed outer fibre strain for $a/W = 0.1$ edge cracked specimens loaded in pure bending. The results conform to the expected pattern of elevated J in undermatched welds and reduced J in overmatched welds.

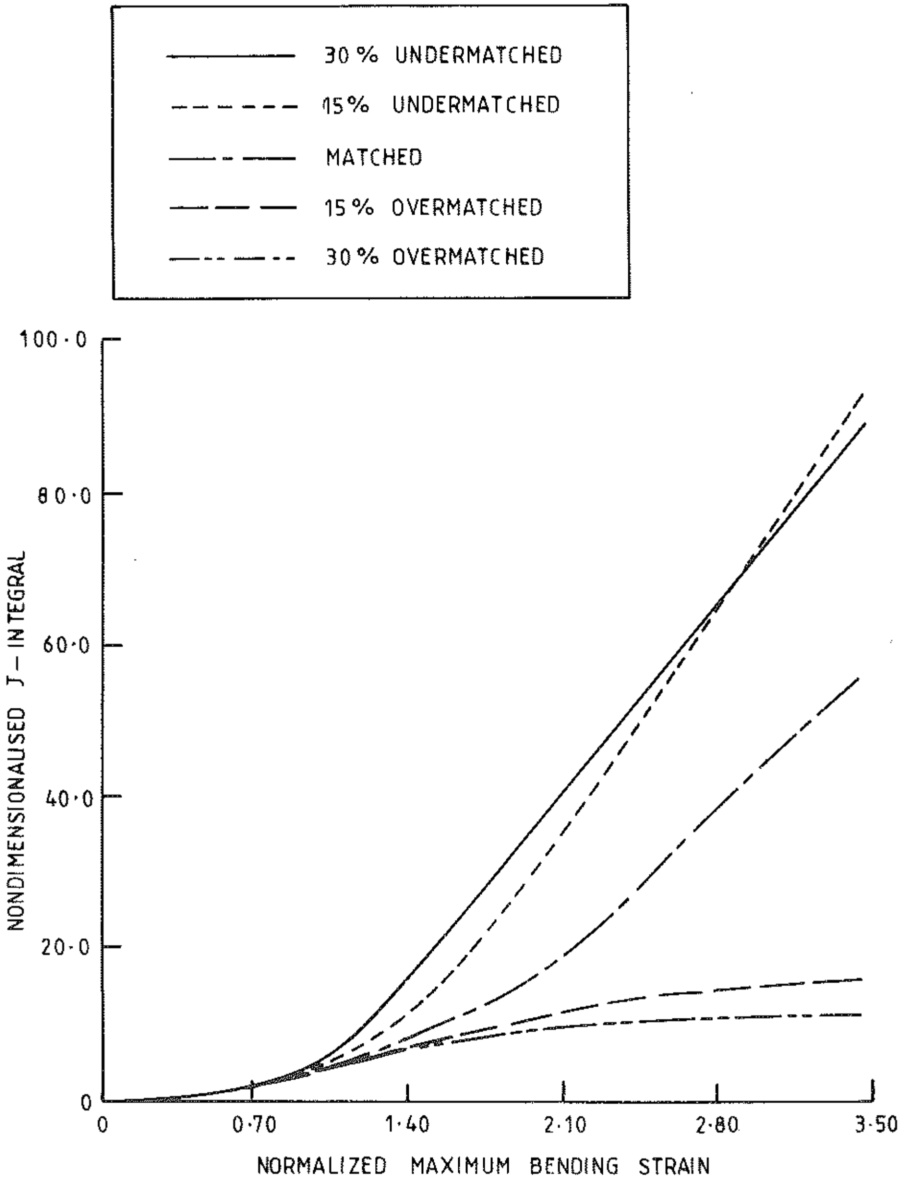
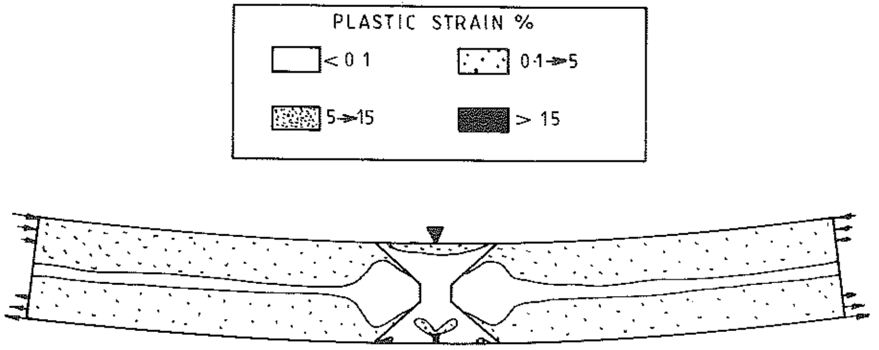
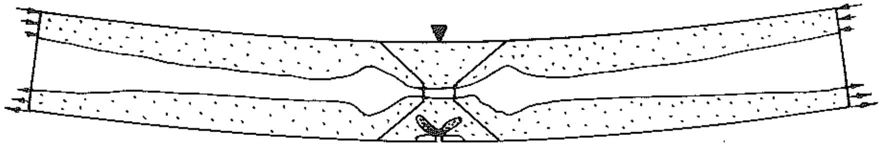


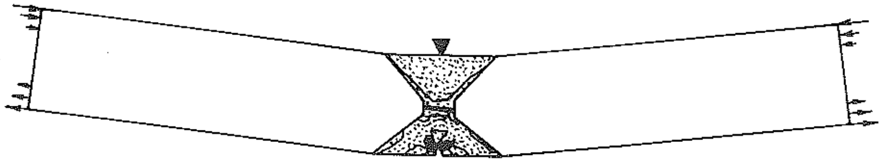
Fig 6 **Non-dimensional J ($JE/\sigma_y^2 a$) versus non-dimensional (e/e_y) for pure bending and various degrees of weld mismatch. Double 'V' weld, edge crack, $a/W = 0.1$**



30% OVERMATCHED



MATCHED



30% UNDERMATCHED

Fig 7 **Plastic zone development in bending**

The plastic zone development for 30 percent overmatched and 30 percent undermatched welds is shown in Fig. 7. In the undermatched case all plasticity is concentrated in the weld. Conversely, with the overmatched weld all plasticity is forced into the plate.

Relationship between J and strain

Fully developed plasticity

The relationship between J and imposed strain is very dependent on yield pattern. The important distinction is between:

(i) net section yield where plasticity is concentrated on the plane of the crack, which leads to rapid J increase; and

(ii) general, or gross, yield where plasticity spreads throughout the whole specimen or structure, and the rate of J increase is low.

Table 1(a) **Computed and theoretical values of C in equation (2) after fully developed yield for specimens in tension (σ_y is the yield stress of the plate in all cases)**

a/W	*30 percent undermatched*	*Matched*	*30 percent overmatched*	*Theoretical (equation (3))*
0.05	10	148	3	184
0.075	11	113	10	122
0.1	13	90	43	92
0.2	21	47	52	46

Table 1(b) **Computed and theoretical values of C in equation (2) after fully developed yield for specimens in bending (σ_y is the yield stress of the plate in all cases)**

a/W	*30 percent undermatched*	*Matched*	*30 percent overmatched*	*Theoretical (equation (4))*
0.05	51	30	0.5	71
0.075	43	30	0.5	59
0.1	35	30	0.5	52
0.2	21	30	12	36

After the yield pattern is established the relationship between increments in J, ΔJ, and increments in applied strain, Δe, becomes nearly linear for low work hardening materials. In non-dimensional form

$$\left(\frac{\Delta J E}{\sigma_y^2 a}\right) = C\left(\frac{\Delta e}{e_y}\right) \tag{2}$$

where C is dependent on geometry, loading, yield pattern, and work hardening behaviour.

Tables 1(a) and 1(b) summarise the final slopes of the non-dimensional J versus strain plots obtained from the present finite element analyses. Shown for comparison are theoretical C values derived for net section yield, perfect plasticity, and homogeneous material.

For tension

$$C = 1.155\,\frac{L}{a} \tag{3}$$

For bending

$$C = \{0.363 + 11.7\,a/W - 24.6\,(a/W)^2 + 12.5\,(a/W)^3\}L/2a \tag{4}$$

Where the polynomial is obtained by differentiating the plastic limit load expression in (2) with respect to crack length. L is defined in Fig. 3.

In comparing the theoretical and computed C values, and in reviewing the effect of crack length and weld mismatch between the different computed cases in Table 1, it is useful to keep the following points in mind.

(i) The theoretical C values in Table 1 are for net section yield only. Gross yield significantly reduces the C value compared to net section yield. No exact methods are available to calculate C for a low work hardening material in gross yield, but evidence in **(3)(4)** suggests that C is likely to be around 10.

(ii) In net section yield J is directly proportional to specimen length and is independent of absolute crack size (in bending, but not in tension, J depends on a/W). As a consequence C is a function of specimen length divided by crack size. That is, if a specimen half the length had been analysed C would have been halved. In gross yield, however, C is a constant independent of crack size and specimen dimensions (the crack is too small to be sensed at the specimen boundaries) and J scales directly with crack size.

With this background the following relevant conclusions can be drawn from a study of Tables 1.

Matched specimens

Computed results for the $a/W = 0.075$, 0.1, and 0.2 tension specimens conform closely to the theoretical net section yield predictions. This is encouraging because it confirms the adequacy of the finite element mesh design. At $a/W = 0.05$ tension, and in bending, C falls below its expected net section yield value indicating a trend towards gross yield. In these cases C is still higher than would be expected for pure gross yield. This is not surprising since yielding away from the plane of the crack is not favoured by the very low work-hardening coefficient used.

Overmatched specimens

For the most deeply cracked ($a/W = 0.2$) specimen in tension, overmatching confers no advantage. Net section yield still occurs in spite of the overmatch and C is similar to that for a matched specimen. In all other cases overmatching provides a considerable advantage in transferring yield outside of the weld region and hence shielding the crack from deformation. This is most evident in bending where the crack virtually ceases to open (C tends to zero) at $a/W \leqslant 0.1$.

Undermatched specimens

The surprising reduction of J in tension loaded undermatched welds has already been noted. It is clear from the values in Table 1(a) that none of the tension specimens are in net section yield. The value of C is close to that which would be expected under gross yield. In undermatched welds under bending C is elevated above the matching weld case at $a/W \leqslant 0.1$ although the elevation is less dramatic than might be expected. The effect of undermatching is more apparent in small scale yielding. From Fig. 6, at $e/e_y = 1.5$, J for the 30 percent undermatched weld is twice that for the matched weld. J values in small scale yielding are discussed in more detail below.

Small scale yielding

For strains up to $e/e_y = 1.2$ Turner **(3)** has proposed

$$\frac{JE}{\sigma_y^2 a} = Y^2\pi\left(\frac{e}{e_y}\right)^2\left[1 + 0.5\left(\frac{e}{e_y}\right)^2\right] \tag{5}$$

At $e/e_y = 1.2$ this gives, for $a/W = 0.1$

$$\frac{JE}{\sigma_y^2 a} = 8.4 \quad \text{for bending, where } Y = 1.04$$

$$\frac{JE}{\sigma_y^2 a} = 10.3 \quad \text{for tension, where } Y = 1.15$$

Equation (5) was derived predominately from finite element analyses of tension specimens with sufficient work hardening to go into gross yield.

It is unconservative for net section yield in tension, as can be seen from Fig. 4, where values as high as $JE/\sigma_y^2 a = 28$ result at $e/e_y = 1.2$. In the strongly over and undermatched welds, which have yield patterns closer to gross yield, $JE/\sigma_y^2 a$ is about 15 at $e/e_y = 1.2$.

In bending, where e represents outer fibre yield strain, equation (5) is conservative enough to cover even the 30 percent undermatched weld where $JE/\sigma_y^2 a = 9$ at $e/e_y = 1.2$. In the matched weld $JE/\sigma_y^2 a$ is 5.5. In the 30 percent over-matched weld $JE/\sigma_y^2 a$ reaches a virtual plateau value of $JE/\sigma_y^2 a = 9$ at $e/e_y = 2$.

Relationship between J and δ

Tables 2(a) and 2(b) illustrate the extremely consistent relationship between J and δ obtained in the present study. The fact that J and δ have a close relationship of the type

$$J = m\sigma_y \delta \tag{6}$$

where m usually lies between 1 and 2 is well known. It is usually fairly easy to deduce the value of m appropriate to a particular geometry once the value of the plastic limit load and the rotational constant are known.

What is surprising in Tables 2(a) and 2(b) is the small effect on m of a wide range of different crack depths, yield patterns, and weld to plate yield ratios (σ_y) in equation (6) is taken to be the yield stress of the weld).

In strongly mismatched welds it is clearly impossible to deduce J in the usual way from the total energy absorbed. For instance, in the 30 percent overmatched weld in bending shown in Fig. 7, considerable energy is expended in the plate remote from the weld. The close link between J and crack displacement irrespective of weld mismatch and crack size suggests that this would provide a valuable alternative method of J determination which might

Table 2(a) **Computed values of m in equation (6) after fully developed yield for specimens in tension (σ_y is the yield stress of the weld in all cases)**

a/W	*30 percent undermatched*	*Matched*	*30 percent overmatched*
0.05	1.15	1.25	1.20
0.075	1.15	1.25	1.19
0.1	1.19	1.26	1.21
0.2	1.24	1.26	1.18

Table 2(b) **Computed values of m in equation (6) after fully developed yield for specimens in bending (σ_y is the yield stress of the weld in all cases)**

a/W	*30 percent undermatched*	*Matched*	*30 percent overmatched*
0.05	1.39	1.25	1.21
0.075	1.42	1.27	1.19
0.1	1.39	1.29	1.23
0.2	1.52	1.46	1.24

be of practical use in specimen or structural element testing of mismatched welds.

Conclusions

Finite element derived J versus remotely applied strain data have been presented for a double 'V' weld with a strength mismatch in a low work hardening material. The relationship between J and applied strain has been found to be very dependent on yield pattern. This is a function of: bending to tension ratio, percentage weld strength mismatch, material work hardening exponent, and crack depth to section width ratio. Each of these parameters will, if changed in isolation, reach a critical value which causes a transition between gross yield and net section yield, or between yield in the weld and yield in the plate. This causes discontinuous changes in the relationship between J and applied strain, which are evident in the data presented here as percentage mismatch is changed (Figs 4 and 6) and as crack length is varied (Tables 1). These trends have so far prevented the derivation of any general conclusions on the consequences of weld mismatch, but the following points are worth highlighting.

(i) Undermatched double 'V' welds under plane strain in tension behave in a surprising way with strain concentrating at the fusion boundaries shielding a central edge crack from a large proportion of the remotely applied deformation.

(ii) In overmatched double 'V' welds in pure bending almost all plastic deformation takes place in the plate. A central edge crack in the weld shows little increase in J for increases in applied strain. In the undermatched case, strain is concentrated in the weld and J is elevated compared to a matched weld. However, because the weld region is in gross yield the elevation is less than might be expected.

(iii) There is a very close relationship between J and crack opening which applies to all the geometries studied irrespective of weld mismatch, loading, yield pattern, and crack depth.

It is recommended that to isolate the trends most important in a real structure, any future two-dimensional analyses should concentrate on configurations which give gross yield in the absence of a weld. Ultimately, three-dimensional analyses may be needed to produce definitive conclusions.

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