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On the Problem of Transferability in the CTOD Concept

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ABSTRACT A realistic assessment of the integrity of a structure requires comparable parameters used in the determination of the critical material states as well as in the design of components. The paper presents the results of a Finite Element calculation of the standard bending specimen showing the difference between CTOD according to the BS 5762 and determined as the local opening at the crack tip for an austenitic steel. The study reveals that the extrapolated CTOD of the BS 5762 can lead to an appreciable overestimation in comparison with the local CTOD particularly for materials with a distinct hardening effect. As a consequence the critical CTOD values indicate a higher fracture toughness than available and so reduce the effective safety factor in the CTOD design curve.

Notation

CTOD	Crack tip opening displacement
a	Crack length
W	Specimen width
B	Specimen thickness
z	Knife edge thickness
E, ν	Elastic modulus, Poisson's ratio
K	Elastic stress intensity factor
J	J contour integral
σ, ε	Uniaxial stress, strain
σ_0, ε_0	Reference stress, strain
σ_Y	Yield stress
n	Hardening exponent
d_n	Function of n and σ_0/E relating CTOD to J
δ	CTOD
δ_{BS}	Crack tip opening displacement according to British Standard
δ_R	Local crack tip opening displacement defined by Rice
V, V_e, V_p	Total crack opening displacement at specimen edge, elastic component, plastic component

Introduction

In the CTOD concept the crack opening at the crack tip is considered to be a representative measure of the local loading intensity of a cracked structure.

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Obviously the quantity corresponds closely with the strain field at the crack tip and may be understood as a special integral of the strain field.

Applying the CTOD concept the calculated crack tip opening of a present component has to be compared with a limiting opening generally defined by BS 5762 **(1)** using SEN bending specimens. This standard correlates the critical material states with an experimentally determined parameter derived from crack opening far off the crack tip and corrected by a term for the range of small scale yielding (SSY).

The specimen-bounded extrapolation scheme based on the global kinematics of the standard bending specimen cannot be transferred to other structures. In practice treating a real component one may have to determine CTOD numerically by FE solutions or other theoretical methods as the locally measured displacement in the crack tip opening profile.

The different definitions will cause some uncertainties and an interesting question is how far the CTOD values determined by BS 5762 correspond with theoretically defined local CTOD. To quantify the deviations and to discuss the effects a FE analysis for an austenitic steel was performed. Since the global displacements of both the real structure and the numerical solution generally agree very well the evaluation of the FE solution by the Dawes formula can be taken as obtained from a corresponding experiment.

The formula of Dawes

The British Standard 5762 defines the crack tip opening displacement by the formula of Dawes **(2)** related to the three-point bending specimen, Fig. 1

$$\delta_{\mathrm{BS}} = \frac{K_{\mathrm{I}}^2}{E \cdot \sigma_{\mathrm{Y}}} \frac{1-\nu^2}{2} + \frac{0.4(W-a)}{0.4W + 0.6a + z} V_{\mathrm{p}}. \tag{1}$$

Equation (1) formulates the geometric relationship between the plastic component V_{p} of the crack opening at the edge of the specimen and the crack tip opening δ_{BS} based on the plastic hinge model, Fig. 2, assuming a rotation point at a distance of $0.4(W-a)$ from the crack tip. The shortcomings of the extrapolation below general yielding are corrected by the term

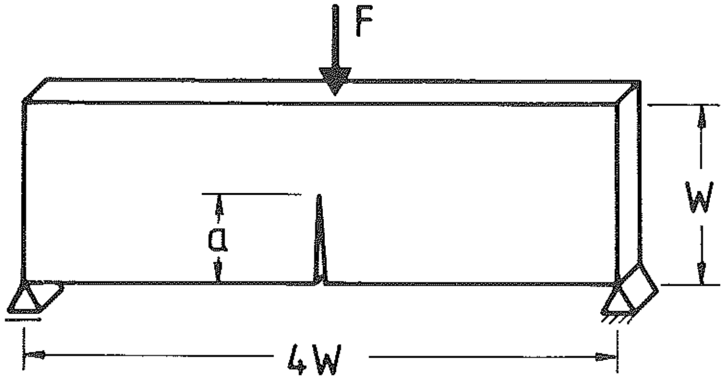

Fig 1 SEN three-point bending specimen

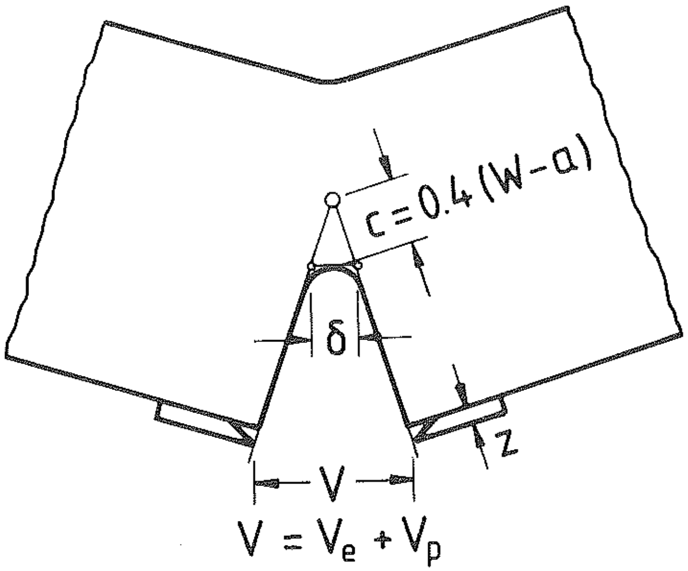

Fig 2 Plastic hinge model

$$\frac{K_{\mathrm{I}}^2}{E \cdot \sigma_{\mathrm{F}}} \frac{1-\nu^2}{2} \tag{2}$$

The factor $(1-\nu^2)/2$ shall account for the constraint from a proposed specimen thickness B of $W/2$ but neglects any hardening effect. The extrapolation term

$$\frac{0.4(W-a)}{0.4W + 0.6a + z} V_{\mathrm{p}} \tag{3}$$

does not include any local constraint at the crack tip but involves averaged material hardening in the resulting ligament deformation.

Comparison of local CTOD and evaluation according to BS

Plane stress state solution

To ensure sufficient accuracy of near tip displacement fields a high subdivision of structure and load sequence were necessary. Owing to the expense caused by an adequate mesh refinement in the thickness direction, numerical analysis

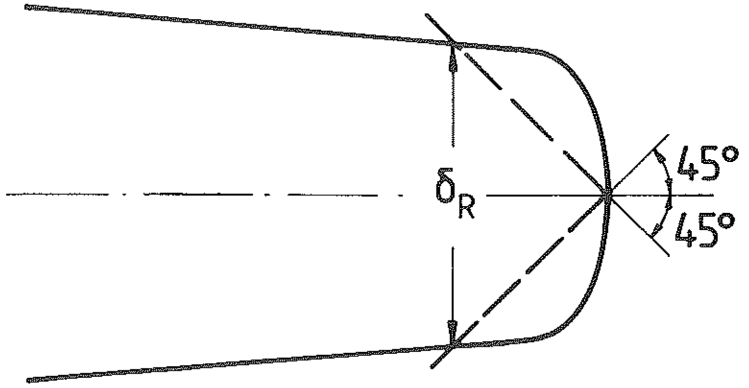

Fig 3 CTOD definition by Rice

had to be restricted to plane stress state. In this case Dawes's formula becomes

$$\delta_{BS} = \frac{K_I^2}{E \cdot \sigma_F} + \frac{0.4(W-a)}{0.4W + 0.6a + z} V_p \tag{4}$$

FE analysis refers to the σ–ε curve of an austenitic steel X6CrNi18 11, Fig. 4, and a corresponding piece-wise power law, Fig. 5, to prove the accuracy of near tip displacement fields and the resulting local CTOD by comparison with the analytical HRR-field solution (3)–(5). Regarding the HRR-field, the section of the σ–ε curve for $\sigma \leqslant \sigma_0$ only influences the magnitude of J but does not affect the characterisation of stress and strain distribution.

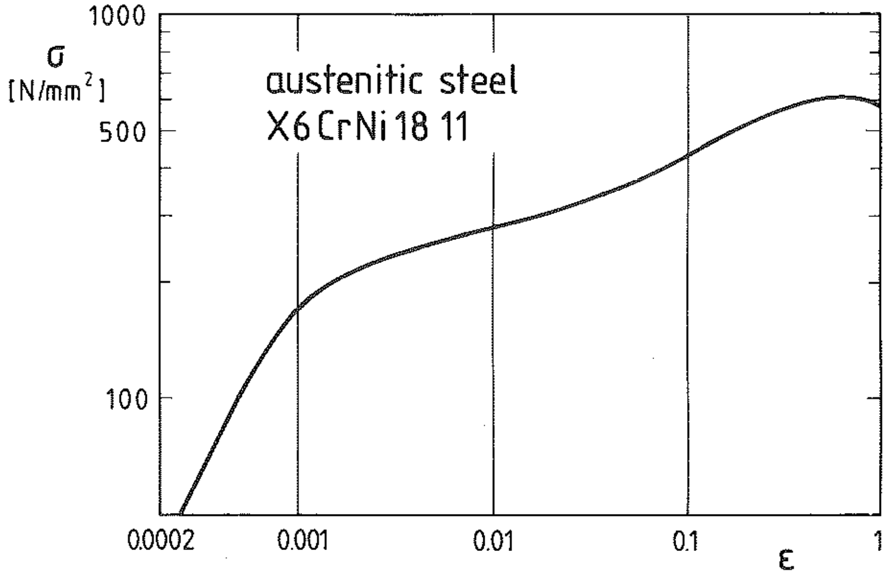

Fig 4 Material law of the investigated austenitic steel X6CrNi18 11

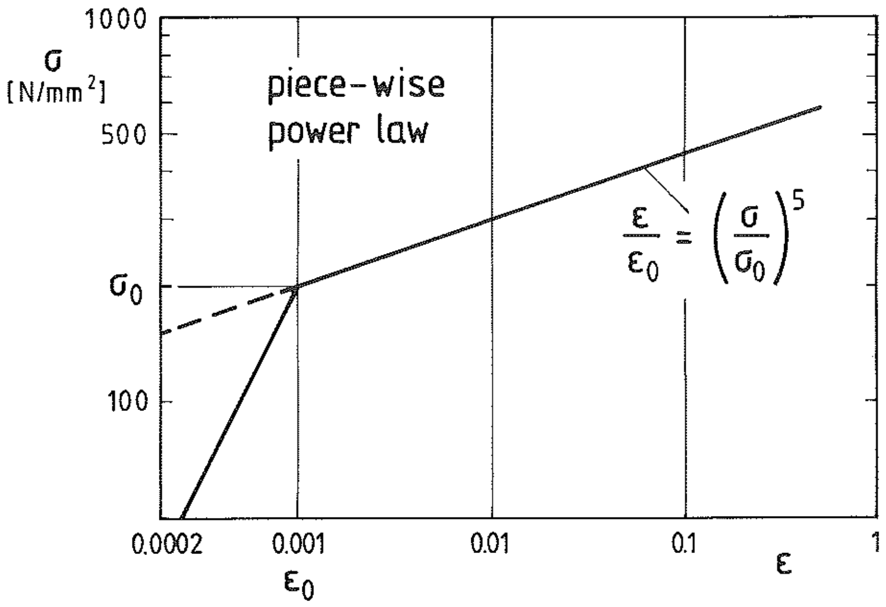

Fig 5 Piece-wise power law investigated

The relationship between the local crack tip opening displacement in the definition suggested by Rice, Fig. 3, and the J integral is given by Shih (6) as

$$\delta_R = d_n \cdot \frac{J}{\sigma_0} \tag{5}$$

where d_n is a function of the hardening exponent n and σ_0/E. Current d_n values can be taken from the diagrams in (6).

Figure 6 shows two crack tip opening profiles of the deformed FE structure as the configuration of the displaced nodes. The plot includes the opening distance according to Rice and BS 5762 and the part corresponding to the extrapolation term. The dependence of the CTOD upon load and plastic extent can be seen in Figs 7 and 8.

As Fig. 7 illustrates, the local CTOD of the power law solution agrees very well with the opening displacement determined by equation (5) on the basis of numerical J values. This indicates a converged solution with a good approximation of the near fields.

Comparing BS 5762 with the local CTOD in part considerable deviations have been found. In the lower load range the differences are mainly caused by the overestimation of the SSY term excluding the hardening influence. Here the strong effect of austenitic hardening reduces the real crack tip opening to about 40 percent.

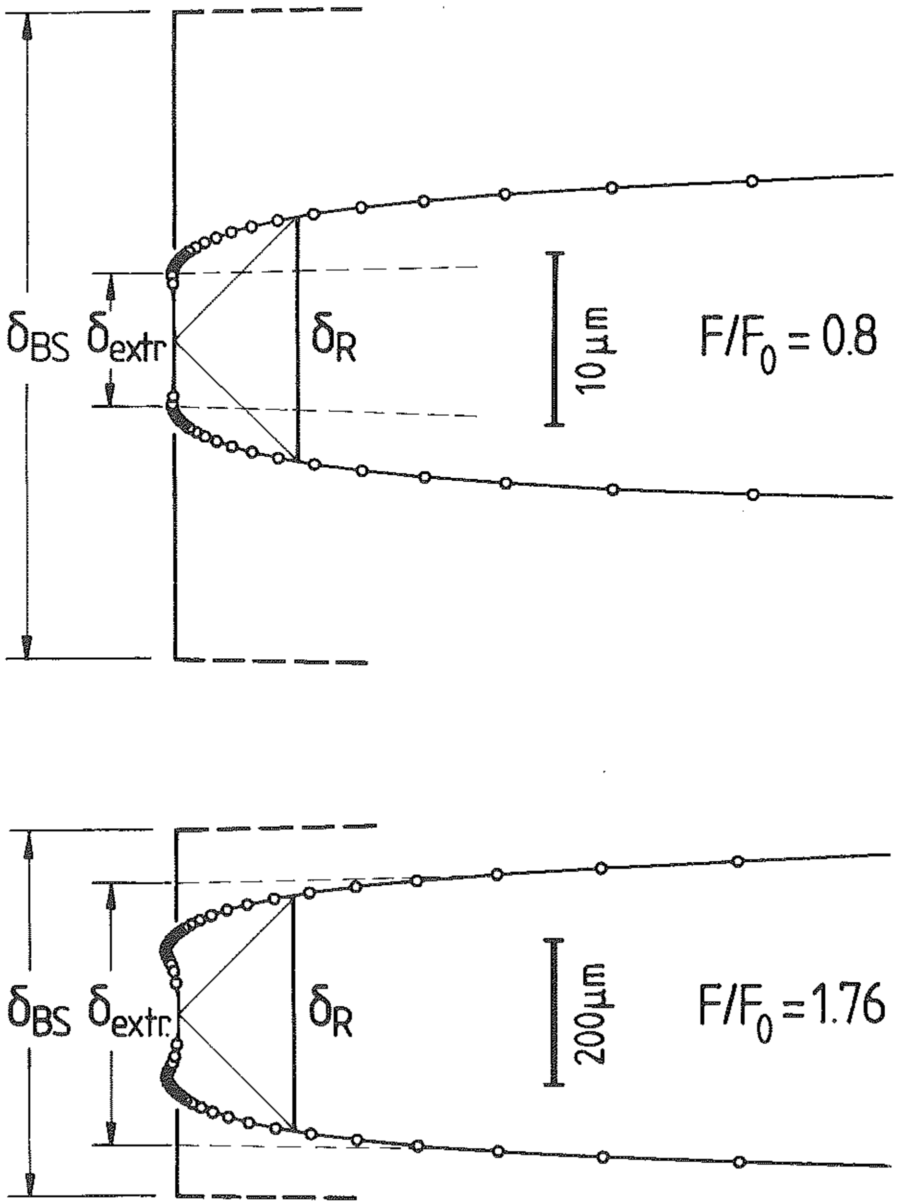

Fig 6 Crack tip opening profiles from FE solution with local CTOD and the CTOD evaluated by BS 5762

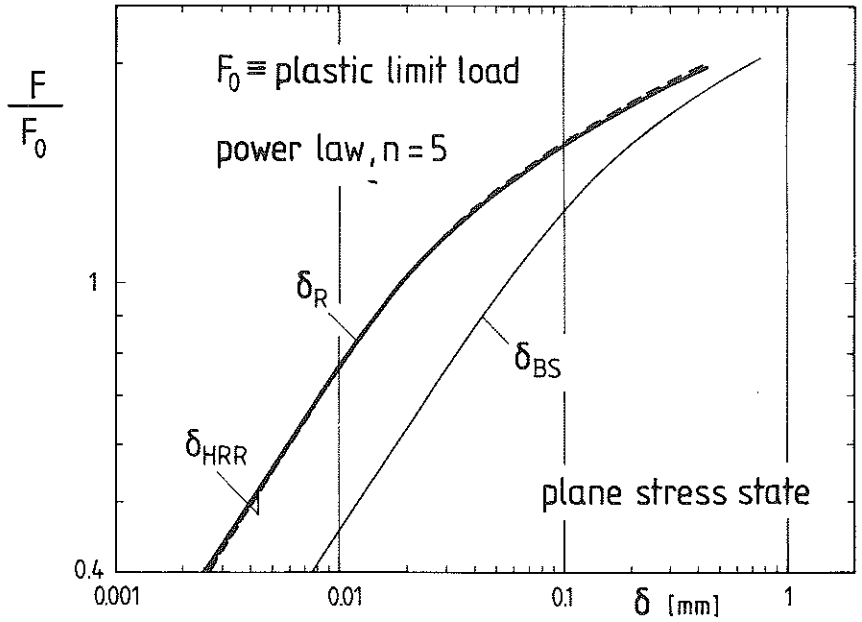

Fig 7 CTOD versus load for piece-wise power law ($n = 5$) – plane stress state

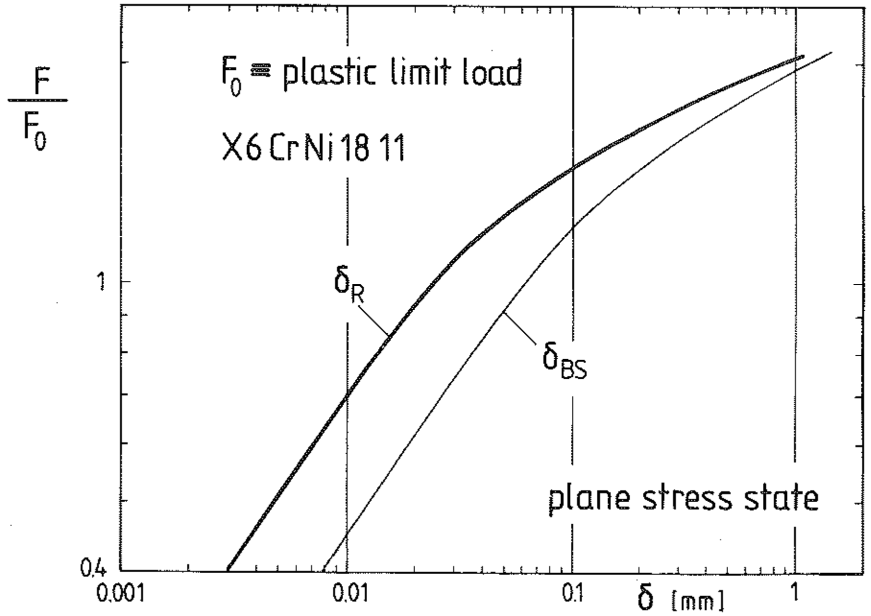

Fig 8 CTOD versus load for X6CrNi18 11 – plane stress state

In the general yielding range increasing load brings the plastic hinge model of the structure into effect and the extrapolation term takes over the dominating part of CTOD. As a result the significant discrepancies related to the lower load range decrease but they obviously diminish to small differences only for rather high load stages.

Projection of solution for finite thickness

The situation shown for plane stress state may essentially hold for structures of finite thickness, too. The CTOD then varies over the thickness. The mid-section opening is considered to be the relevant parameter. When plastic regions are small (relative to the thickness dimension) reduced transverse contraction in the near field results in a state close to plane strain.

Dawes's formula accounts for constraint in SSY by a factor of nearly one half. A (fictitious) FE solution of a specimen with finite thickness might result in shifting the CTOD curve of Fig. 8 by the magnitude of the difference between the HRR-field solutions of plane stress and plane strain state. Assuming a mean hardening exponent $n = 5.5$ for the austenitic steel Shih's diagrams lead to a factor of 0.7 between the two states. This means that the differences between BS 5762 and local CTOD (compared to plane stress) decease but considerable deviations should remain from the missing hardening influence, see Fig. 9 (lower load range).

Since the global deformations of the specimen are determined by a ligament state close to plane stress the extrapolation term does not include a noticeable constraint effect. But according to Rice the reduction of the local constraint at the crack tip in large scale yielding depends mainly on the characteristic deformation kinematics of the structure. Because the plastic extent in the bending specimen is limited by the elastic domain, sliding on planes at ±45 degrees with the crack line is prevented and a high hydrostatic stress can persist. Consequently the local constraint will reduce the crack tip opening also above the plastic limit load contrary to the extrapolation by Dawes's formula, Fig. 9 (upper load range). The differences by a factor of 1.5–2 can be understood as an upper limit, but distinct deviations might exist in reality.

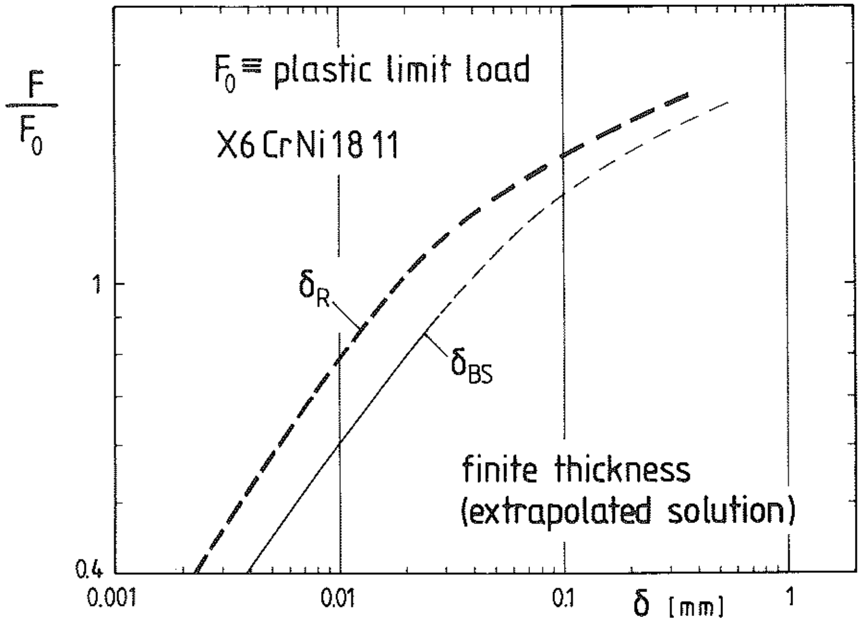

Fig 9 CTOD versus load for X6CrNi18 11 – estimation for a specimen of finite thickness

Conclusions

(1) The study has shown that Dawes's formula has a tendency to overrate the crack tip opening of the numerical solution. Considering that the relation is exclusively used for the determination of critical states this aspect gets some importance because too high CTOD values may lead to an over-estimation of the material toughness. In view of deficient FE calculations generally resulting in smaller displacements – and thus under-estimation of the real CTOD – a further disposition to an unconservative assessment has to be envisaged.

(2) Using different specimen sizes, see Fig. 10, the non-uniform deviations possibly induce a pseudo size effect because it makes a difference whether the critical states are reached on a load level with limited or large plastic deformations.

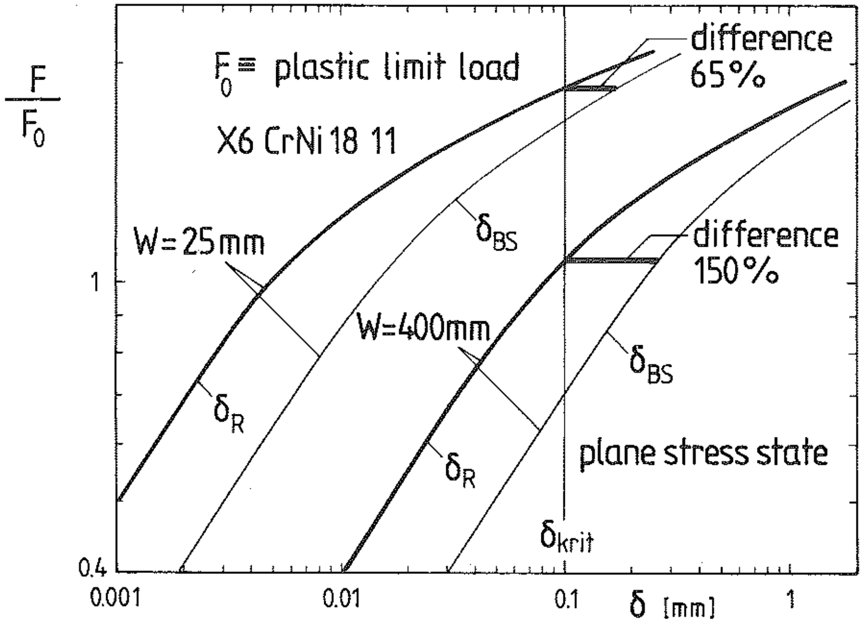

Fig 10 Comparison of local CTOD and evaluated by BS 5762 for different specimen sizes

(3) Comparing different materials the non-uniform deviations make it difficult to get a proper classification with respect to the fracture toughness.

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