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On Stress Relaxation at the Tip of a Crack under Normal Tension

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ABSTRACT Considering the localization of plastic deformation in separate yield bands to be a fundamental property of metals, the authors, using a suggested criterion of the appearance of such bands, conclude that, at crack tip under normal opening (where plastic deformations at a certain stage of deformation localize in two yield bands – 'direct layers' – symmetrical to the crack plane), the stress state reaches a uniform omnidirectional tension. The described test of tension and torsion of specimens containing soft interlayers supports such an assumption. The test shows a possibility of a complete relaxation of tangent (shear) stresses in metals when normal (tensile) stresses are above the yield strength.

In attempting to give a physical description of metal fracture we are always compelled to study, in one form or another, the plastically strained region at the crack tip or at some other stress concentrator. Stress relaxation in this region plays an important role in the formation of the stress–strain state there and, consequently, in the development of crack resistance.

It is observed that in many cases, at certain stages of deformation, plastic deformations localize in separate flow layers. Under homogeneous loading these layers are known as the Luders–Chernov bands. The appearance of the Luders–Chernov bands is always abrupt though their length is quite considerable. Let us assume localization in flow surfaces to be a fundamental property of metals manifesting the fact that plastic deformations as a result of irreversible shear-slip taking place in a finite volume limited from below. A possible condition for the appearance of plastic deformations in the form of a separate flow surface $l(x, y) = 0$ (a plane case is considered) will acquire the following form

$$\int_0^{d_0} \tau_{nl} \, dl = \tau_s d_0 \tag{1}$$

where d_0 is a characteristic of the metal, a constant, depending on the loading rate and testing temperature; τ_{nl} is the tangent stress at the dl element of the assumed slip band (before the appearance of the slip band the value of τ_{nl} is determined by linear elasticity theory); $n = n(x, y)$ is the normal to the slip band $l(x, y) = 0$; and τ_s is the yield strength of the metal under shear (at the given loading rate and testing temperature).

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Shear stress τ_{nl} cannot be higher than τ_s, and equation (1) is satisfied when $\tau_{nl} = \tau_s$, so at first sight equation (1) becomes identity. It is obvious that equation (1) is an identity for homogeneous stress state. At inhomogeneous stress state, the stresses reach the value only in some point (points), but neighbouring regions, where the shear stresses are less the τ_s value, hinder the development of plastic deformations, which can be realized in the form of irreversible shears of body particles. In that situation the regions, where the stresses are higher than τ_s must deform in a special way. But how? Here we suppose that shear load along the element Δl is $T_{\Delta l} = \int_0^{\Delta l} \tau_{nl}\, dl$, defined by external load parameters and does not depend on the situation: $0 < \tau_{nl} < \tau_s$ or $\tau_{nl} > \tau_s$, that means elastic deformation of the body up to achieving the condition (1). Considering the potential energy of the deformed body to be constant and the impossibility of $\tau_{nl} > \tau_s$ situation, it can be concluded that in such metal regions ('that must deform in a special way') the shear load transform into normal and at 'abrupt' appearance of the yield bands the 'abrupt' reverse transformation takes place.

Naturally, the yield line $l(x, y) = 0$ will appear at the point where the value of the functional

$$T(l(x, y)) = \int_l (\tau_s - \tau_{nl})\, dl, \qquad |l| \geqslant d_0, \tag{2}$$

is minimal and equal to zero.

Tangent stress τ_{nl} may be expressed in the following form

$$\tau_{nl} = (\sigma_y - \sigma_x)y'/(1 + y'^2) + \tau_{xy}(1 - y'^2)/(1 + y'^2), \tag{3}$$

where σ_x, σ_y, τ_{xy} are the components of the stress tensor; $y' = dx/dy$, $y = y(x)$ is the equation of the yield line $l(x, y) = 0$. If x_1, x_2 are the coordinates of the beginning and the end of the just appeared yield layer under loading in plane strain conditions, then, with account for (3), functional (2) will be transformed into

$$T(y(x)) = \int_{x_1}^{x_2} [\tau_s - (\sigma_y - \sigma_x)y'/(1 + y'^2) - \tau_{xy}(1 - y'^2)/(1 + y'^2)]\sqrt{(1 + y'^2}\, dx. \tag{4}$$

Thus, the problem of determination of the yield band is limited to determination of bands, where functional (4) values are minimal and equal to zero. Determination of the extremes of the functional of described type (4) is one of the tasks of the variational calculation. Eiler's variational equation (1)

$$\frac{d}{dx}\left(\frac{\partial p}{\partial y'}\right) - \frac{\partial p}{\partial y} \equiv \frac{\partial^2 p}{\partial y'^2} y'' + \frac{\partial^2 p}{\partial y\, \partial y'} y' + \frac{\partial^2 p}{\partial y'\, \partial x} - \frac{\partial p}{\partial y} = 0$$

for functional (4) where the subintegral function

$$p = [\tau_s - (\sigma_y - \sigma_x)y'/(1 + y'^2) - \tau_{xy}(1 - y'^2)/(1 + y'^2)]\sqrt{(1 + y'^2)},$$

has the form of

$$\frac{\partial}{\partial y}[y'^3(\sigma_y - \sigma_x) + (1 + 3y'^2)\tau_{xy}] - \frac{\partial}{\partial x}[(\sigma_y - \sigma_x) - (y'^3 + 3y')\tau_{xy}] + [\tau_s + 3y'(\sigma_y - \sigma_x)/(1 + y'^2) + 3(1 - y'^2)\tau_{xy}/(1 + y'^2)]y'' = 0. \tag{5}$$

Under a known stress field $\sigma_x = \sigma_x(x, y)$, $\sigma_y = \sigma_y(x, y)$, $\tau_{xy} = \tau_{xy}(x, y)$ equation (5) may be solved and thus extremes of the functional (4), i.e., yield bands $y = y(x)$, $x \in [x_1, x_2]$ can be found. Inserting $y'(x)$ into equation

$$\int_{x_1}^{x_2} \tau_{nl}\sqrt{(1 + y'^2)}\, dx = \tau_s \int_{x_1}^{x_2} \sqrt{(1 + y'^2)}\, dx, \tag{6}$$

enables us to determine loads, corresponding the appearance of plastic deformations in the form of yield layer $y = y(x)$, $x \in [x_1, x_2]$.

Under loading in plane strain conditions of a body with a crack along mode I (Fig. 1) it is observed that plastic strains are localized in two straight line yield bands generating from the crack tip and symmetric to its plane. Let us clarify conditions of the appearance of the yield bands in terms of the proposed approach.

Let us assume planes $y(x) = k(x) + q$ to be the extremes of functional (4), i.e., to be solutions of equation (5). Extreme $y(x) = kx + q$ will cross the fixed point (x_0, y_0) when one of the two parameters will be fixed: either $k = \text{const}$ or $q = \text{const}$. Under fixed k or q in the extremes $y(x) = kx + q$ will form a field.

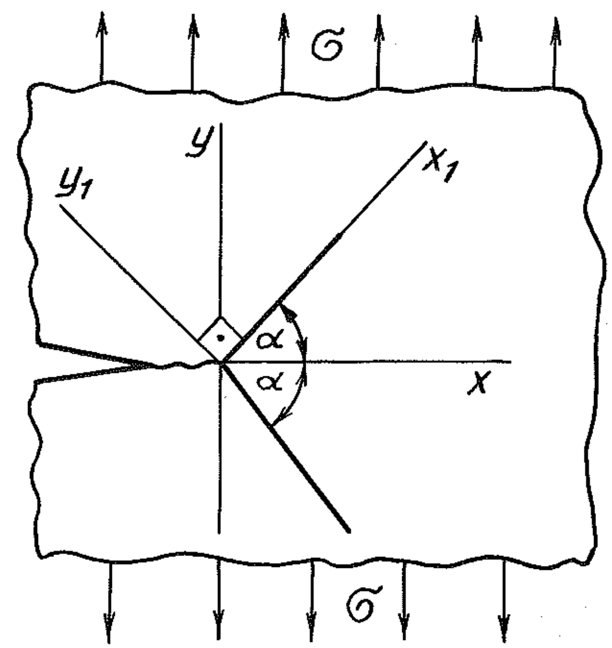

Fig 1 Characteristic view of the development of the yield band near crack tip under normal tension at plane deformation

Besides, the multiplier at y'' in equation (5) is equal to $\tau_s + 3\tau_{nl}$ and is positive, if $\tau_{nl} > -\tau_s/3$, along the whole assumed yield line. Thus the solution $y(x) = kx + q$ is correct for extremes, satisfying the strong minimum conditions **(1)**.

In order to obtain $y(x) = kx + q$ as a solution of equation (5) it is evident that satisfying the following equation is necessary and sufficient

$$\frac{\partial}{\partial y}[k^3(\sigma_y - \sigma_x) + (1 + 3k^2)\tau_{xy}] - \frac{\partial}{\partial x}[(\sigma_y - \sigma_x) - (k^3 + 3k)\tau_{xy}] = 0. \tag{7}$$

Let us represent the components of the stress tensor satisfying the differential equilibrium equations via a stress function $\phi(x, y) - \sigma_x = \partial^2\phi/\partial y^2$, $\sigma_y = \partial^2\phi/\partial x^2$, $\tau_{xy} = -\partial^2\phi/\partial x\, \partial y$ and transform equation (7) into

$$\frac{\partial^3\phi}{\partial x^3} + 3k\frac{\partial^3\phi}{\partial x^2\,\partial y} + 3k^2\frac{\partial^3\phi}{\partial x\,\partial y^2} + k^3\frac{\partial^3\phi}{\partial y^3} \equiv \left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^3\phi = 0. \tag{8}$$

The factor k may be taken equal to zero, since it is always possible to choose the coordinates so as to have the OX axis along the yield band. In this case equation (8) will acquire the form

$$\frac{\partial^3}{\partial x_1^3}\phi(x_1, y_1) = 0, \tag{9}$$

where new coordinates x, y are obtained by rotation of the initial coordinates by the angle $\alpha = \text{arc tg } k$ and by displacement of the orogon of the coordinates into the point $(0, q)$ due to the known expressions: $x_1 = x\cos\alpha + (y - q)\sin\alpha$, $y_1 = -x\sin\alpha + (y - q)\cos\alpha$.

Thus, the moment of the appearance of the yield band $(y = 0)$ on the OX axis is preceded by a stress state described by a stress function, which satisfies equation (9). It should be noted, that equation (9) was obtained using only the central extremes field ($q = \text{const}$). It is easy to see (inserting $k = 0$ into equation (7) and directing the yield band along the OX axis, i.e., $q = 0$) that under investigation of the extremes field ($k = \text{const}$) we also obtained equation (9).

The following function is the solution of the differential equation (9)

$$\phi(x_1, y_1) = C_1(y_1)x_1^2 + C_2(y_1)x_1 + C_3(y_1), \tag{10}$$

where $C_i(y_1)$ $(i = 1, 2, 3)$ are arbitrary, twice differentiable functions.

According to (10) the following will be the stress tensor components

$$\begin{aligned}\sigma_{x_1} &= \partial^2\phi/\partial y_1^2 = C_1''(y_1)x_1^2 + C_2''(y_1)x_1 + C_3''(y_1),\\ \sigma_{y_1} &= \partial^2\phi/\partial x_1^2 = 2C_1(y_1)\\ \tau_{x_1}y_1 &= -\partial^2\phi/\partial x_1\,\partial y_1 = -2C_1'(y_1)x_1 + C_2^1(y_1).\end{aligned} \tag{11}$$

Using the expressions (10) and (11) it is possible to formulate final conclusions on redistribution (relaxation) of stresses at the tip of the crack under normal opening (Fig. 1) when plastic deformations are localized in two yield bands, symmetrical to the crack plane. However, such analysis requires further assumptions: on the crack tip geometry, conditions on the yield bands etc. Realizing certain conventionality of the way expressions (10) and (11) were obtained, it would be unwise to base conclusions on pure assumption. Since stresses (11) do not correspond to the known solution of the elastic problem for a body containing a crack (Fig. 1), where the stress tensor components are presented via the stress intensity factor and contain singularity $1/\sqrt[4]{(x^2 + y^2)}$ we need an experimental evidence of stress redistribution (relaxation) at the crack tip.

Under loading of a body containing a crack (Fig. 1) local yield criterion is reached initially at the point directly near the crack tip and under the increase of loading (σ, Fig. 1) it expands over a certain small region. Since plastic deformation is always a result of shear-slip of some parts of the body relatively to others, then the neighbouring areas of the material, where the yield criterion is not reached, prevent the appearance and development of plastic deformations which should result in stress redistribution.

The plastic deformation role in fracture process, that is examined as the crack propagation process and the development of plastic deformation near stress concentrators, cracks in particular, is widely reported **(3)–(11)**. The classic theory of plasticity calculations by finite element method show that the elastic–plastic zone at the crack tip under normal opening at plane strain is a compact formation, in which maximum displacements make an angle $\alpha = 66°$ ... $82°$ with crack plane; the presentation of plastic zones like plastic bands, using this or that admissions gave the α values: $\alpha = 45°$(8); $\alpha = 57°$(9); $\alpha = 59°$(10); $\alpha = 63°30'37''$(11); for different ways of loading $\alpha \in [58°, 105°]$(5); for different plastic bands lengths, angles of plastic band propagation $\alpha \in [22{,}5°; 112{,}5°]$(6). Determination of the angles of plastic band propagation according (1) was made in (12), where, depending on the plastic band length–crack length relation (d_0/L) $\alpha \in [54°, 67°]$, for $d_0/L \in [0,3;\ 1,0]$ $\alpha \in [54°, 57°]$ and for $(d_0/L) \in [0,01;\ 0,3]$ $\alpha \in [57°, 67°]$. The calculations were made using the solution of the problem of uniaxial strain in plane strain conditions of the plate, containing elliptical cut of minor axis length $(2b)$ to major axis length (L) relation: 0, 01 and 0, 001. This results are in good accordance with **(3)–(10)** data.

Let us discuss the problem of stress relaxation at the crack tip. 'One of the main reasons of stress variations in resting medium, are the migrations of microscopic defects in crystal lattice, that are called dislocations. Such investigations cannot be made without careful study of its kinetics. But in practice, there are rather many cases where the attitude of Maxwell can be used, developing Poisson's ideas. In an isotropic Maxwell medium without any macroscopic displacement and without thermal inflow and withdrawal from the element of the medium, the stress state is changed in such a way as to cause shear stresses decrease' **(13)**. Maxwell's conception is easily understood when one tries mentally to put up the following experiment. Let us take a thin

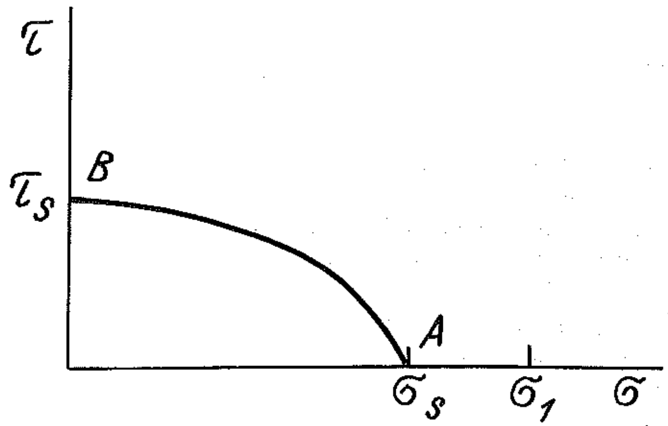

Fig 2 Under tension stresses higher than the yield stresses ($\sigma_1 > \sigma_s$), tangent stresses fully relaxed ($\tau = 0$)

walled tube made of ideally elastic–plastic material. Under torsion, plastic deformations will appear if $\tau = \tau_s$ (τ_s – yield strength under shear). Under simultaneous tension and torsion the moment of appearance of plastic deformations will be described by a certain convex curve connecting points A(σ_s, 0) and B(0, τ_s) (Fig. 2). As shown in Fig. 2 under $\sigma_1 > \sigma_s$ no force ($\tau = 0$) is required for torsion of the tube, which is evidence of complete relaxation of tangent stresses.

In real materials under uniaxial tension of homogeneous specimens it is impossible to obtain $\sigma > \sigma_s$. And if we take into consideration the fact that all real materials show deformational strengthening, it is impossible either to confirm or to disprove the conclusion about the complete tangent stress relaxation.

We believe we have been successful in obtaining this condition by testing specimens with soft interlayers. It is known **(2)**, that under uniaxial tension of cylindrical welded specimens with soft interlayers, limited by regions with a higher yield strength, plastic deformations in the interlayer appear if

$$\sigma \geqslant \sigma_S(\kappa) = \sigma_{SM}(1 + \kappa^2)/2\kappa, \qquad \kappa \leqslant 1, \tag{12}$$

where $\sigma = P/F$; P is the tension; $F = \pi r^2$ is the cross-section of the specimen; σ_{SM} – is the yield strength of interlayer; $\kappa = h/d$ is the relative thickness of interlayer (Fig. 3).

The specimens, 16 mm in diameter, were made of P6M5 steel ($\sigma_{ST} > 2000$ MPa) with interlayers of 20 steel ($\sigma_{SM} = 210$ MPa) of height $h = 9 \ldots 10$ mm (Fig. 3). In the performed experiment $\kappa = 0.56 \ldots 0.63$, consequently $\sigma_s(\kappa) = (1.1 \ldots 1.2)\sigma_{SM}$. The tension curve P vs Δh (Δh is interlayer height increase), was linear. Under $\sigma = 1.1\sigma_{SM}$ tension was removed and the specimen was subjected to torsion. We then found that twisting the specimen through an angle of $\phi \lesssim \pi/6$ (Fig. 3) required no force – the twisting component, measured with an accuracy of 0.5 Nm was equal to zero. For $\phi \gtrsim \pi/6$ some specimens

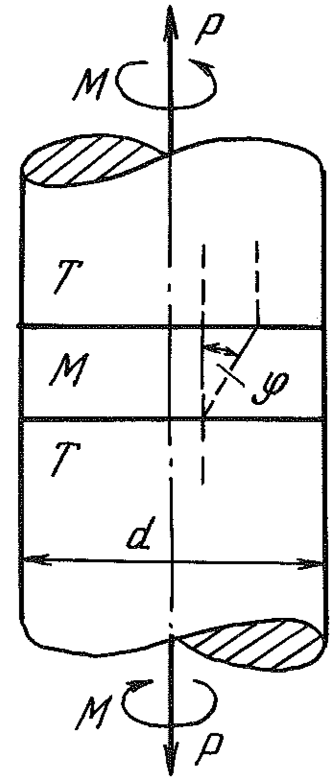

Fig 3 Tension and torsion of the sample with a soft interlayer

showed fracture by ductile shear in steel 20 close to the welded contact zone; specimens twisted by $\phi \gtrsim \pi/6$ which did not fail started resisting torsion. Evidently, deformational strengthening took place. If the twisting moment increased (under $\phi = \text{const}$), the tensile stress decreased (under $\Delta h = \text{const}$). The effect accompanying the tension and subsequent torsion process is difficult to explain. The principle conclusion, that tangent stresses are equal to zero under $\sigma > \sigma_s$ has been confirmed, i.e., under tensile stresses larger than the yield strength, tangent components are absent from it.

In our opinion, under $\sigma > \sigma_{SM}$ the stress state of the soft interlayer in the welded specimen (Fig. 3) corresponds to the stress state of the metal at the crack tip (Fig. 1) and applying the conclusion on complete relaxation of the tangent stresses ($\tau_{x_1y_1} = 0$, also $\tau_{xy} = 0$) to the expression (11) we obtain $C_1(y_1) = 0$, $C_2(y_1) = 0$ from which follows that $C_1(y_1) = a$, $C_2(y_1) = b$, a, b are constants. Consequently, the stress function (10) acquires the form of

$$\phi(x_1, y_1) = ax_1^2 + bx_1 + C_3(y_1). \tag{10a}$$

It should be noted, that bx_1 is neglected, since this component does not influence the stress state.

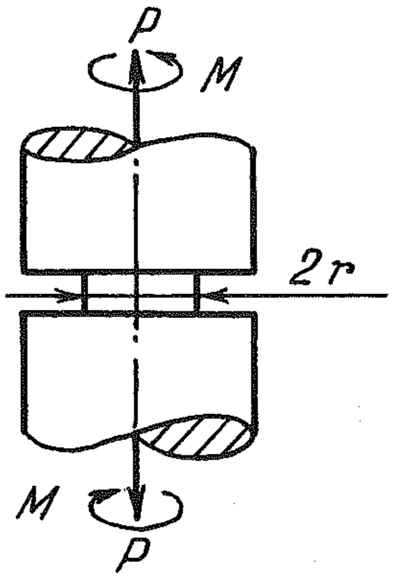

Fig 4 Sample of steel 20 with a groove, d = 20 nm, d′ = 10 nm, and h = 3–10 nm

Coming back to the original coordinates ($x_1 = lx + my$, $y_1 = -mx + ly$, $l = \cos\alpha$, $m = \sin\alpha$, Fig. 1) we obtain

$$\phi(x, y) = a(lx + my)^2 + C_3(-mx + ly). \tag{10b}$$

From condition $\tau_{xy} = 0$ follows $C_3(-mx + ly) = a(-mx + ly)^2$. Thus

$$\phi(x, y) = a(x^2 + y^2), \tag{13}$$

i.e. at the crack tip under normal tear, before the appearance of the yield bands, relaxation processes lead to the stress state of omnidirectional uniform tension. ($\sigma_x = \sigma_y = 2a$).

Conclusions

(1) We corroborated the known, but rarely used (equation (9)) conclusion, that resistance of metal to plastic displacement is a very capacious physical characteristic, reflecting some values under macrohomogeneous strain.

(2) When the metal flow criteria ($\tau = \tau_s$) is reached in the small region and there are neighbouring regions hindering the plastic deformation, where this criteria is not reached, the relaxation of shear stresses take place and that process can occur without any visible displacements in deformed medium.

(3) Testing of specimens with soft interlayers enlarges the possibilities of experimental investigations of metal fracture process. It was possible to reproduce the metal 'overstressed' state in volumes of sufficiently large dimensions, that are, supposedly, homogeneously stressed.

(4) It seems that the state of a uniform omnidirectional tension near the normal opening crack tip does not always take place. Under tensile and torsion of the steel 20 specimens with grooves (Fig. 4) the phenomenon of full relaxation of tangent (shear) stresses was not registrated under $\langle\sigma\rangle > \sigma_s$, where $\langle\sigma\rangle = P/F$, $F = \pi r^2$ is an area of the specimen in the groove region. But it is possible to believe that the increase of the rate of obtaining the stress state $\langle\sigma\rangle > \sigma_s$ may cause the fuller relaxation.

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