P. Agatonovic, H. Kordisch†, and E. Sommer†*

Justification of Dimensioning Criteria for a Rocket Motor Case using Fracture Mechanics

REFERENCE Agatonovic, P., Kordisch, H., and Sommer, E., **Justification of dimensioning criteria for a rocket motor case using fracture mechanics,** *Defect Assessment in Components – Fundamentals and Applications,* ESIS/EGF9 (Edited by J. G. Blauel and K.-H. Schwalbe), 1991, Mechanical Engineering Publications, London, pp. 1095–1109.

ABSTRACT For the purposes of quantifying the structural integrity criteria of the rocket motor (booster) case for Ariane 5, the yield and ultimate strength material properties are used. The presence of defects, however, may influence the structural response. Therefore the dimensional criteria based on safety margins referring to material strength properties i.e. yield and ultimate strengths, which neglect any possible defects, are not sufficient to guarantee the structural integrity of the booster. Additional considerations based on fracture mechanics analysis are necessary. In this paper the concept for the safety assessment of booster case based on fracture mechanics methods is presented. The concept is based on the given requirements and necessary preliminary assumptions and therefore must be verified. For its verification extensive experimental and numerical investigations are planned. A part of this programme has been already carried out. The results contribute to the better understanding of the relevant particular problems. These results show that the conventional procedures based on EPFM such as J_{Ic}, COD, the two criteria approach, etc. do not sufficiently cover all necessary requirements for the given case. For this reason, in addition to the present experimental investigation and numerical analysis, theoretical research capable of supporting the proposed concept is now required.

Introduction

Linear elastic fracture mechanics (LEFM) enables a quantitative description of the fracture initiation process to be made in terms of applied stress, defect size, and material toughness. However, it is limited to a regime in which section size and material properties combine to give failure at stresses well below the yield strength. This is insufficient in the case of the Ariane 5 booster. Particularly by minimizing the weight the possibility of a failure dominated by crack tip events has to be prevented and the full material carrying capacity (i.e., without defects) achieved. The resulting failure behaviour, if this requirement is fulfilled, should be controlled rupture with a ductile failure mode and no fragmentation of the cylinder. For the purposes of the analysis it is assumed that, as a result of the manufacturing and processing operations, cracks or flaws below the NDI (non-destructive inspection) detectability level always exist in the component after inspection. These cracks or flaws may create conditions for catastrophic failure under critical load conditions during initial loading or after some incubation period with corresponding crack extension under environmental and load conditions of the 'life envelope' of the structure. Consequently, the requirements concerning prevention of failure caused by defects

---

* MAN Technologie AG, FRG.
† Fraunhofer-Institut für Werkstoffmechanik, Freiburg, FRG.

concentrate on the specification of a 'sufficient' fracture toughness to prevent any reduction in material strength properties due to non-detectable defects. For cracks of this size, therefore, the demonstration of the component fracture load insensitivity based on elasto-plastic fracture mechanics (EPFM) analysis has to be performed.

For the treatment of the defect severity in the non-linear region at stress levels up to the plastic collapse limit a number of EPFM parameters and prediction methods have been proposed. They include J integral, COD, the two criteria approach, etc. The selection between these different methods and parameters and the reliable assessment of the defect severity varies according to the actual load conditions and is therefore quite a complex task. Because of this, fracture control is an important area of technology that demands close attention in the booster case development.

From the above discussion it becomes evident that for the assessment of booster cases current methods of fracture mechanics have a number of limitations and sources for inaccurate estimation of significant influences, acting both on the safe and the unsafe side. Characteristic examples are fracture resistance by the ductile mechanism of crack growth and the surface crack 'shortness'. Because of this, further experimental investigations of combined short crack and plasticity effects, supported by numerical analysis, have to be performed.

Concept for assessment of booster cases

Expected material and loading conditions in service

The concept for the safety assessment of booster cases is based on the following conditions and assumptions.

- Manufacturing procedure; the cases are manufactured by flow turning using the steel D6AC.
- Material data; the material is heat treated and has to achieve the guaranteed properties shown in Table 1.
- Geometry of a booster segment; cylinder with internal diameter $D = 3000$ mm, length $h = 3600$ mm, and wall thickness $t = 8$ mm.
- Loading situation; although during the service life of the component time-dependent loading and environmental effects cannot be completely excluded, in this paper as a first approach only the quasi-static loading conditions will be considered.

Table 1 Material Properties

Strength properties			
Yield $R_{p0.2}$ (*MPa*)	*Ultimate* R_m (*MPa*)	*Elongation* A_5 (%)	*Fracture toughness* K_{Ic} ($MPam^{1/2}$)
1300	1500	9	78

To improve fracture control, each booster segment shall be subjected to an overpressure proof test. The aim of this test is to guarantee, with sufficient probability, that the structure will hold its functional capability during the envelope lifetime and to check that there is no dangerous drift in the manufacturing cycle. It is well known that this kind of test demonstrates the absence of a defect of a critical size with greater certainty than other methods. This certainty is, however, justified only if the applied overpressure is sufficient to cover the different influences tending to reduce the safety margin achieved during this test. The beneficial effects of proof testing, aside from the pure integrity demonstration, are the mechanical stress relief and crack growth retardation in the presence of non-critical defects. However, this margin can also change or disappear with time. Under environmental conditions ageing will occur causing embrittlement of the pre-stressed material at the crack tip. Combining this effect with the eventual crack extension by tearing during the proof test, the above mentioned beneficial effects of proof testing may disappear. Then the resulting situation can be even worse, and the fatigue crack growth rate can effectiely be accelerated by the proof test if a large amount of ductile crack extension takes place. During the proof test the resulting circumferential stress in the component is close to the yield strength.

- Defect situation; An important objective of fracture control in case of the Ariane 5 booster is to establish an adequate NDI procedure. Figure 1 shows e.g. surface cracks with different aspect ratios and nearly the same criticality (stress at fracture) relying on LEFM methods. It can be seen that for aspect

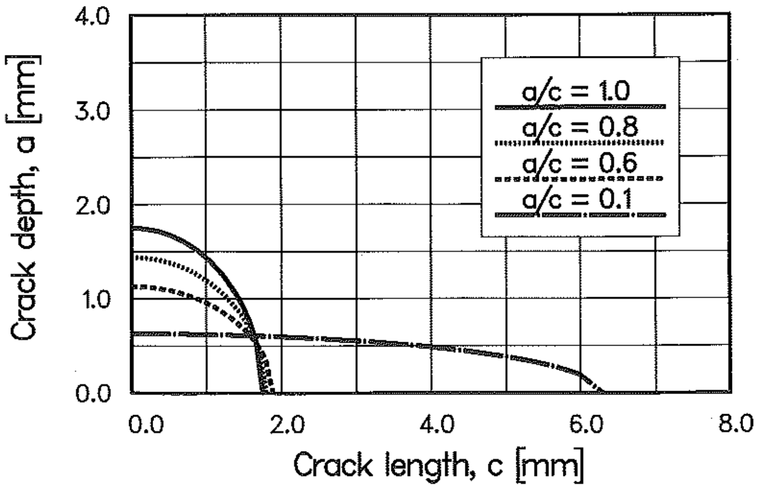

Fig 1 Critical crack profile for K_{Ic} = 80 MPa.m$^{1/2}$

ratios between 0.6 and 1 the crack depth 'a' can significantly reduce, in spite of maintaining nearly the same length. This important feature has to be considered in the selection of the sensitivity of the NDI method. Following typical assessment procedures based on fracture mechanics methods, any defect will be described by a similar but more dangerous crack configuration. As the most dangerous situation surface cracks of different depth to width ratios (a/c) under mode I condition will be considered in the first step, although the investigation of oblique cracks under superposed normal and shear loading should not be completely excluded.

According to these conditions first calculations show that the assessment concept cannot be based on LEFM- and EPFM-methods only, since the resulting cracks to be treated are embedded in plastically deformed material and are very short in absolute and relative terms when compared to the wall thickness.

Outline of the concept

First goal: definition of the size of tolerable defects

In order to define the size of tolerable defects curves of the type: strength s as a function of crack depth a (parameter a/c) will be considered. In the fully plastic regime as the resulting strength a value between yield strength $R_{p0.2}$ and tensile strength R_m is to be expected. In the elastic regime according to LEFM the corresponding strength values are determined by the following equation

$$\sigma = \frac{K_{Ic}}{f(\text{geometry}) \cdot \sqrt{a}}$$

The validity of this curve is restricted to 'long' cracks only. The expected strength in the transition region between plastic and elastic behaviour is shown as dotted line in Fig. 2. Since the proof test conditions lead to a circumferential stress in the booster case, which is very close to the yield strength, the tolerable defect depth will be defined as the intersection of the dotted part of the curve with the flow stress. This definition of the tolerable defect depth still includes a dependence from the geometry-function f(geometry). In addition a correction for the 'short' crack behaviour is needed. Due to the crack geometry the defect depth varies between an upper bound value given for a nearly semi-circular crack ($a/c = 1$) and a lower bound value given for $a/c = 0$. The two corresponding complete curves of fracture stress versus crack depth are plotted in Fig. 2. Experimentally these curves can be established by carrying out tensile tests of plates containing cracks of the outlined dimensions as a reasonable approximation for the configuration of the component. The construction of these curves has to be performed on the basis of fracture mechanics concepts. As a supporting approach a strain criterion will be used. This concept proposed by Soete **(1)** for the evaluation of an 'acceptable defect' states that 'no fracture may initiate at the tip of the defect before the defect free zone reaches the yield stress'. This implies the achievement of gross section yield before fracture. Moreover, he states that the application of classical fracture mechanics concepts such as K_{Ic}, COD, J integral, R curve, etc., have no practical sense for these conditions. On the other hand, close qualitative agreement with EPFM exists with Soete's concept since the four regimes of elastic, contained, net section, and gross section yield, that he firstly identifies, can be also established in J based theory **(2)**. The Soete approach underlines, however, the importance of a good overall elongation capacity, which can be crucial if defects appear in the peak stress areas. The presence of defects under these conditions can be even more severe, giving the possibility of a reduction in the actual safety margin against catastrophic failure, for example due to an inability of the material to reduce local stress raisers by plastic accommodation, and the development of brittle failure at section stresses well below the nominal material strength values.

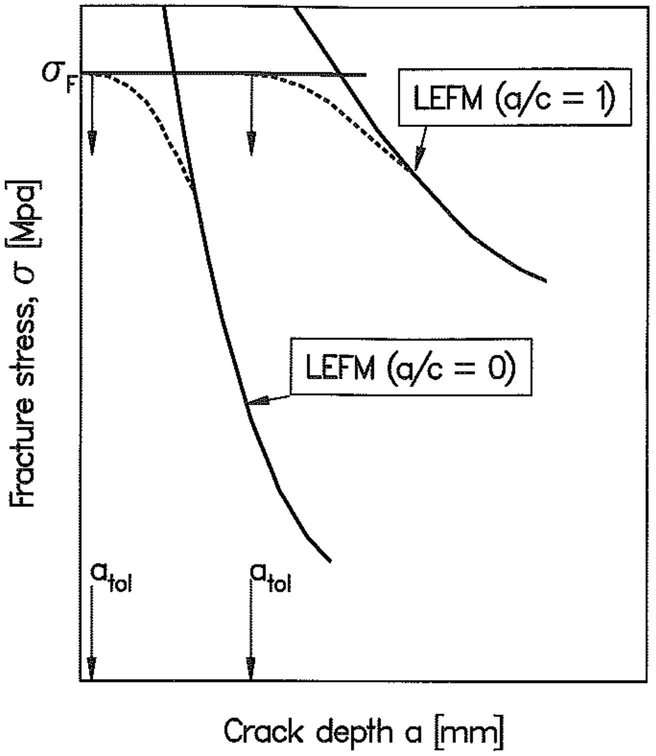

Fig 2 Fracture stress dependence on crack depth

As explained in Fig. 3, the global fracture strain of the surface crack tension (SCT) specimen containing a crack is plotted in dependence of the corresponding crack depth. For small cracks this curve is expected to be approximately

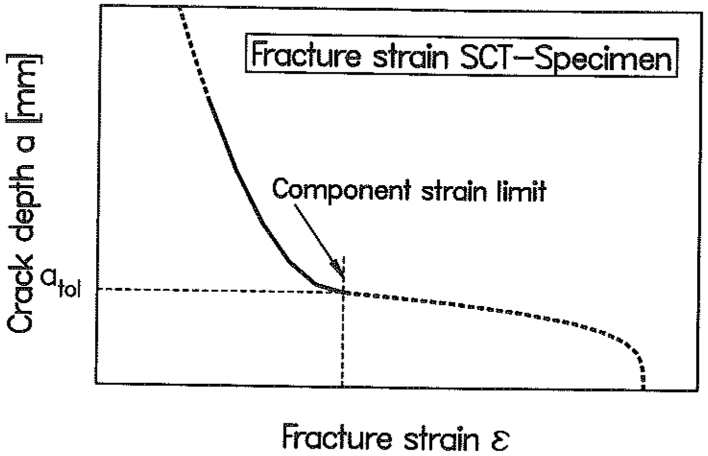

Fig 3 Fracture strain dependence on crack depth

insensitive to variations in the crack geometry (*a*/*c*). It can be assumed that in the component under service strain values greater than a certain limiting value can be excluded. As a consequence tolerable defect depths can be evaluated and related to those obtained by the first approach.

Second goal: material characterisation
Since the assessment concept is based on the assumption that booster cases may – beyond the limits determined by NDI techniques – contain defects and cracks randomly distributed with respect to their size, location, and orientation, a defined fracture resistance of the component has to be guaranteed in order to prevent fracture under all circumstances. Therefore an extensive material characterisation program is needed which accounts for the specific conditions. Its foundation is twofold.

- Due to the fact that only material limited with regard to the quantity and the wall-thickness of the booster case is available and that the through-the-thickness-direction given by axial cracks is most severe, only small specimens should be investigated. Therefore, specimens of the three-point-bending type seem to be most suitable **(3)**. In order to meet the actual requirements the depth of the cracks has to be varied from 'long' to 'short' with sharp and blunt notch tips. Since tests of this kind can be performed with low effort, another advantage is that information on the scatter of material parameters easily can be obtained.
- In addition to the investigations described before: the tensile tests of plates containing surface cracks with varying dimensions will serve to characterise the material condition as well.

In order to ensure the transferability of results obtained for the different types of test specimens to components numerical calculations of the non-linear stress–strain behaviour in the various process zones have to be performed and evaluated.

Planned investigations

The material to be tested in this program is D6AC. The heat treatment conditions were selected to achieve minimum yield and ultimate strengths as given in Table 1. Typical measured values lie around 1495 and 1625 MPa for each with corresponding elongation 11 percent. The test equipments and specimens used in this study correspond to established standards and will not be described here. All tests are carried out at room temperature with the same lot of material in three different laboratories, so that methodical errors can be considered small.

(1) Fracture toughness measurements and fracture testing with long fatigue cracks on standard CT and three-point-bending specimens: concerning these investigations the question arises as to whether the fracture toughness values determined by standard methods using through-cracked specimens are transferable to the real structure with predominantly surface or embedded defects.
(2) Fracture testing with 'short' cracks (fatigued and blunt notches). The precracking procedure for 'short' cracks by fatigue is complicated due to allowable size and may be rationalised if blunted cracks are permissible. Futhermore, if the yielding can occur at a position of high stress concentration, in spite of a large extent of the plasticity the crack is still contained within a large elastic zone. This complex condition has to be examined for specimens and components.
(3) Fracture testing with SCT specimens: this kind of specimen gives the most relevant applied loading conditions compared to those experienced by the booster wall. These tests will be performed for varying crack sizes and specimen thicknesses to cover a large range of possible situations.
(4) Numerical investigations by FE analysis of three-point-bending specimens, of SCT-specimens and of a cylindrical booster case section containing a surface crack: The aim of these investigations is to find supporting arguments for the transferability of results – mainly by analysing the stress–strain behaviour of the corresponding process zones.

Preliminary results

Experimental results

At time of writing only a part of the planned program has been performed. Even the preliminary results, however, are found to contribute to the better understanding of the relevant problems and will be presented here for further discussion.

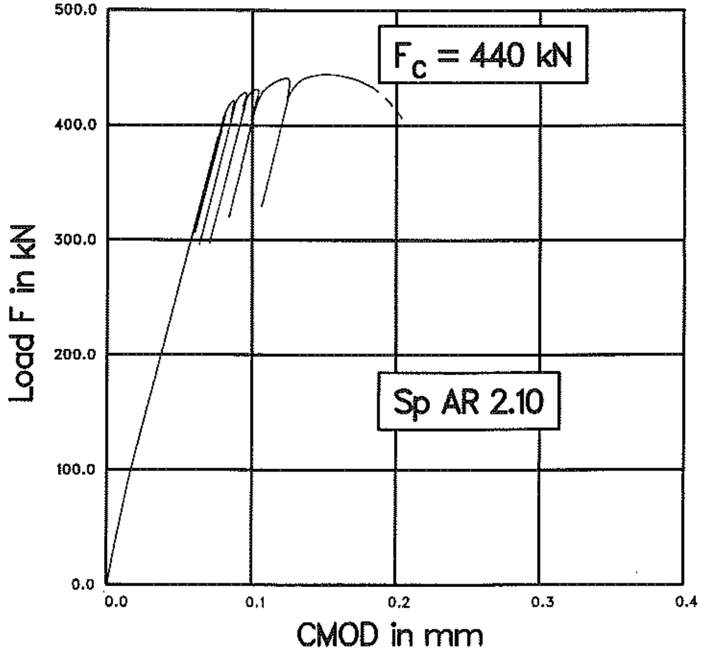

Fig 4 Load versus CMOD measurements

Material characterisation

All CT specimens gave sufficiently high values of K_{Ic} with a mean at about 100 MPa.m$^{1/2}$. Similar results were achieved with three-point-bend specimens with normal crack length **(4)**. Reduction of the crack size from 4 to 0.5 mm with this kind of specimen, however, produced further increase of K_r values (defined here as the stress intensity value at specimen fracture) in contrast to SCT specimens where the crack size reduction gave lower values. The K_r values are formally evaluated on the base of LEFM concepts though considerable plastic deformation has occurred.

Shallow cracks in small specimens hardly can be produced according to the requirements set by fracture mechanics standards. Therefore, specimens containing differently fabricated cracks and notches are investigated. The obtained K_r values are higher for the notched specimens except for specimens with 'short' cracks and notches, showing only small differences. The crack or notch effect appears too low to influence markedly the failure load in the fully plastic regime.

SCT tests

The SCT specimen crack sizes are varied from 1 to 7 mm in depth **(5)(6)**. For all the tests stable crack growth, based on compliance measurements could not be observed also after increased partial unloading. For this purpose CMOD measurements were performed using a MTS 377 extensometer attached at a distance of 1.55 mm from the crack. A typical CMOD curve is shown in Fig. 4. There was some doubt as to whether the sensitivity of the applied measurement method was sufficient for reliable evaluation of the CMOD under this circumstances. For this reason additional measurements were performed by load interuption and 'heat tinting'. The plateau load after repeating the test increases however by 5 to 10 percent due to the ageing effect. Crack extension during first part of the test marked by tinting (Fig. 5) was below 0.1 mm. Also careful SEM investigations of the cracked surfaces were not able to define more precisely the amount of the crack extension before instable conditions prevail.

In Fig. 6 SCT specimen results are shown indicating the net section failure stress for different crack depth. For the values above 4 mm the results follow LEFM predictions giving K_r values slightly above the CT specimen results. Under these conditions predictions based on a standard K_{Ic} evaluation are conservative. At yield strength level the results bend down for crack depths below 3 mm increasing the failure stress with more or less constant slope up to the ultimate strength at crack depth of about 1.2 mm. Between the 8 to 10 mm thickness variation there are no significant differences. The results with 6 mm plate thickness are, however, slightly lower. Only two results to date with a different aspect ratio ($a/c = 0.7$) show higher stress values than the others. The results are, however, in contradiction to the theoretically expected in Fig. 1, which estimates this aspect ratio to be more critical than $a/c = 1$.

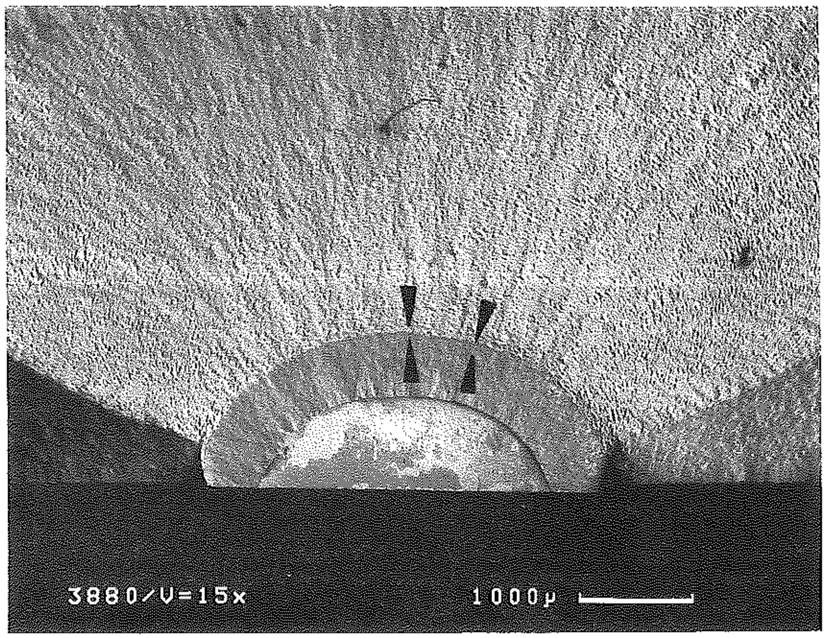

Fig 5 Crack extension during the first part of the test

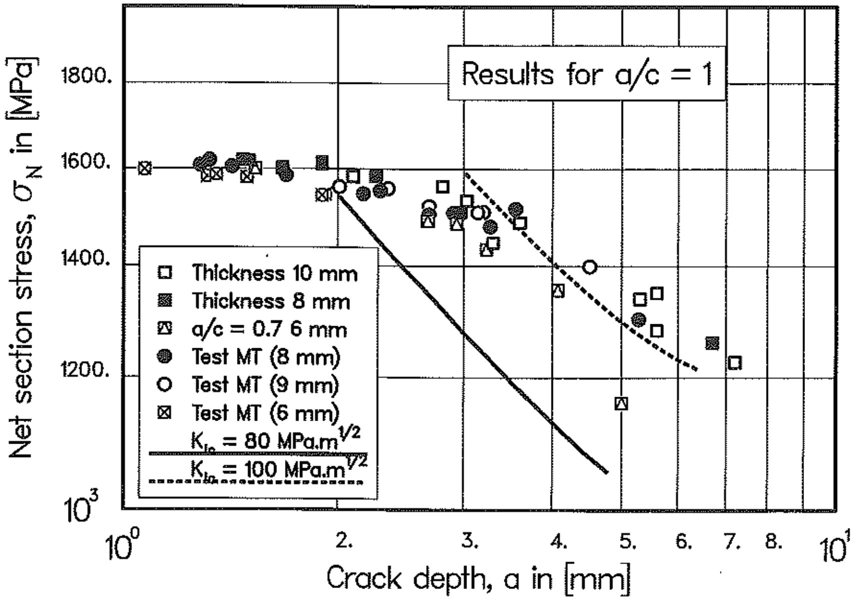

Fig 6 Fracture stress of SCT specimens

For one series of specimens of 6 and 8 mm thickness, strain measurements were performed. For this purpose three strain gauges were placed on the back of each specimen in the plane of the crack, at centre, edge, and intermediate positions. All strain gauges gave within the measurements accuracy limits identical strain measurements. The results are presented in Fig. 7 and show fracture strain dependence on crack size. The measurements were, however, limited to strain values of 2 percent. This level is exceeded for crack depths below about 1.2 mm. The coincidence with the results in Fig. 6 is obvious. It is interesting that the value of 2 percent strain is also recommended by Soete **(1)** for the experimental evaluation of an 'acceptable defect'. A crack depth of 1.2 mm is, however, very small and the given conditions may be too rigorous. For a strain value of 1 percent, for example, the 'acceptable' crack size increases above 2 mm.

Numerical results

Finally, the results of the first numerical calculations are shown. In this case a plate with a surface crack of aspect ratio $a/c = 0.7$ and crack depth 1.5 mm has been analysed. To achieve the $1/r$ singularity the crack front is modelled using collapsed volume elements. The mesh used is shown in Fig. 8. All calculations were performed using the non-linear FE MARC program. The J integral distribution at crack tip was evaluated using the virtual crack extension method. Figure 9 shows that at lower load levels the distribution is nearly uniform, as predicted by theory (based on the surface crack solution of Neuman/Raju). An increase in load level produces an uneven J integral distribution at the crack front due to the plasticity effect, so that above yield strength the difference between the apex and the surface position becomes more than 50 percent. These results may explain the above experimental results with specimen with aspect ratio $a/c = 0.7$. The results for remote stress conditions 15 MPa below and above yield strength level are shown in Figs 10 and 11. It can be seen that before reaching the yield stress level the yielded zone is still surrounded by an 'elastic' strain zone which corresponds equivalent stress below the yield stress or an inelastic strain less than 0.2 percent. Therefore, the condition for net section yield does not appear and plasticity conditions are immediately followed by gross section yielding, implying net section work-hardening and high ductility, by inelastic strain development throughout a specimen. These conditions can, however, be only achieved if crack extension by tearing does not begin during the application of the load, a feature this numerical model does not include.

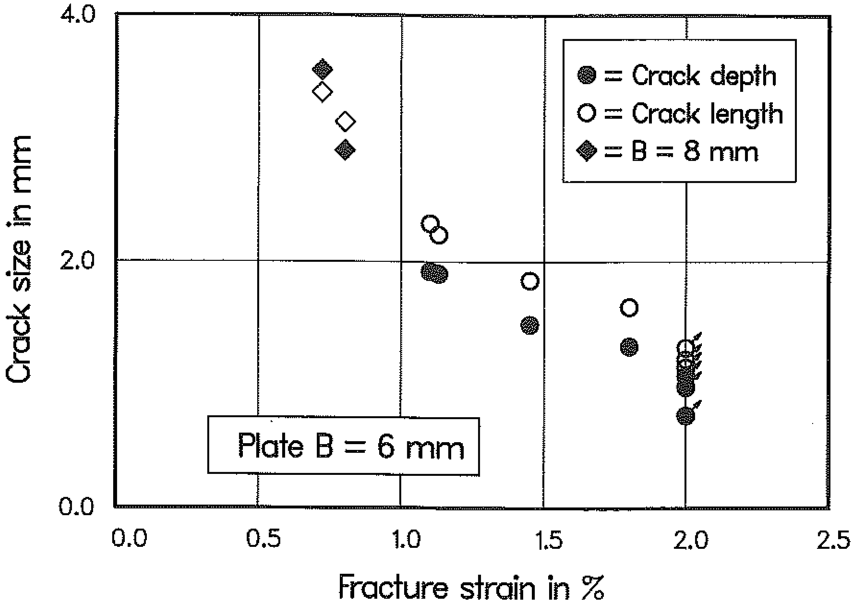

Fig 7 Fracture strain of SCT specimens

Discussion of results and conclusions

The results of this study clearly show that the currently applied assessment procedures based on EPFM do not sufficiently consider the importance of maintaining the overall plastic properties of the material under some conditions. Redcution of the crack size allows stress increase and general yielding conditions in the critical crack containing section of the component. Under such conditions the plastic zone extends from the crack tip to the opposite

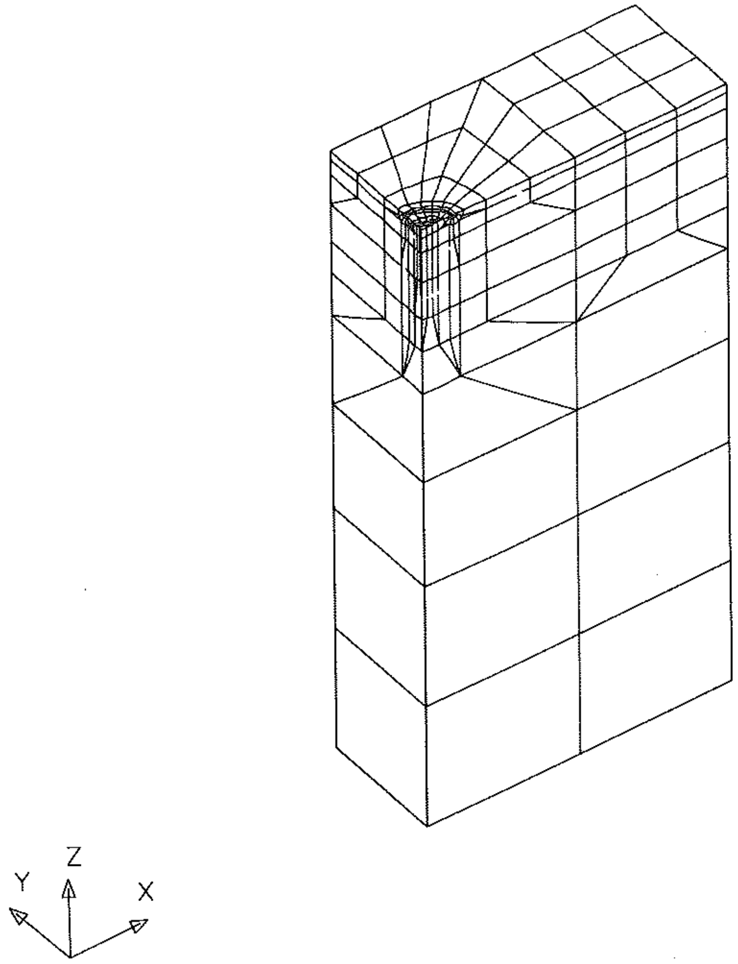

Fig 8 The FE mesh for numerical analysis of a surface crack

edge ahead of the crack, and is thus uncontained. There are two types of general yielding: net and gross section yielding. In net section yielding, extensive plasticity is limited to the usual deformation bands that emanate from the crack tip. Under these conditions, the crack still dominates the plastic flow pattern. Therefore, a limited fracture strain and correspondingly low ductility is produced. If these conditions dominate, the structure's failure mode is still brittle. This situation has to be avoided if the booster structure is to be considered reliable. For very short cracks, of about 1 mm, gross section yielding occurs before net section yielding can develop. Since the applied stress is greater than the material yield strength, extensive plasticity originates over the full length of the specimen, both far from the crack as well as across the crack

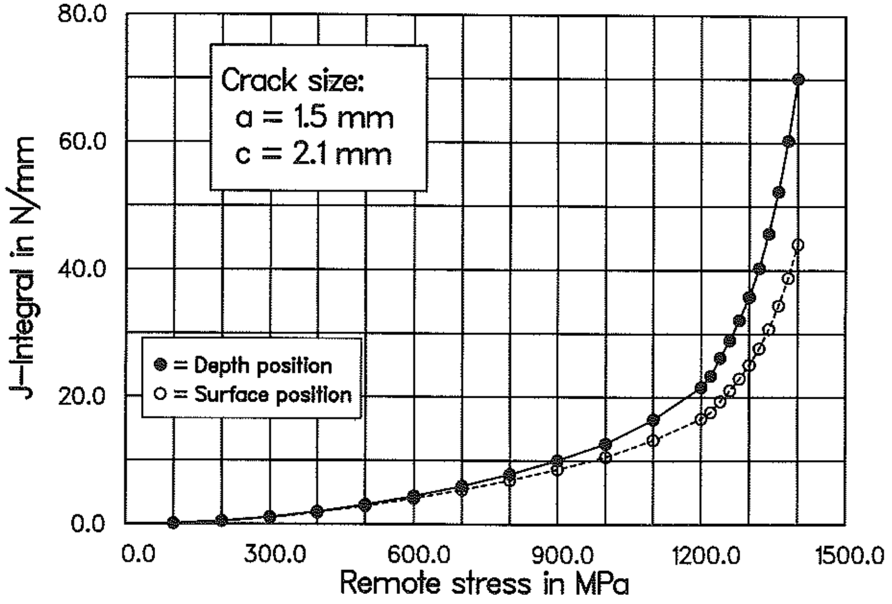

Fig 9 *J* integral distribution for different load levels

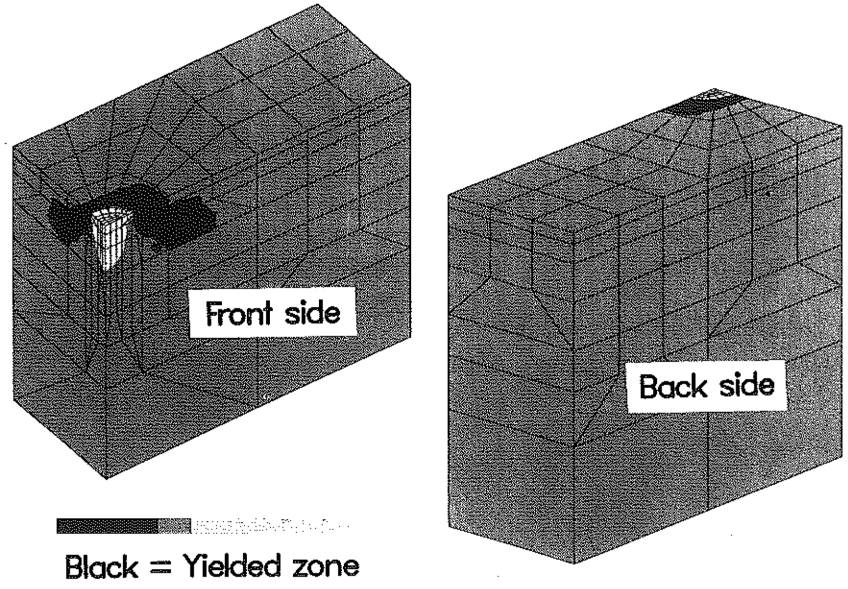

Fig 10 The yielded zone for a nominal stress approaching the yield stress level

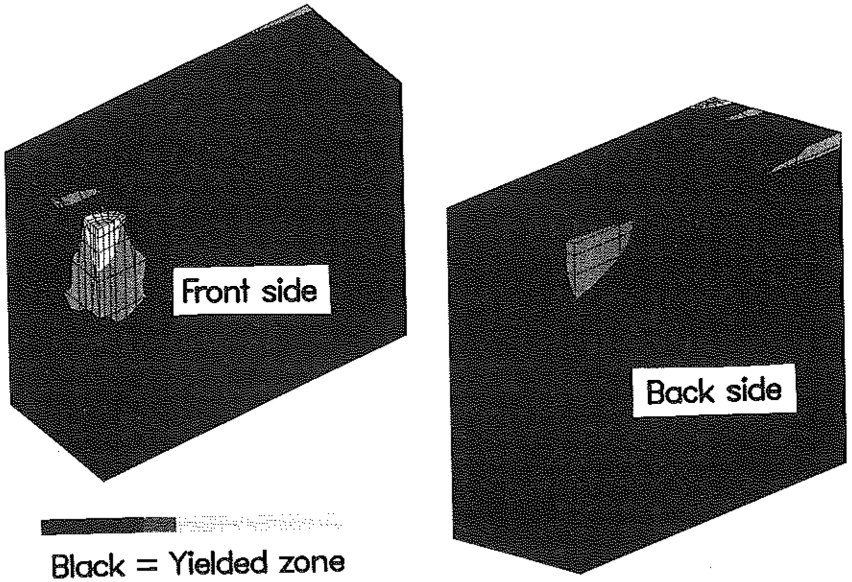

Fig 11 The yielded zone for a nominal stress exceeding the yield stress level

containing section itself. Clearly, from the viewpoint of achieving maximum structural safety this condition is desirable **(7)–(9)**.

In the transition region, failure may well be by plastic collapse, but microvoid coalescence (tearing) can also be the failure mode. This kind of ductile instability usually occurs after extensive stable crack extension depending on the energy of loading system, with the applied load increasing as the crack extends. Consequently, the crack growth can also be stopped if the rate of increase in the applied crack growth driving force is lower than that of the material resistance. Thus, stable crack extension can also provide useful structure resistance beyond the J_{Ic} point. Because of uncertainties concerning the prediction of ductile instability, however, the use of tearing initiation is very often recommended as a conservative failure criterion. The corresponding restriction can result in excessive conservatism when applied to the prediction of the behaviour of an actual structure. Using this criterion the relevant failure mode would not be really considered.

An understanding of this transition region in which either mode between linear elastic and fully ductile fracture is possible is essential for the assessment of small defects such as surface cracks. In this respect the method of Soete makes an important contribution (see Fig. 7). This concept should be, however, treated in terms of the J integral and for the present case the corresponding solution has to be developed. The advantage of the J integral approach over the purely empirical Soete approach would be to offer continuity with the well-defined theoretical fracture mechanics background.

Summarising the findings of the preliminary test results and theoretical investigation, it would appear that a defect size of 2 mm is required to cause fracture at a nominal stress in excess of the yield strength for the given test and material conditions. The results of these first investigations have to be compared with the lower bound values to be obtained from tensile and bending specimens with short cracks with an a/c ratio approaching zero, in order to obtain reliable data for the assessment procedure.

Generally, the results in this study raise many questions regarding the role of the crack aspect ratio, the remote fracture strain, possible crack extension, relevance of the data to the actual structure etc. Therefore, the problem is still far from resolved and further experimental and theoretical investigations should follow. It should also be recalled that the use of any single criterion and/or evaluation procedure must in this case consider the effects of the dimensioning life and the proof test conditions.

References

(1) SOETE, W. (1977) An experimental approach to fracture initiation in structural steels, *Advances in Research on the Strength and Fracture of Materials (ICF 4)* (Edited by Taplin), Pergamon Press, Oxford, vol. 1, pp. 775–804.
(2) TURNER, C. E. (1979) Methods for post-yield fracture safety assessment, *Post-Yield Fracture Mechanics*, (Edited by D. G. H. Latzko) Applied Science Publishers, London, pp. 23–210.
(3) MUSCATI, A. and TURNER, C. E. (1976) Post-yield fracture behaviour of shallow-notched alloy steel bars in three point bending, *Fracture Mechanics in Engineering Practice*, (Edited by P. Stanley) Applied Science Publishers, London, pp. 289–304.
(4) VOSS, B. and GÖRTZEN, M. (1988) Untersuchung der Bruchzähigkeit von 48 CrMoNiV 4 10 (D6AC) mit Proben mit Kurzen Rissen, IWM-Bericht V 54/88, Interim Report.
(5) OBERPARLEITER, W., Untersuchung der Bruchzähigkeit von Proben mit Oberflächenrissen, Interim Report IABG TF-2522.
(6) FROMMANN, A. and GRUNMACH, R., Fracture mechanics investigation on material 48 CrMoNiV 4 10 using SCT-specimens, MAN MT Interim report, MT-A-016.
(7) KING, R. B. *et al.* (1983) J-integral analysis of surface cracks in pipeline steel plates, elastic–plastic fracture: *Second Symposium, Inelastic Crack Analysis, ASTM STP 803*, (Edited by C. F. Shih and J. P. Gudas) ASTM, Philadelphia, vol. 1, pp. I-444–I-457.
(8) TOWERS, O. L. and GARWOOD, S. J. (1982) Fracture assessment in ductile tearing situation, *Advances in Frac. Mech. (ICF5)*, (Edited by D. Francois) Pergamon Press, Oxford, vol. 4, pp. 1731–1739.
(9) McHENRY, H. I. *et al.* (1981) Fitness-for-purpose research at the national bureau of standards, *Fitness for Purpose Validation of Welding Constructions*, (Edited by R. W. Nichols) The Welding Institute, Cambridge, Paper 19.