

Elastic-Plastic Handbook Solutions: Experimental Evaluation and User Guide

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ABSTRACT The Handbook of elastic-plastic solutions generated by EPRI and General Electric gives a method for calculating crack tip parameters and deformation behaviour used in elastic-plastic analysis. Although this Handbook covers many representative geometries and loading conditions and can be used by an engineer employing only a hand calculator, there is some difficulty in using it. This paper addresses two concerns with using the Handbook; first the problem of interpolating input variables, and second a need for experimental verification of results. Guidelines are given for the interpolation of three input variables; one is the crack length to width ratio, a/W , the second is the material hardening exponent, n , and the third is constraint, plane strain versus plane stress.

The method proposed in this paper bases the input upon a calibration specimen that models a geometry with a Handbook solution. The success of this approach is demonstrated for some A533B steel specimens of various geometries by predicting experimental results from Handbook solutions. Based on these results a series of guidelines are proposed for using the Handbook. These eliminate need for interpolation of a/W and constraint, requiring only linear interpolation for n , the hardening exponent.

Introduction

A ductile fracture methodology requires two separate inputs in order to predict the behaviour of structural components containing crack-like defects. The first is a set of calibration functions relating load (or stress), displacement (or strain) and defect size (1)(2). The second is fracture behaviour; an example of this is an R curve which relates a fracture parameter to ductile crack extension (3). The fracture behaviour can only be measured experimentally. The calibration curves can be developed experimentally or analytically. Usually experimental calibration curves require testing of model component sections; so this technique may not be suitable for many cases. Analytical development of calibration curves usually requires non-linear analysis and numerical methods are most suitable although frequently these solutions may require long and expensive computations. An alternative is to use tabulated calibration functions. Such a source exists for several common geometrical shapes required for ductile fracture analysis in the form of a Handbook of elastic-plastic solutions (4). This Handbook of elastic-plastic solutions, titled *An Engineering Approach for Elastic-Plastic Fracture Analysis*, generated by EPRI and General Electric contains solutions for crack tip parameters and calibration functions used in elastic-plastic analysis.

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Although this Handbook has solutions for many representative geometries and loading conditions and can be used by an engineer employing only a hand calculator, there is some difficulty in using it. The solutions are based on numerical results which use ideal material behaviour. Many engineers feel more comfortable with solutions which are verified by experiment and these Handbook solutions have little experimental verification. The solutions are given in a tabular format which requires some judgement in using. The elastic-plastic behaviour requires a number of input variables; the tabulated solutions are at discrete values of these inputs which usually do not match the input variables exactly and an interpolation or extrapolation must be made. When interpolations must be made simultaneously over several variables, no guidelines for doing this are given and the calculated parameters may be influenced by the methods of interpolation. Despite these problems, the Handbook is valuable to engineers working on ductile fracture problems because it represents the only source of tabulated calibration functions. Therefore it is important to try to solve these difficulties so that it can be used more easily.

This paper addresses these two problems with using the Handbook; first, that of providing guidelines for interpolating input variables and, second, an experimental verification of results. Guidelines are given for the interpolation of three input variables; one is the crack length to width ratio a/W , the second is the material hardening exponent, n , and the third is constraint, plane strain versus plane stress. Finally a set of guidelines for general Handbook use is given.

Handbook inputs

The Handbook of elastic-plastic calibration functions needs input of material properties and geometrical quantities (4). From these the displacement and a fracture parameter can be predicted as a function of load and crack length. The displacement is usually a load line or crack mouth displacement and the fracture parameter can be J or crack tip opening displacement, CTOD. Typically calibration functions can be written as

$$\begin{aligned} v &= v(P, a) \\ J &= J(P, a) \end{aligned} \quad (1)$$

These describe the solution capability of the Handbook where P is load, a is crack length, v is displacement, and J is the J integral fracture parameter (5)(6).

The inputs fall into the categories of geometry and material inputs. The geometry inputs include geometrical shape, crack position, and loading type. For example, single notched tension would describe this set of inputs. For the solution the correct table or graphs corresponding to both shape and loading mode must be chosen. Once the correct set of solutions is chosen further geometry inputs are needed including overall dimensions, height,

width, and thickness; and crack size in the form of crack length to width ratio, a/W . Finally the constraint of the structure must be selected based upon thickness and other geometrical features. This can be either plane strain (high constraint) or plane stress (low constraint).

The material inputs come from a tensile test of mechanical properties for the material. These would include a yield or flow stress, σ_0 , and values n and α which come from a Ramberg-Osgood material power hardening characterisation

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0} \right)^n \quad (2)$$

where $\varepsilon_0 = \sigma_0/E$; E is the elastic modulus. All Handbook solutions are based upon a non-linear stress-strain behaviour as given by equation (2). To develop the material properties a true stress-true strain tensile test must be conducted. This requirement is often a problem. Many laboratories do not have the instrumentation to develop the power hardening parameters n and α . Tests which have the instrumentation often demonstrate a hardening behaviour which does not follow equation (2) over a significant strain range.

Once all of the input values are gathered, solutions can be generated in the format of equations (1). Note that these equations do not describe the complete material behaviour. With four variables and two equations, one equation is missing; in this format the relationship between load, P , and crack length, a . This must come from an additional condition, usually the fracture toughness test result. Because the calibration functions cannot describe the complete material behaviour, it is usual to generate families of solutions for stationary values of crack length. For fixed crack length, load P , can be incremented and v and J determined for the complete range of P . Crack length is then changed and the procedure is repeated. This is done until families of curves covering the desired range of stationary crack lengths are generated. The solutions contain two additive parts, the first is a linear elastic part modified slightly by a plastic zone adjustment. The second part is the fully plastic part. The elastic part is essentially linear; however, at higher loads the plastic zone adjustment causes some non-linearity. The fully plastic part is non-linear and gives a power law type relationship between v and P . Because input of material stress-strain is a power law, the fully plastic solution is always a power law having the shape of $v = AP^n$ where A is a parameter incorporating all of the other inputs. As illustrated in the next section, this power law shape is not always appropriate.

Power law assumption

The first aspect of the Handbook solutions that can be evaluated experimentally is the assumption of power law stress-strain input. One problem in supplying this input is that standard tensile tests do not usually generate the

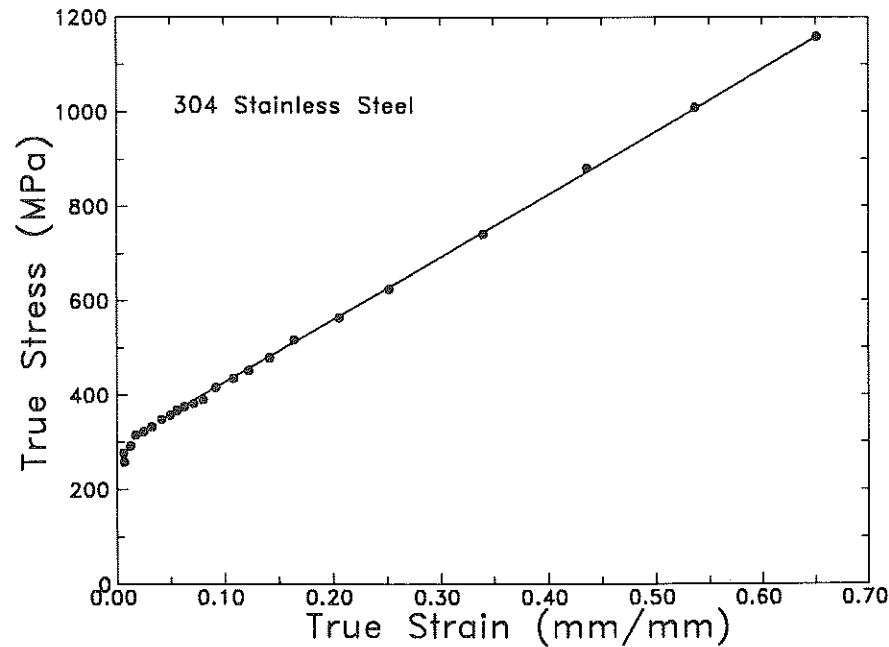


Fig 1 Stress-strain curve for type 304 stainless steel showing non power hardening behaviour

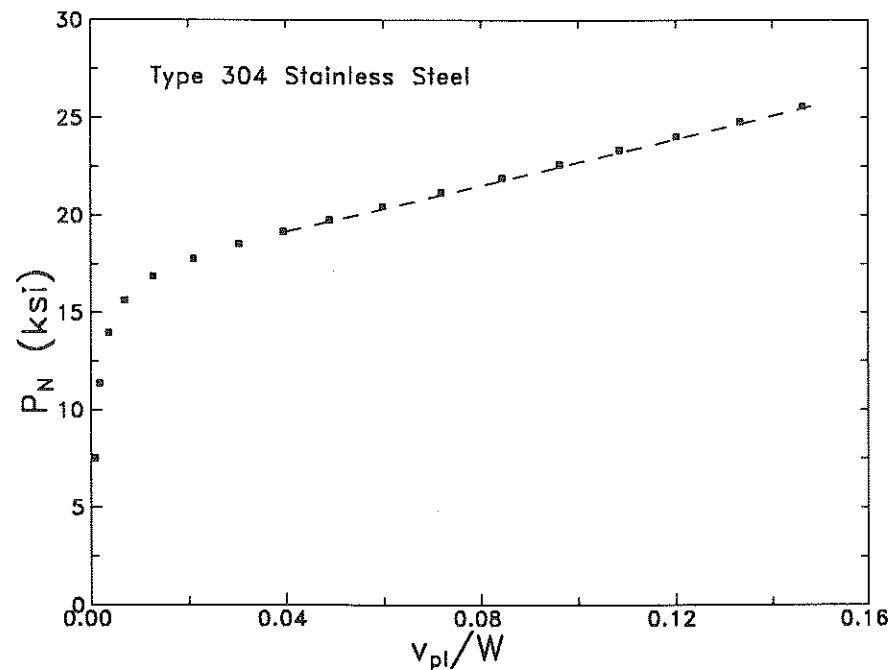


Fig 2 Normalised load versus plastic displacement for a 304 stainless steel showing behaviour similar to the stress-strain

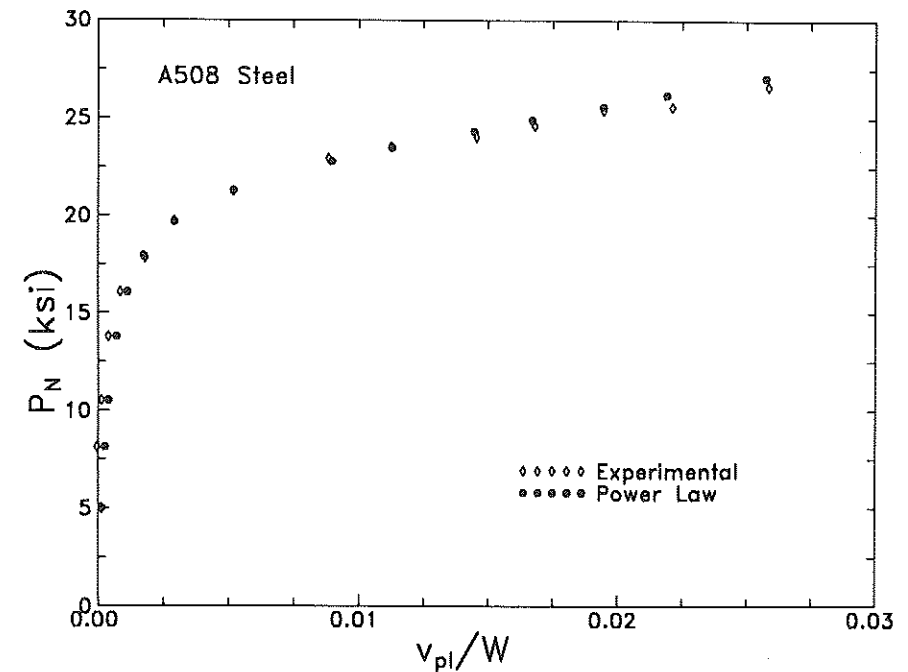


Fig 3 Normalised load versus plastic displacement for A508 steel showing power law behaviour

values of n and α needed. This can be solved by using an approximation suggested by Bloom and Malik which uses yield strength and ultimate tensile strength to estimate n and α (7).

Another problem with the power law assumption is that many materials do not exhibit a power law stress-strain behaviour. An example of this is given in Fig. 1 for a 304 stainless steel material (8). Here the true stress-true strain curve is not a power law. An alternative approach within the Handbook framework is to use several power laws added to get a combined result. The curve in Fig. 1 clearly is best fit by a power law for small strain and a straight line for larger strain. The experimental load and displacement curve developed for a compact specimen has a similar form, Fig. 2. Separating the curve into elastic and plastic parts, the plastic part has initially a power law region which quickly becomes a straight line for larger displacements. For this case, as in every case, the fully plastic Handbook solution would be a power law over the entire range of displacement. The conclusion from this example is that experimental load displacement curves generated for various structural shapes model the stress-strain behaviour for the material. The Handbook predicts only one type, that of the power hardening shape. Further examples illustrate that this is appropriate whenever the stress strain fits a power law. Figure 3 is normalised load versus plastic displacement for an A508 steel. This is an

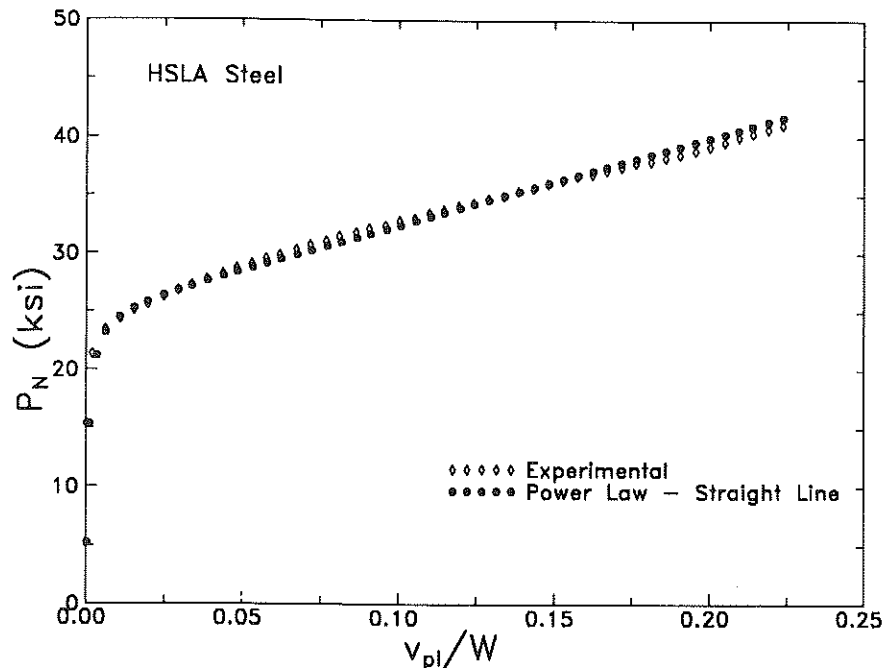
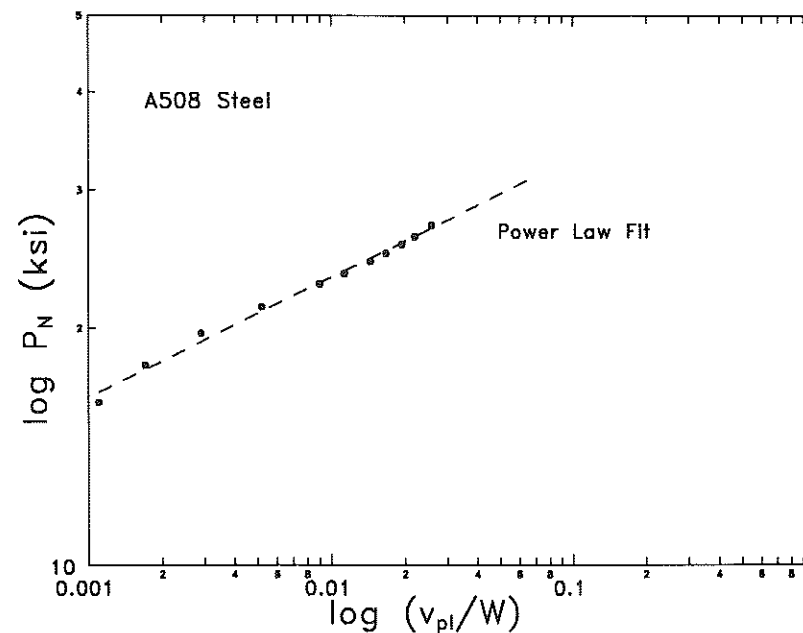


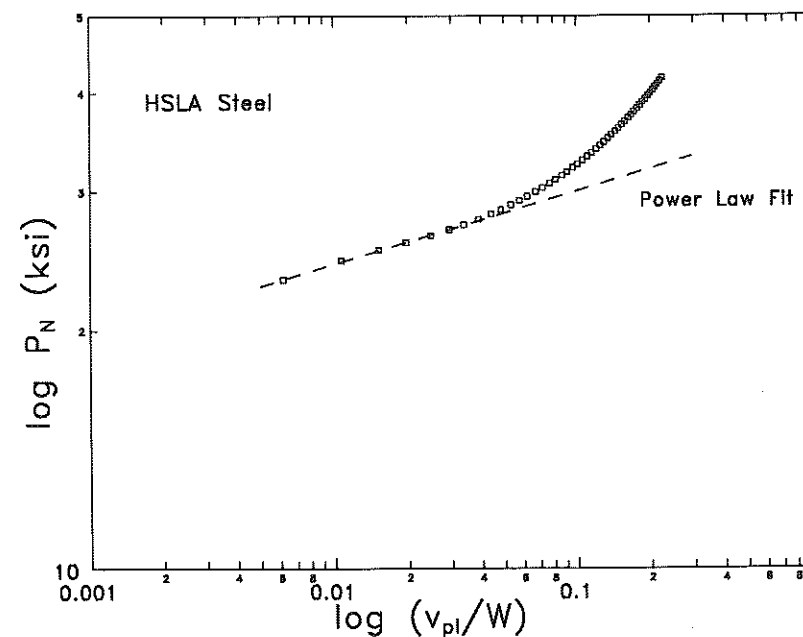
Fig 4 Normalised load versus plastic displacement for HSLA steel showing non power law behaviour

example where the experimental load displacement is a power law. Figure 4 is the same for an HSLA steel; it shows a power law plus a straight line in a way similar to Fig. 2. Whether or not a material follows a power law can be examined on a log-log plot of load versus plastic displacement. Figure 5 shows a log-log plot of load versus plastic displacement for the A508 steel and HSLA steel. The A508 steel data lie on a straight line, and are suitable for a power law representation. It is clear from Fig. 5 that a power law will not accurately model the HSLA steel over the entire range of displacement.

One problem then with using the Handbook solutions to predict deformation behaviour is that the correct shape for the plastic region may not be predicted by the Handbook. In general nearly all materials exhibit an initial power law behaviour followed by a linear load displacement curve at fixed values of crack length. Materials which are not very ductile may go through the failure process before reaching the linear portion; hence the Handbook solution may be completely appropriate. Materials which are very ductile develop the straight line behaviour before any fracture process begins and a power law Handbook solution is not appropriate. A method for determining whether or not the power law shape is appropriate will be suggested later; first the problem with using interpolation of inputs for the Handbook will be addressed.



(a)



(b)

Fig 5 Log-log plot of A508 steel and HSLA steel normalised load versus displacement

Interpolation of variables

As stated in a previous section, the Handbook solutions require a number of input variables. These include geometrical variables, material variables and a choice of two-dimensional constraint, plane strain or plane stress. The solutions are given in a tabular format at discrete values of these input variables. Interpolation is required whenever the inputs do not match the values in the tables. Three input variables frequently require interpolation; these are the crack length to structural width dimension, a/W ; the exponent, n in the power hardening law and the constraint. Interpolating over one of these input variables is not difficult; however, sometimes interpolation is required over three variables simultaneously. This is often a difficult task, especially since solutions do not necessarily vary linearly (or even monotonically) with these input variables. Therefore a scheme to aid with the interpolation is very useful. Methods for interpolating a/W and hardening exponent have already been suggested (2), these will be reviewed and revised here. Interpolating the constraint is more difficult but can be solved by considering an overall approach to the use of the Handbook.

Interpolation of a/W

The interpolation of a/W can be solved most easily by normalising the load with a geometry factor which makes the solution continuous in a/W , therefore essentially requiring no interpolation. The method for normalising the load with a geometry function comes from the work of Ernst *et al.* (9). This can be written in a general format as

$$P_N = \frac{P}{G\left(\frac{a}{W}\right)} \quad (3)$$

where P_N is normalised load and $G(a/W)$ is a function of crack length to width ratio which can be different for each specific geometry. For example

$$G(a/W) = BW(b/W)^2 e^{0.522 b/W} \quad (4)$$

has been a traditional form for the compact test specimen geometry, here $b = (W - a)$ is the uncracked ligament length.

Using the Handbook solution requires first that a value of P_N be developed as a function of another variable, for example, load line displacement. This value of P_N can be obtained at a tabulated calibration value of a/W usually the one closest to the a/W of interest. Once the P_N is determined from the Handbook it becomes the standard normalised load which then can be used to get load P at any a/W using equation (3). This solution is continuous hence a Handbook value of P can be determined at any a/W without interpolation; the function $G(a/W)$ does the interpolation.

The $G(a/W)$ value is known for several geometries. However when this is not known the Handbook itself can be used. A recent discussion of J calibration from load separation concepts is useful here (10). The load separation study suggests that $G(a/W)$ can be written in a form using $b = (W - a)$

$$G\left(\frac{b}{W}\right) = BW\left(\frac{b}{W}\right)^\eta \quad (5)$$

where η is the coefficient in the J solution

$$J = \frac{\eta}{Bb} \int_0^v P dv \quad (6)$$

The form in equation (5) holds when η is essentially constant with a/W . This was demonstrated to be the case for four different geometries (10).

Since the Handbook solutions give tabulated output for several values of a/W the separation technique can be used to generate the η values in this form for any geometry. This is illustrated for the compact specimen using the following steps. In Fig. 6 the load versus plastic displacement, v_{pl} , is generated for all tabulated crack lengths. Separation constants are calculated at fixed

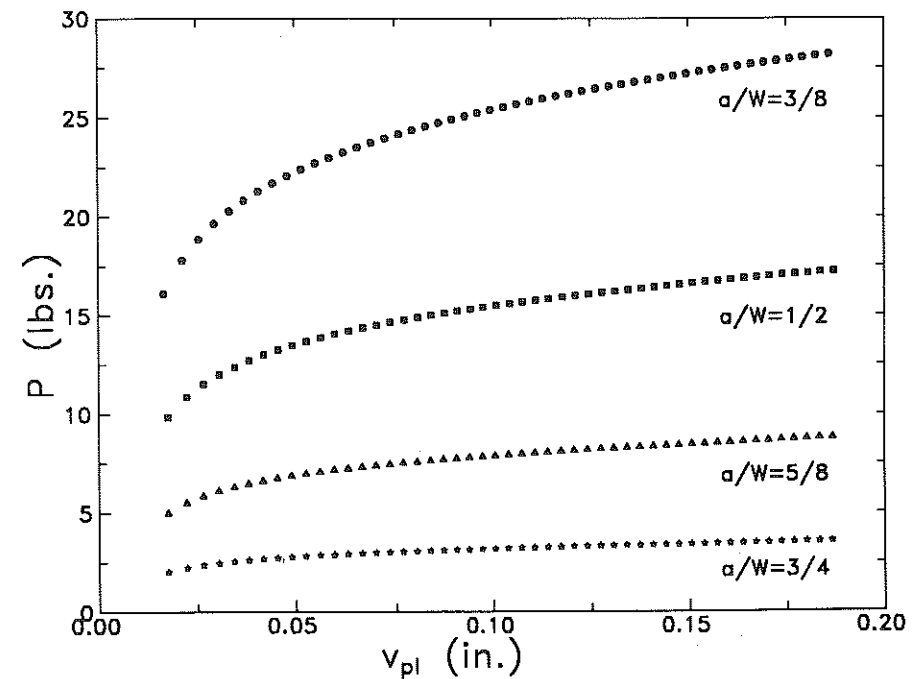


Fig 6 Load versus displacement for compact geometry Handbook solutions for all tabulated a/W values

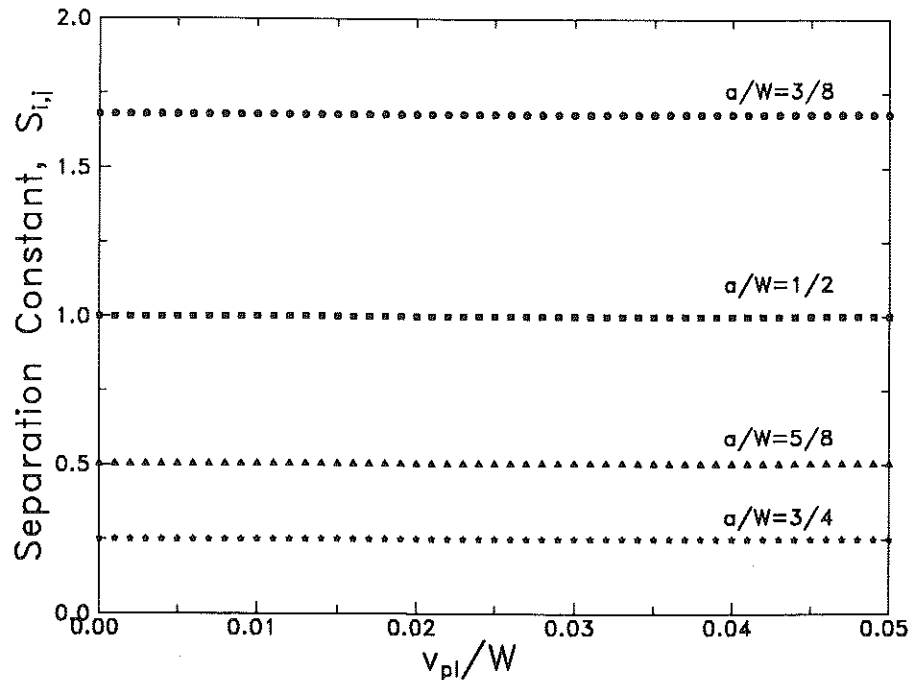


Fig 7 Separation constant S_{ij} versus plastic displacement for all tabulated a/W values

values of plastic displacement in the form

$$S_{ij} = \frac{P(a_i/W)}{P(a_j/W)} \quad (7)$$

S_{ij} is plotted as a function of v_{pl} , Fig. 7. If it does not vary as a function of v_{pl} , as is the case for the compact specimen, load separation exists. For the form of equation (5) to be correct the separation constants S_{ij} must be a power law function of b/W . Figure 8 shows S_{ij} versus b/W with a power law fit. (Empirically this fit can be done on a log-log plot.) The exponent of the fit is the value of η in equations (5) and (6). Since every geometry can be subjected to this analysis, the function $G(a/W)$ can be obtained from the Handbook solutions, hence the solutions become continuous in a/W and need no interpolation.

Interpolation of n

The second input which requires interpolation is the hardening exponent n . The suggestion made previously is that n does not really require an interpolation (2). The approach suggested was as follows. When n and α are not available from the material tensile test, generate them from the yield and ultimate tensile strengths using the Bloom analysis (7). Use the tabulated values

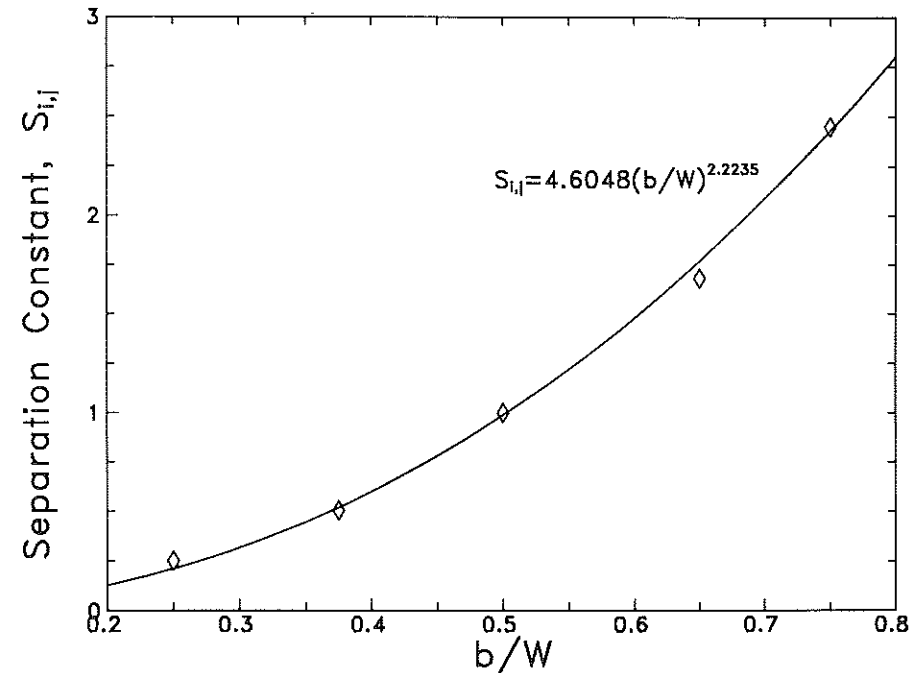


Fig 8 Separation constant S_{ij} versus b/W with power law fit

for n from the column that most closely fits the approximated n , without interpolation. This seemed adequate at the time because the value was likely to be an approximation which may only be correct within ± 20 percent or so. However, an experimental approach will be suggested later for which the value of n may be determined more accurately and a better approach can be suggested. As explained in the previous section, a/W no longer requires an interpolation. In the next section a suggestion will be made that constraint does not need interpolation. This leaves n as the only input parameter requiring interpolation. Hence the suggestion is made here that a linear interpolation be made between the tabulated n values. For most geometries the values of n are tabulated between 1 and 20. The value of 1 corresponds to elastic behaviour and is not non-linear plastic behaviour. The value of 20 is nearly perfectly plastic. For any value of n greater than 20 the behaviour is not likely to be predicted well by the Handbook and the suggestion here is not to extrapolate to determine a solution; rather only interpolations in n should be made when using the Handbook solutions.

Interpolation of constraint

The interpolation of constraint is more difficult. Two dimensional numerical solutions are usually determined either under plane strain or plane stress conditions. These solutions often bracket the experimental results but do not

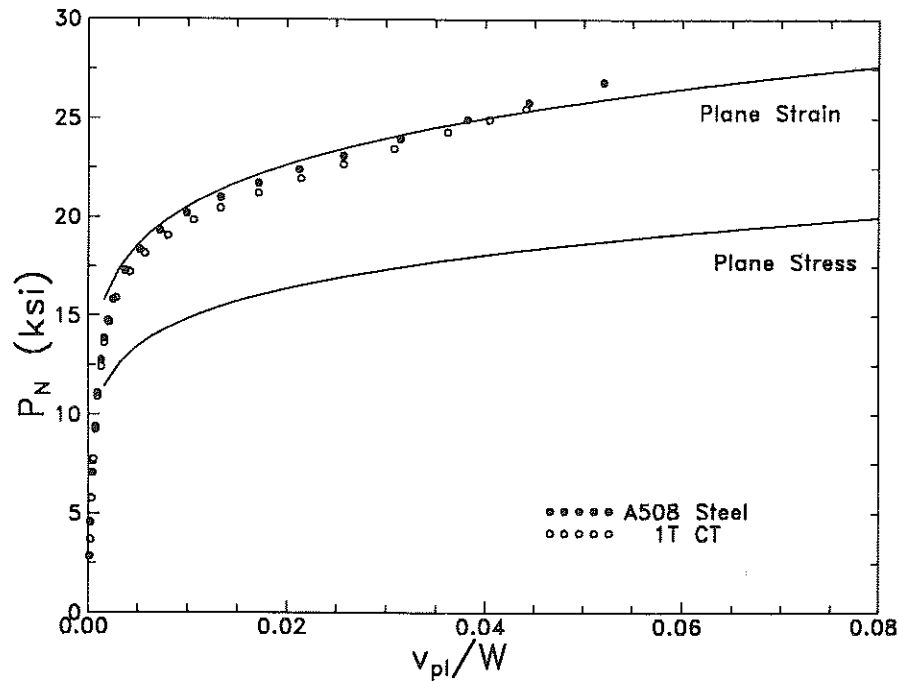


Fig 9 Normalised load versus plastic displacement comparing Handbook predictions with experimental results

accurately predict them (11). Therefore an interpolation appears to be necessary. However, there is no constraint indicator on a structural geometry to assist with such an interpolation. Rather than making an interpolation an alternate approach is suggested here which couples yield strength and constraint.

Figure 9 presents an example of a Handbook prediction for A508 steel where the plane strain solution predicts the experimental result very closely. This result is somewhat fortuitous because the yield strength was not precisely known and a value of 56 ksi was selected. The same experimental result is given in Fig. 10 where a range of yield stress values, σ_0 , were chosen for the plane strain Handbook prediction to compare them with the experimental result. This result shows that strength and constraint are coupled, a conclusion that can be used to avoid an interpolation of constraint.

An approach to dealing with constraint can then come from an experimental calibration specimen. This not only can be used to eliminate the interpolation of constraint but can be used to develop all of the material inputs needed, thus eliminating both a need for a true stress-true strain tensile test or an estimated value of n .

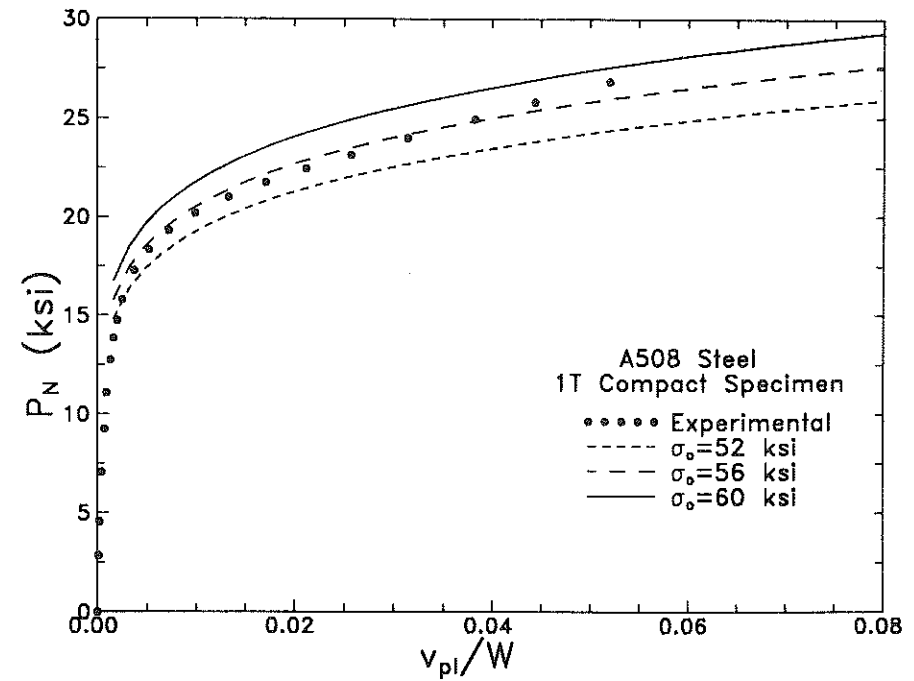


Fig 10 Normalised load versus plastic displacement looking at effect of yield strength on Handbook prediction

Experimental calibration specimen

Many problems involved in using the Handbook can be solved by using an experimental calibration specimen. This represents an additional step which may be perceived as a hardship, but it is not difficult for most laboratories which do fracture testing. In fact it could be easier than conducting a true stress-true strain tensile test; a test for which some laboratories do not have instrumentation. The result of the calibration test is a set of material properties which give more accurate input to the Handbook than an actual tensile test (8).

The calibration test must correspond to one of the geometries in the Handbook. For laboratories doing fracture testing the compact specimen or single edge notched bend geometry may be most convenient. What is needed is a plot of load versus displacement for a stationary value of crack length. This can be obtained by testing a blunt notched specimen which does not exhibit crack length change during the test. It is especially convenient to have the notch length correspond to a tabulated a/W value from the Handbook. Alternately a pre-cracked specimen with a crack length monitoring system can be used. Here the crack length values can be used to refer load to a value that would be obtained at a stationary crack length.

The load versus displacement data from the blunt notched calibration test can be used to evaluate all of the material input values. In addition the appropriateness of the power law stress-strain assumption can be evaluated. The following approach could be used. The displacement should be separated into elastic and plastic components. This can be done by taking the original loading slope as a measure of compliance, Fig. 11. The load can be plotted versus plastic displacement in a log-log format Fig. 12. If the power law is appropriate, a predominately straight line plot will result. From this the value of n can be determined from the slope of the plot. This determines n directly from an experimental result so the true stress-true strain tensile test is no longer required.

The constraint and yield or flow strength, σ_0 , can be directly determined. A constraint type, plane strain or plane stress, should be chosen which best fits the structural types being analysed. The calibration specimen can be made accordingly. For this constraint the Handbook solutions can be generated for various input values of σ_0 until the match is made. This value of σ_0 with the constraint chosen is then the appropriate one for all other geometries, namely the one representing the structural component which is to be predicted.

The Handbook requires other material properties besides n and σ_0 , namely α and ϵ_0 . Since equation (2) can be rewritten in the format

$$\epsilon = \frac{\sigma}{E} + k\sigma^n \quad (8)$$

where $k = \alpha\epsilon_0/\sigma_0^n$, the values of ϵ_0 and α can be evaluated in terms of the parameter k , rather than σ_0 , because this combination appears in all of the Handbook solution equations.

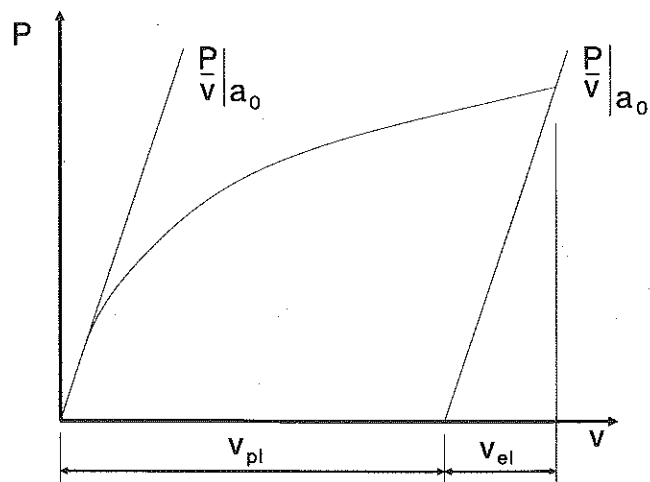


Fig 11 Schematic showing separation of displacement into elastic and plastic components

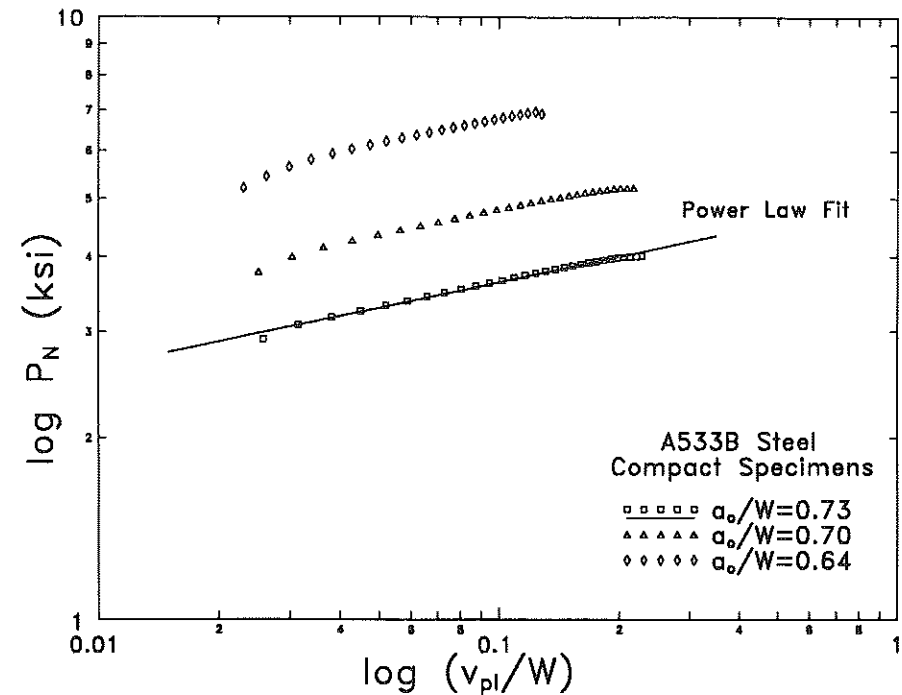


Fig 12 Log-log plot of blunt notched specimens load and plastic displacement showing power law definition

The method of using a blunt notched calibration specimen can be easier and more accurate than using a tensile test. In addition it may require less instrumentation than the true stress-true strain tensile test and the results are often more appropriate. Tensile results in the past have sometimes been found to be inaccurate for predicting material flow in other geometries. In particular the exponent n obtained from a true stress-true strain test does not represent the exponent of the power representing the load versus plastic displacement curve of a compact specimen. In addition σ_0 can be taken as yield strength, flow strength or even ultimate tensile strength. The σ_0 chosen from a tensile test usually gives solutions which are too high in load for plane strain and too low for plane stress. Matching constraint and σ_0 to a calibration test is more accurate. It appears that the component structure may choose its flow properties in a way not always represented well by a uniaxial tensile test. The success of the calibration test principle is demonstrated next.

Structural predictions

The principle presented in the previous section for solving the problem of interpolating constraint was applied to test results from four different geometries. These include a compact geometry (which was used for calibration),

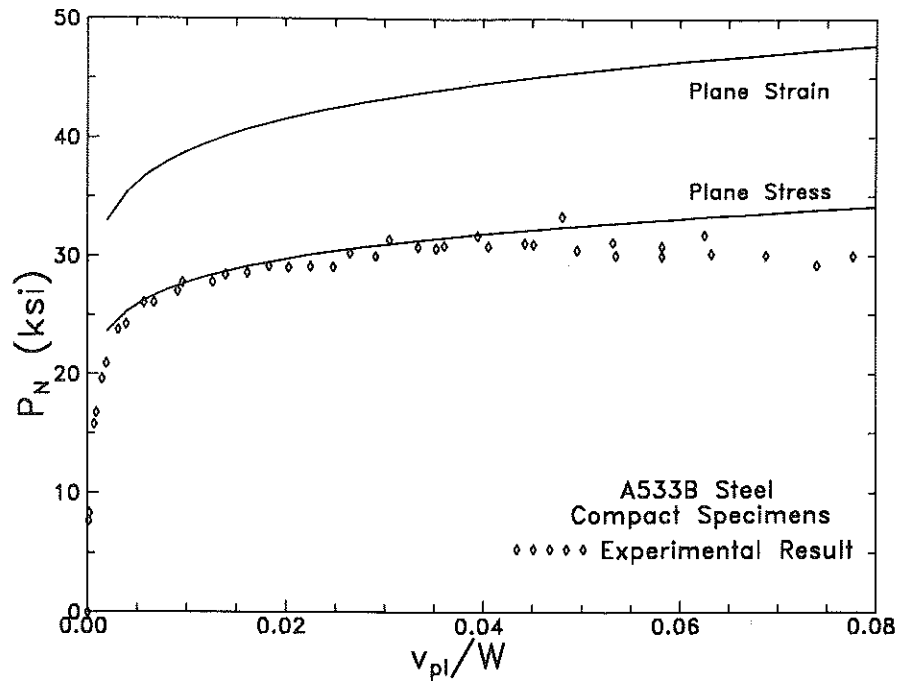


Fig 13 Normalised load versus plastic displacement for A533B compact specimen

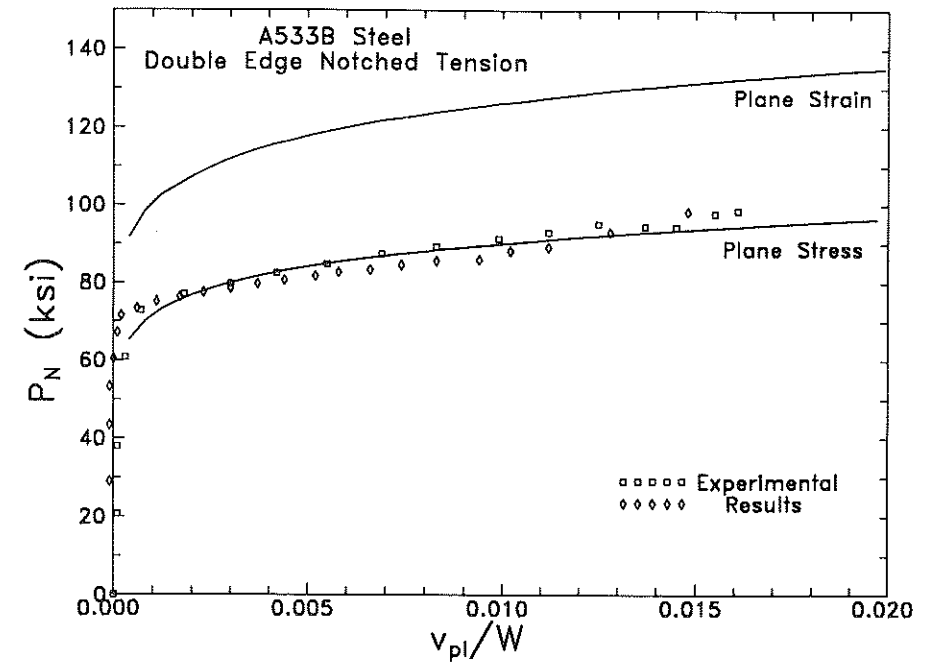


Fig 15 Normalised load versus plastic displacement for A533B double edge notched specimen

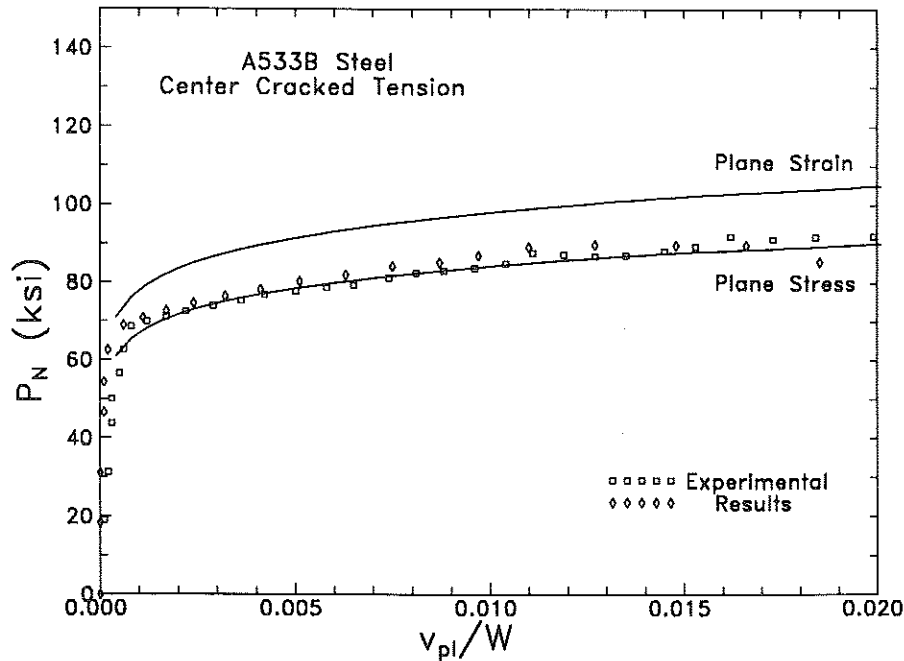


Fig 14 Normalised load versus plastic displacement for A533B centre cracked tension specimen

centre cracked tension, double edge notched tension and single notched tension geometries all taken from an A533B steel plate (11). The tests were conducted for these geometries using thin sections so that plane stress constraint is most appropriate. The result from the compact specimen was used with the plane stress Handbook solution to determine the appropriate yield stress to fit experimental results, Fig. 13. That yield stress was used in the plane stress Handbook solutions for the other three geometries. The predictions were compared in each case with experimental results, Figs 14, 15, and 16.

These predictions show a very good fit to the experimental results for the centre cracked and double edge notched geometries, Figs 14 and 15. The single edge notched fit was not as good, Fig 16. It could be that the loading system for the experiment did not match well with the way the boundary conditions were applied in the Handbook solution. It would be good to try the same approach for other materials and geometries. Some pressure boundary geometries would be especially interesting. To try these different solutions, results from experiments would be needed to match with the predictions.

Guidelines for Handbook use

As a summary to all of the suggestions made for using the Handbook, the following set of guidelines is given.

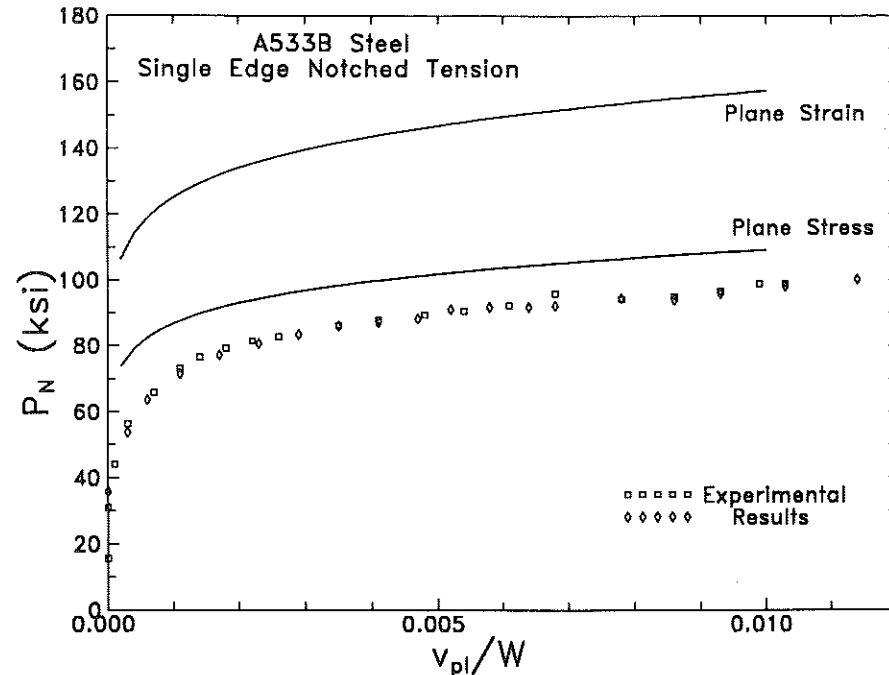


Fig 16 Normalised load versus plastic displacement for A533B single edge notched tension specimen

- (1) The Handbook solutions must be chosen for the appropriate geometry. This means that a two-dimensional model of the actual geometry must usually be postulated.
- (2) A normalising function $G(a/w)$ must be used to interpolate over differences in crack length. For many geometries this function has been obtained experimentally. For some geometries where this function has not been evaluated, the Handbook solutions for the various crack length can be used to develop the $G(a/W)$. This is simplified if the form $G(b/W) = BW(b/W)^n$ is found to be appropriate. The Handbook solution can then be taken for a tabulated value of a/W , $G(b/W)$ makes this solution continuous.
- (3) The calibration specimen should be used to generate the input values of n and $\epsilon_0 \alpha / \sigma_0^n$. These are taken from the experimental load versus displacement of the calibration specimen and the Handbook solution for the appropriate geometry and constraint. Alternately n , α , and σ_0 can come from a true stress-true strain tensile test or from the Bloom approximation.
- (4) The value of n from step 3 is used in the new structural geometry. It can be interpolated linearly in the Handbook solution tables of $h(a/W, n)$ values but should not be extrapolated.

Summary

The Handbook of elastic-plastic calibrations function provides a source of information valuable for failure analysis. However, it has received little experimental verification. The study in this work suggests that it can be used accurately if inputs are supplied in the proper formats. The key to supplying input is the testing of a calibration specimen, preferably a blunt notched specimen, corresponding to a geometry in the Handbook. Using this calibration specimen to supply input, the solutions for many other geometries can be used accurately. This was demonstrated from experimental results for three other geometries.

Continued evaluation of the Handbook solutions would be valuable. The work here considered only uniaxial loading. It would be good to evaluate solutions for geometries which have biaxial loading like pressure boundaries. Further experimental evaluation of the Handbook solutions can give the user more confidence in their accuracy or perhaps suggest techniques which will improve their accuracy.

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