

## An Improved Procedure for Calculating COD from the Plastic Component of Clip-Gauge Displacement

**REFERENCE** Kolednik, O., An improved procedure for calculating COD from the plastic component of clip-gauge displacement, *Defect Assessment in Components – Fundamentals and Applications*,ESIS/EGF9 (Edited by J. G. Blauel and K.-H. Schwalbe) 1991, Mechanical Engineering Publications, London, pp. 285–297.

**ABSTRACT** An improved procedure for calculating COD from the plastic component of clip-gauge displacement is stated. The procedure is valid for bend and CT-specimens. It relies on a recently developed theory which is based on well-known relationships of linear elastic and elastic-plastic fracture mechanics. The theory also allows for material properties which have a strong influence on the opening behaviour of pre-cracked bend specimens.

The new procedure is tested on four different types of materials. In all cases the calculated COD-values come close to the experimental CODs which were measured directly. It is shown that especially for low strength materials with high work-hardening exponent the so-called Dawes' formula can lead to a large overestimate of the physical COD.

### Introduction

For routine COD-testing it is necessary to calculate the crack-tip-opening displacement, COD, from the crack-mouth-opening displacement,  $v$ , which can be easily measured at the outside of the specimen. For bend and CT-specimens a relationship

$$\text{COD} = \frac{r_p b}{r_p b + a} v_p \quad (1)$$

can be applied when it is assumed that the two specimen halves rotate about a 'plastic hinge' during the opening of the crack. The plastic rotational factor,  $r_p$ , gives the position of the 'plastic hinge' in terms of the ligament length,  $b = W - a$ .  $W$  is specimen width and  $a$  the crack length.  $v_p$  is the plastic component of clip-gauge displacement.

For bend specimens in equation (1)  $a$  should be replaced by  $(a + z)$  where  $z$  is the distance between specimen surface and clip-gauge position.

Equation (1) yields correct results only when the 'right' values of  $r_p$  are inserted. The matter is complicated by the fact that the hinge moves during the loading depending on the degree of plasticity of the specimen. Additionally,

\* Erich-Schmid-Institut für Festkörperphysik, Österreichische Akademie der Wissenschaften, 8700 Leoben, Austria.

material properties and specimen geometry have an influence on the opening behaviour of the specimen. We can write

$$r_p = r_p(\text{load, material, geometry}). \quad (2)$$

#### A brief description of a new theory

Recently, a new theory has been developed which allows one to estimate the sizes of the plastic rotational factor for different stages of loading (1)–(3). The first estimates were made for deeply notched bend specimens assuming idealised load–displacement records: linear elastic fracture mechanics (LEFM) is assumed to be valid for loadings up to the general yield load. Beyond the general yield point the material should behave like a perfectly plastic material. For the first region (valid LEFM) COD and the plastic component of the total angle of bend,  $\theta_p$ , can be calculated as functions of load.  $v_p$  is a simple function of  $\theta_p$  and  $r_p$ . Inserting  $v_p$  and COD into equation (1) we get an expression in  $r_p$  which can be solved (1). For the second region of the load–displacement curve  $r_p$  can be deduced as a function of  $v_p$  using the relationship between the  $J$ -integral and COD and the formula to estimate  $J$  from the area under the  $F$ - $v$ -curve (1). The theoretically calculated  $r_p$  decreases rapidly from infinity at the very beginning of loading, passes a value of  $r_p = r_p^{(GY)}$  at the point of general yielding and decreases further asymptotically to a value of  $r_p = r_{pp}^{(a)}$  for large  $v_p$ -values.

It should be emphasised that  $r_p$  is only a fictitious size which combines COD and  $v_p$  in the form of equation (1). It gives something like a ‘mean position’ of the plastic hinge during the whole deformation of the specimen. Physically more important is the ‘actual’ or ‘incremental’ rotational factor,  $r_p^{(a)}$ , which characterises the position of the rotational centre during a considered increment of loading. Similarly to  $r_p$ ,  $r_p^{(a)}$  decreases from an infinite value and remains constant at a value of  $r_p^{(a)} = r_{pp}^{(a)}$  after the beginning of general yielding.

If crack extension occurs, the  $r_p$ -values must be corrected. This is also included in the theory. Crack extension is the reason why the  $r_p$ -values in a real COD-test run through a minimum value (see Table 4).

In (2) the theory was extended to CT-specimens and in (3) it has been shown that the theory is applicable for non-idealised specimens made of strain-hardening materials, too. From these theoretical findings it has been possible to establish a new, improved COD-calculation procedure (3).

The estimated  $r_p$ -values depend on three constants. The constant  $\beta$  which describes the size of the plastic zone was set to  $\beta = 1/6\pi$  according to Irwin’s model of the plastic zone for plane strain condition. The two other constants can be determined experimentally as is given below. In the procedure no fit parameters or empirical formulae are used.

Below a more comprehensive form of the procedure is listed.

#### The new procedure

(1) *Estimates of the constants C and m*

(a) *Determination of C*

(The constant  $C$  describes the maximum load of the load–displacement record.  $C$  is a sort of a constraint factor increased by the effect of work hardening.)

Measure the maximum load,  $F_{\max}$ , of the load–displacement record. For bend specimens calculate the value of  $C$  using

$$C = \frac{F_{\max} S}{\sigma_y B b^2} \quad (3)$$

$S$  is the span of the specimen,  $B$  is its thickness. For CT-specimens use (4)

$$C = \frac{F_{\max}}{\alpha \sigma_y B b} \quad (4)$$

with

$$\alpha = \sqrt{\{(2a/b)^2 + 2(2a/b) + 2\} - (2a/b + 1)} \quad (5)$$

(b) *Determination of m*

(The constant  $m$  arises from the relationship between  $J$ -integral and COD,  $J = m\sigma_y \text{COD}$ .  $m\sigma_y$  can be seen as a mean flow stress during the process of crack-tip blunting.  $m$  depends primarily on the work-hardening exponent,  $n$ , and on the yield strain  $\sigma_y/E$  (5).)

If the work-hardening exponent of the material is not known, determine its approximate size from Fig. 1 (6), or using the relationship (6)

$$\frac{\sigma_y}{\sigma_u} = \frac{1}{1 + \varepsilon_y^*} \left( \frac{e \ln(1 + \varepsilon_y^*)}{n} \right)^n \quad (6)$$

with  $\sigma_u$  as the ultimate tensile strength and

$$\varepsilon_y^* = \sigma_y/E + 0.002 \quad (7)$$

Evaluate the size of  $m$  applying the relationship. (It may be possible to use a second method to estimate  $m$ : determine the constant,  $d_n$ , and the reference stress,  $\sigma_0$ , according to the EGF-recommendations (6) and compute  $m = \sigma_0/(d_n \sigma_y)$ .)

$$m = \frac{\sigma_u}{\sigma_y} \frac{e^n}{(1 + n)n^n} \quad (8)$$

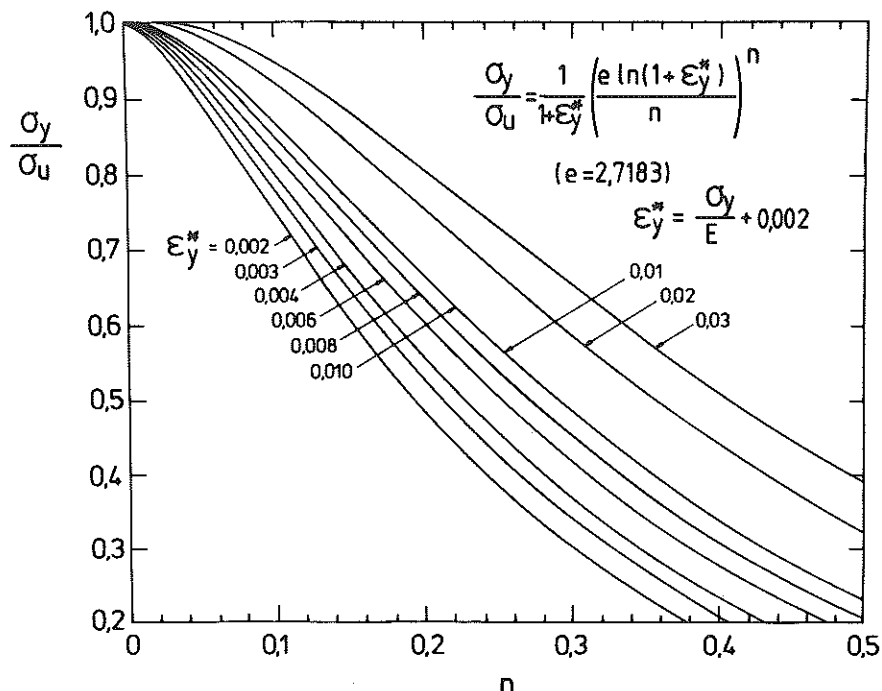


Fig 1 Estimate of the work-hardening exponent,  $n$

(2) Evaluation of  $r_p^{(GY)}$  and  $v_p^{(GY)}$

( $r_p^{(GY)}$  and  $v_p^{(GY)}$  are the plastic rotational factor and the plastic component of clip-gauge displacement at the beginning of general yielding.)

From the values of  $C$  and  $m$  calculate

$$r_p^{(GY)} = \frac{1.78}{mC} \quad (9)$$

and

$$v_p^{(GY)} \approx \frac{\pi \sigma_y}{m E} \left[ b + \frac{a}{r_p^{(GY)}} \right] \quad (10)$$

For bend specimens replace  $a$  by  $(a + z)$ .

(3) Evaluation of  $r_p$  for  $v_p < v_p^{(GY)}$

If in your COD-test one or more  $v_p$ -values are smaller than  $v_p^{(GY)}$ , draw a line at a distance  $v_p^{(GY)}$  parallel to the elastic line to find the point  $A$  of the load-displacement curve (of the maximum loaded specimen), see Fig. 2. Determine  $F_{GY}$ , i.e., the load of point  $A$ .

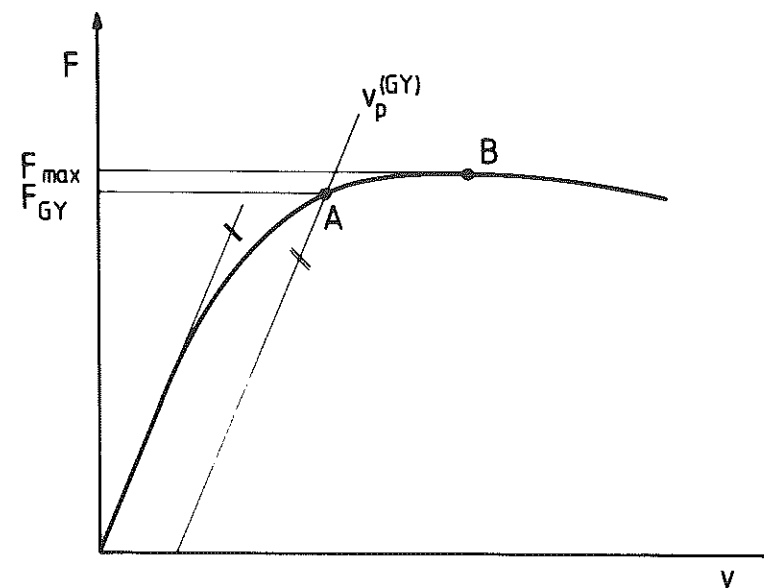


Fig 2 To the determination of the general yield load,  $F_{GY}$

From the actual load-displacement record determine the load,  $F$ , corresponding to  $v_p$ . Calculate

$$r_p = \frac{3\pi}{4mC} \frac{F_{GY}}{F} \frac{(1 - r_y/b)^2}{(1 - r_y/(2b))} \quad (11)$$

where

$$\frac{r_y}{b} = \frac{C^2}{6\pi} \left( \frac{F}{F_{GY}} \right)^2 \quad (12)$$

( $r_y$  is the radius of the plastic zone for plane strain yielding.)

(4) Evaluation of  $r_p$  for  $v_p > v_p^{(GY)}$

(a) Estimate of the actual plastic rotational factor  $r_{pp}^{(a)}$

For  $v_p$ -values larger than  $v_p^{(GY)}$  compute  $r_{pp}^{(a)}$ . For bend specimens use

$$r_{pp}^{(a)} = \frac{C}{2m}, \quad (13)$$

for CT-specimens

$$r_{pp}^{(a)} = \frac{a}{b} \frac{\frac{C}{m} \alpha f_j \left( \frac{a}{w} \right)}{1 - \frac{C}{m} \alpha f_j \left( \frac{a}{w} \right)} \quad (14)$$

with  $\alpha$  according to equation (5) and with

$$f_j\left(\frac{a}{w}\right) = 2 \frac{1 + \alpha}{1 + \alpha^2} \quad (15)$$

(b) *Correction for large crack extensions*

Measure the average crack extension,  $\Delta a$ , and calculate the corrected value of  $r_{pp}^{(a)}$

$$r_{pp}^{(a)*} = r_{pp}^{(a)} + (1 - r_{pp}^{(a)}) \frac{\Delta a}{b} \quad (16)$$

$b$  is the initial ligament length.

(c) *Evaluation of  $r_p$*

Determine  $s = v_p/v_p^{(GY)}$  and compute

$$r_p = \frac{r_{pp}^{(a)*} s [a/b + r_p^{(GY)}] + a/b [r_p^{(GY)} - r_{pp}^{(a)*}]}{s [a/b + r_p^{(GY)}] - [r_p^{(GY)} - r_{pp}^{(a)*}]} \quad (17)$$

For large crack extension insert the corrected value  $r_{pp}^{(a)*}$  instead of  $r_{pp}^{(a)}$ .  
(For some materials with high work-hardening exponent  $[r_p^{(GY)} - r_{pp}^{(a)*}]$  may also be negative.)

(5) *Evaluation of COD*

Calculate COD from the measured  $v_p$  and the estimated  $r_p$  using

$$\text{COD} = \frac{r_p b}{r_p b + a} v_p \quad (18)$$

For bend specimens replace  $a$  by  $(a + z)$ .

**The experimental control of the procedure**

The new procedure shall be tested by comparing the calculated COD-values to the directly measured COSs. Additionally, a comparison shall be made to the results of the Dawes' formula (7),

$$\delta = \delta_e + \delta_p = \frac{K^2(1 - \nu^2)}{2\sigma_y E} + \frac{r_p b}{r_p b + a} v_p \quad (19)$$

with  $r_p = 0.4$ .

This formula is utilised by the British Standard for COD-testing (8).  $\delta_e$  and  $\delta_p$  are (misleadingly) called as 'elastic' and 'plastic components of COD',  $K$  is the stress intensity and  $\nu$  the Poisson's ratio. The EGF-recommendations for determining the fracture toughness of ductile materials (6) also gives a version

of Dawes' formula which is adjusted for crack growth. Somewhat rewritten it looks like

$$\delta = \frac{K^2(1 - \nu^2)}{2\sigma_y E} + \frac{r_p b + (1 - r_p) \Delta a}{r_p b + (1 - r_p) \Delta a + a} v_p \quad (20)$$

with  $r_p = 0.4$ .

For bend specimens in equations (19) and (20)  $a$  should be replaced by  $(a + z)$ .

In (2)(3) the new procedure was already tested. In all cases the calculated CODs came close to the experimentally determined values. For some materials the Dawes' formula produces good results, too, for other material the error was large. In the following it shall be investigated, therefore, for which types of materials the error of the Dawes' formula is especially large. To do this, four types of materials are studied: a high strength steel, an annealed structural steel, a Nickel-based alloy, and a Nitrogen alloyed austenitic steel.

*High strength steel HY 130*

For this material Fields and Miller supplied me with the results of their infiltration measurements (9). They tested bend specimens having  $W = 20$  mm,  $B = 24$  mm,  $S = 80$  mm,  $z = 1$  mm. From the data of the material ( $\sigma_y = 940$  MN/m<sup>2</sup>,  $\sigma_u = 980$  MN/m<sup>2</sup>) the work-hardening exponent can be estimated to  $n \approx 0.05$  (Fig. 1 or equations (6)(7)). From equation (8) follows that the constant  $m = 1.2$ . Such a small value is reasonable for such high strength materials, e.g., see (10). The constant  $C \approx 1.55$  was determined from the maximum load for each specimen (equation (3)).

The experimental data and the calculated COD- and  $\delta$ -values are collected in Table 1. The calculated CODs coincide with the experimental data, the  $\delta$ -values are too small (Fig. 3). The maximum error is 33 percent.

*Annealed structural steel*

From the annealed structural steel ( $\sigma_y = 298$  MN/m<sup>2</sup>,  $\sigma_u = 426$  MN/m<sup>2</sup>,  $n = 0.20$ ) five CT-specimens ( $W = 50$  mm,  $B = 25$  mm) were loaded within the

**Table 1** Measured and calculated data of pre-fatigued bend specimens made of HY130-steel

Spec.	Measured				Calculated			
	a (mm)	F <sub>max</sub> (kN)	F (kN)	v <sub>p</sub> (mm)	COD (mm)	r <sub>p</sub> (I)	COD (mm)	δ (mm)
1	8.8	55.13	55.10	0.295	0.15	0.90	0.150	0.10
2	8.5	56.40	55.40	0.510	0.26	0.78	0.247	0.21
3	9.6	48.85	47.20	0.831	0.32	0.74	0.341	0.28
4	8.5	56.31	53.00	0.930	0.42	0.70	0.427	0.35
C = 1.55    m = 1.2    v <sub>p</sub> <sup>(GY)</sup> = 0.26 mm    r <sub>pp</sub> <sup>(a)</sup> ≈ 0.64								

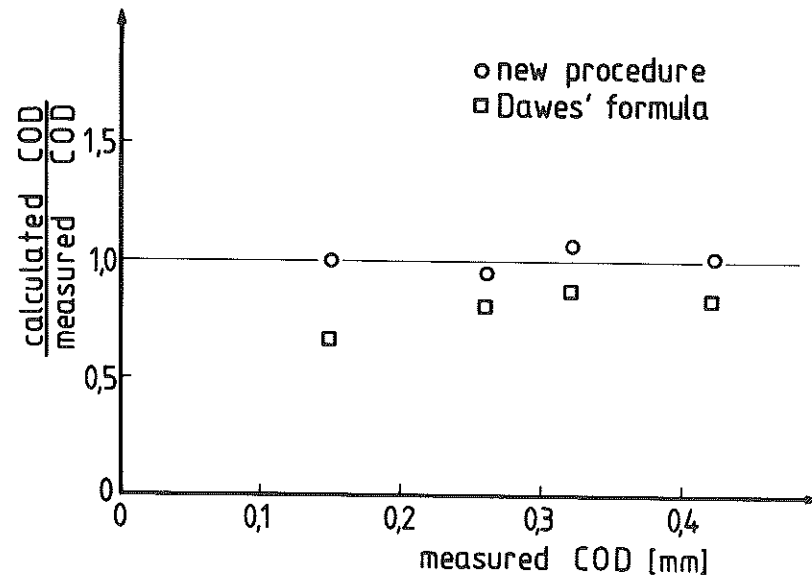


Fig 3 The accuracy of the COD-estimates for the HY130 steel

region of crack-tip blunting and broken in liquid nitrogen. The COD-values were measured applying the method of stereophotogrammetry with the scanning-electron microscope (11)(12).

The constant  $m$  was determined experimentally to  $m = 1.9$ , the estimate according to equation (8) would give  $m = 2.0$ . The constant  $C$  was determined from an additional, maximum loaded specimen,  $C = 1.47$ . In Table 2 the experimental values of COD are compared to the calculated ones. Again the new procedure yields good results, the  $\delta$ -values are generally too large (Fig. 4). The error decreases from nearly 100 percent for Spec. 1 down to 20 percent for the COD of Spec. 5 which comes very near to the critical COD of this material,  $COD_1 = 73 \mu\text{m}$ .

Table 2 Measured and calculated data of the CT-specimens made of the annealed structural steel

Spec.	Measured				Calculated		
	a (mm)	F (kN)	$v_p$ (mm)	COD (mm)	$r_p$ (I)	COD (mm)	$\delta$ (mm)
1	27.05	25.03	0.045	0.015	1.01	0.021	0.029
2	27.56	27.68	0.070	0.027	0.83	0.028	0.040
3	27.33	29.03	0.115	0.038	0.80	0.046	0.052
4	27.65	30.14	0.145	0.057	0.78	0.056	0.061
5	26.67	34.75	0.210	0.071	0.57	0.070	0.086
$C = 1.47 \quad m = 1.9 \quad v_p^{(GY)} = 0.16 \text{ mm} \quad r_{pp}^{(a)} = 0.40$							

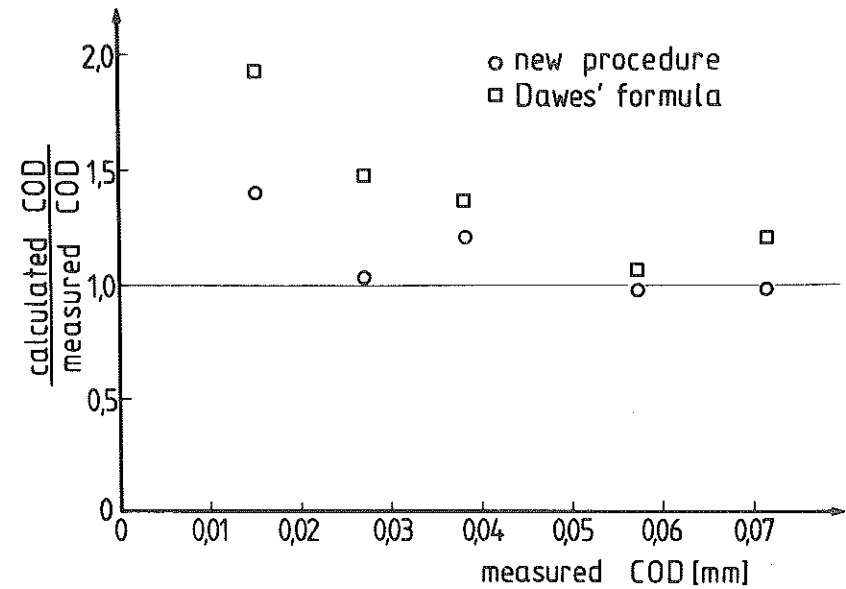


Fig 4 The accuracy of the COD-estimates for the annealed structural steel

#### Alloy 600

Only three CT-specimens ( $W = 37.5 \text{ mm}$ ,  $B = 18.85 \text{ mm}$ ) were available of the Alloy 600 ( $\sigma_y = 318 \text{ MN/m}^2$ ,  $\sigma_u = 680 \text{ MN/m}^2$ ). From one maximum loaded specimen the constant  $C$  was determined to  $C = 2.35$ . From Fig. 1 and equation (8) we get  $n = 0.25$  and  $m = 3.11$ .

The two other specimens were loaded within the region of crack-tip blunting and subsequently fatigued to mark the amount of crack tip deformation.

The COD-values were measured making use of the small depth of focus of a light microscope. From the midsection region of each specimen half a series of photographs were taken in the scanning-electron microscope. The photographs were necessary to find the 'same' regions on both specimen halves. These corresponding stretched-zone regions were examined in the light microscope.

The data of the two specimens are listed in Table 3. The COD of Spec. 1 is

Table 3 Measured and calculated data of the CT-specimens made of Alloy 600

Spec.	Measured				Calculated		
	a (mm)	F (kN)	$v_p$ (mm)	COD (mm)	$r_p$ (I)	COD (mm)	$\delta$ (mm)
1	20.86	25.4	1.24	0.246	0.37	0.281	0.348
2	20.04	28.0	1.71	0.408	0.37	0.419	0.491
$C = 2.35 \quad m = 3.1 \quad v_p^{(GY)} = 0.16 \text{ mm} \quad r_{pp}^{(a)} = 0.39$							

overestimated by both calculation procedures. The new procedure makes an error of 14 percent, the error of Dawes' formula is even 42 percent. The COD of Spec. 2 is only slightly smaller than  $COD_i$ . Here the calculated COD is rather accurate (3 percent error) whereas the  $\delta$ -value is too high by 20 percent.

#### X3CrNiN2315

This is a 0.3 percent Nitrogen alloyed austenitic steel ( $\sigma_y = 364 \text{ MN/m}^2$ ,  $\sigma_u = 760 \text{ MN/m}^2$ ,  $E = 203 \text{ GN/m}^2$ ) which was used in an investigation of Werner (13) where the influences of grain size and cold deformation on the fracture toughness were studied.

From the data of a single-specimen  $J_{Ic}$ -test on a CT-specimen ( $W = 50 \text{ mm}$ ,  $B = 25 \text{ mm}$ ,  $a = 30.97 \text{ mm}$ ) the COD- and  $\delta$ -values were calculated (see Table 4). From these values COD- $\Delta a$ -curves were drawn (Fig. 5). The crack extension  $\Delta a$  was measured using the unloading compliance method. The two curves differ significantly.

To make a check, a second specimen was loaded to about the point of initiation of stable crack growth and analysed in the same way as described for the Alloy 600. The analysis gives a measured COD of  $COD \approx COD_i = 176 \pm 11 \mu\text{m}$ .

The work-hardening exponent of the material was determined to  $n = 0.24$  and the constants are  $C = 1.67$  and  $m = 3.01$ . The measured and calculated data are presented in Table 5. Dawes' formula overestimates the real COD by 44 percent whereas the new procedure is underestimating it by 9 percent.

From the COD- $\Delta a$ -curves of Fig. 5 one can deduce that the Dawes' formula can lead to a dangerous overestimate of the critical COD ( $\delta_i = 340 \mu\text{m}$ ,  $COD_i = 180 \mu\text{m}$ ).

Table 4 Measured and calculated data to the single-specimen COD-test of the X3CrNiN2315

Point	Measured			Calculated		
	F (kN)	$v_p$ (mm)	$\Delta a$ (mm)	$r_p$ (I)	COD (mm)	$\delta$ (mm)
1	29.88	0.87	0.06	0.289	0.131	0.210
2	30.81	1.09	0.12	0.287	0.163	0.258
3	31.58	1.35	0.13	0.285	0.201	0.312
4	32.32	1.65	0.21	0.285	0.246	0.375
5	32.78	1.97	0.41	0.291	0.299	0.445
6	33.18	2.37	0.56	0.295	0.364	0.531
7	33.46	2.85	0.74	0.300	0.444	0.636
8	33.47	3.37	1.02	0.310	0.539	0.755
9	33.29	3.96	1.46	0.325	0.659	0.900
10	32.93	4.57	1.70	0.334	0.778	1.043
11	32.49	5.12	2.09	0.348	0.902	1.185
12	31.71	5.91	2.53	0.365	1.083	1.389
13	30.62	6.87	3.15	0.388	1.323	1.653

$C = 1.67$   $m = 3.0$   $v_p^{(GY)} = 0.20 \text{ mm}$   $r_{pp}^{(a)} = 0.275$

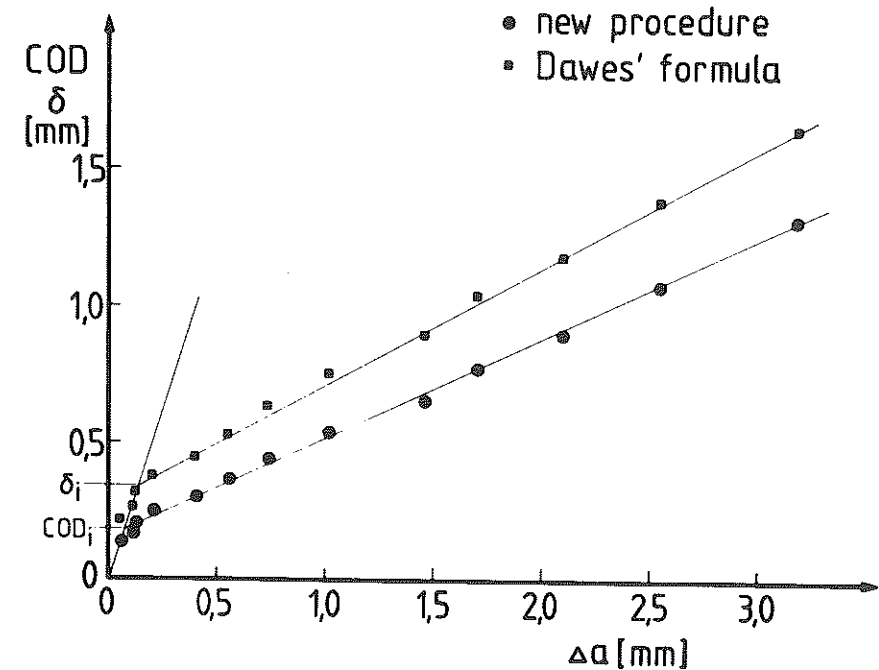


Fig 5 The COD- $\Delta a$  curves for the X3CrNiN2315

## Discussion

### On the accuracy of COD-measurements

For the testing of the procedure reliable and accurate COD-data are necessary. In (12) the COD-measuring methods are discussed. Indirect measuring methods are inappropriate. For example, the double clip-gauge method which is frequently used may give totally wrong results because the crack flanks of loaded specimens are bent elastically and plastically.

In the current work direct measuring methods are used. The method of stereophotogrammetry is the most accurate for rather small COD-values. The relative error of the measured COD is not larger than 10 percent. This

Table 5 Measured and calculated data of the CT-specimen made of X3CrNiN2315

Spec.	Measured			Calculated			
	a (mm)	F (kN)	$v_p$ (mm)	COD (mm)	$r_p$ (I)	COD (mm)	$\delta$ (mm)
1	31.91	29.20	1.14	0.176	0.29	0.160	0.254

$C = 1.67$   $m = 3.0$   $v_p^{(GY)} = 0.20 \text{ mm}$   $r_{pp}^{(a)} = 0.275$

includes the error of the analysing method as well as the scatter of the COD-values along the crack front in the midsection region of the specimen (12). For the infiltration method Fields and Miller claim their measurements to be accurate within  $\pm 20 \mu\text{m}$  (9). Depth-of-focus measurements have an error of about 10 percent, if the same regions on both specimen halves are analysed.

#### The comparison of the two calculation methods

The opening behaviour of a pre-cracked bend specimen under load is strongly influenced by material properties. For a given specimen geometry the limiting value of the actual or incremental plastic rotational factor,  $r_{pp}^{(a)}$ , is determined by the constants  $C$  and  $m$ , e.g.,  $r_{pp}^{(a)} = C/(2m)$ . This is important because with increasing displacement,  $v$ , the integral value of the plastic rotational factor,  $r_p$ , approaches asymptotically the value of  $r_{pp}^{(a)}$ . The constant  $m$  depends mainly on the materials work-hardening exponent,  $n$ : generally, large  $n$  means large  $m$ , and vice versa. So for different types of materials the size of  $r_{pp}^{(a)}$  can vary significantly: from the very high value of  $r_{pp}^{(a)} = 0.64$  for the high-strength steel with the low work-hardening exponent down to  $r_{pp}^{(a)} = 0.275$  for the austenitic steel.

The Dawes' formula works with a constant value of the plastic rotational factor,  $r_p = 0.4$  to calculate  $\delta_p$ . Additionally, a value  $\delta_e$  is added which makes a large contribution to the total  $\delta$  for small displacements. For large  $v_p$ -values  $\delta_e$  becomes less and less important. So this formula will yield good results for the annealed structural steel ( $r_{pp}^{(a)} = 0.40$ ) and the Alloy 600 ( $r_{pp}^{(a)} = 0.39$ ), if the displacement is large enough. But even for these materials the error will be substantial near the beginning of loading: in both cases COD<sub>i</sub> is overestimated by 20 percent (see Table 2 and Table 3).

The opening behaviour of soft materials having large work-hardening exponents is very badly described by Dawes' formula. Here the over-estimate of COD can be more than 50 percent (Table 4). On the other hand, for high strength, low work-hardening materials COD may be considerably underestimated (Table 1).

In this work it could be shown that the new procedure yields rather accurate values of COD for very different types of materials. In all cases tested experimentally the new procedure produced better results than the Dawes' formula.

#### Acknowledgements

I would like to thank Dr E. Werner, Dr Anne Fields, and Professor K. J. Miller for supplying me with their experimental data, and to Dipl. Ing. A. Gerscha from the Kraftwerks Union AG, Erlangen, for the Alloy 600.

#### References

- (1) KOLEDNIK, O. (1988) On the calculation of COD from the clip-gauge displacement in CT and bend specimens, *Engng Fracture Mech.*, **29**, 173-188.
- (2) KOLEDNIK, O. (1989) Plastic and overall rotational factors in bend and CT specimens, *Int. J. Fracture*, **39**, 269-286.
- (3) KOLEDNIK, O. (1989) An improved procedure for calculating COD in bend and CT-specimens, *Engng Fracture Mech.*, **33**, 813-826.
- (4) MERKLE, J. G. and CORTEN, H. T. (1974) A  $J$ -integral analysis for the compact specimen, considering axial force as well as bending effects, *Trans. ASME, J. Press. Vess. Technol.*, **96**, 286-292.
- (5) SHIH, C. F. (1981) Relationships between the  $J$ -integral and the crack opening displacement for stationary and extending cracks, *J. Mech. Phys. Solids*, **29**, 305-326.
- (6) (1987) EGF Recommendations for Determining the Fracture Resistance of Ductile Materials, *1st Draft*.
- (7) DAWES, M. G. (1979) Elastic-plastic fracture toughness based on the COD and  $J$ -contour integral concepts, *ASTM STP*, **668**, pp. 307-333, ASTM, Philadelphia.
- (8) BS 5762 (1979) Methods for crack opening displacement (COD) testing, *The British Standards Institution*.
- (9) FIELDS, B. A. and MILLER, K. J. (1977) A study of COD and crack initiation by a replication technique, *Engng Fracture Mech.*, **9**, 137-146.
- (10) ROBINSON, J. N. and TETELMAN, A. S. (1974) The measurement of  $K_{Ic}$  on small specimens using critical crack tip opening displacement, *ASTM STP*, **559**, pp. 139-158, ASTM, Philadelphia.
- (11) KOLEDNIK, O. (1981) A contribution to stereophotogrammetry with the scanning electron microscope, *Pract. Metallogr.*, **18**, 562-573.
- (12) KOLEDNIK, O. and STÜWE, H. P. (1985) The stereophotogrammetric determination of the critical crack tip opening displacement, *Engng Fracture Mech.*, **21**, 145-155.
- (13) WERNER, E. (1988) Der Einfluß der Korngröße, des Legierungsgehaltes und einer Kaltverformung auf die Bruchzähigkeit, *Z. Metallkde.*, **79**, 585-590.