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# The Effect of Notch Radius on the Fatigue Notch Factor and the Propagation of Short Cracks

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**ABSTRACT** Strength reductions in fatigue due to sharp notches are less severe than indicated by elastic stress analysis, even when notch root strains are nominally elastic. This investigation evaluates the reduction in fatigue concentration for central circular notches in thin plate specimens of a 2024-T351 aluminum alloy and a SAE 1045 steel. The specimens were tested in uniaxial, fully reversed stressing ($R = -1$) until fracture, using a closed-looped servo-controlled electrohydraulic testing system. Successively smaller diameter notches: 3 mm, 1 mm, 0.5 mm, and 0.24 mm, were tested at different stress levels to establish the stress–life curves. Smooth specimen stress–life data was obtained from previous investigations. The fatigue notch factor was observed to decrease with decreasing notch radius from its theoretical maximum of 3 to values approaching its minimum of 1. The observed fatigue notch factors were in agreement with the predictions of short crack fracture mechanics.

## Introduction

The majority of fatigue failures in engineering components occur due to local stress concentrations at notches which promote crack initiation and subsequent crack propagation. An elastic notch solution defines the theoretical stress concentration factor, $K_t$, as the ratio of the local stress at the notch root, $\sigma_{notch}$, to the nominal stress in the component, $\sigma_{nom}$.

$$K_t = \frac{\sigma_{notch}}{\sigma_{nom}} \quad (1)$$

The theoretical stress concentration factor depends on the geometry of the component, the geometry of the notch and the type of loading. It is well established that $K_t = 3$ for a central circular notch in a plate of infinite width subjected to pure uniaxial tension/compression(**1**)(**2**).

Strength reductions in fatigue due to sharp notches are generally less severe than is indicated by the elastic solution, even when notch root strains are nominally elastic. Therefore, $K_t$ is replaced by the fatigue notch factor which is the ratio of the unnotched fatigue strength of the material to the notched

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component fatigue strength at a given fatigue life. In this paper the fatigue notch factor is defined at the fatigue limit as:

$$K_f = \frac{\Delta\sigma_e}{\Delta S_{fat}} \qquad (2)$$

where $\Delta\sigma_e$ is the fatigue limit stress range of an unnotched specimen, and $\Delta S_{fat}$ is the nominal fatigue limit stress range on the net area of a notched specimen. Several empirical relationships **(1)–(5)** have been suggested for the prediction of $K_f$ in the general form

$$K_f = \text{fn}\,(K_t, \rho, \alpha) \qquad (3)$$

where $\rho$ is the notch root radius and $\alpha$ is a material constant. It has been known for some time that the reduced fatigue concentration at sharp notches is associated with the existence of non-propagating cracks **(6)–(10)**. Several investigators **(11)–(16)** have developed analytical expressions for the fatigue notch factor based on traditional fracture mechanics. However, such expressions appear to be valid only when the size of the notch plastic zone and the length of the non-propagating crack are small relative to the depth of the notch.

Topper *et al.* and El Haddad *et al.* **(17)–(22)** showed that a modified form of fracture mechanics that they had developed for short cracks could predict both the geometries in which non-propagating cracks would appear and the fatigue notch factors for these geometries. In their analysis notches are divided into two groups: 'blunt' notches, which have radii greater than a critical radius, $\rho_{cr}$, and 'sharp' notches, which have radii less than $\rho_{cr}$. The critical radius is defined by a length parameter, $l_o$, which was thought to be a material constant. They also showed that for blunt notches the fatigue strength depends on the resistance to crack initiation and that $K_f \approx K_t$, while for sharp notches the fatigue strength depends on the resistance to crack propagation to a critical length, equal to $\sqrt{(c/l_o)}$, and they suggested the equation

$$K_f = \frac{1}{F}\{1 + \sqrt{(c/l_o)}\} \qquad (4)$$

where F is a geometric constant of the order of unity and $c$ is the notch depth. This expression appears to have general validity.

The objectives of the present investigation are to determine the manner in which the fatigue notch factor varies with the root radius of sharp circular notches, and to determine whether short crack fracture mechanics can predict the fatigue notch factors for these notches.

## Materials, equipment, and methods

The materials used in this investigation were a 2024-T351 aluminum alloy and a SAE 1045 steel. The chemical compositions and the mechanical properties of



**Table 1 Chemical compositions (% by weight)**

*2024-T351 aluminum alloy*

| Si | Fe | Cu | Mn | Mg | Cr | Zn | Ti |
|---|---|---|---|---|---|---|---|
| 0.50 | 0.50 | 4.35 | 0.60 | 1.50 | 0.10 | 0.25 | 0.15 |

*SAE 1045 steel*

| C | Si | Mn | P | S | Fe |
|---|---|---|---|---|---|
| 0.46 | 0.17 | 0.81 | 0.027 | 0.023 | Remainder |

**Table 2 Mechanical properties**

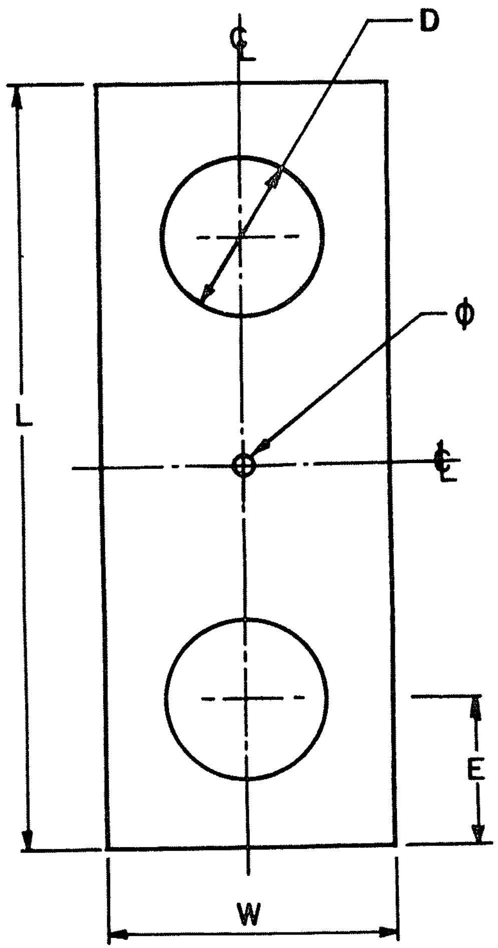
| | *2024-T351 aluminum alloy* | *SAE 1045 steel* |
|---|---|---|
| Elastic modulus (MPa) | 72 400 | 203 500 |
| Yield stress (MPa) (0.2% offset) | 356.5 | 471.6 |
| Ultimate tensile strength (MPa) | 466.1 | 744.7 |
| True fracture stress (MPa) | 623.3 | 1046.0 |
| Hardness (HB) | — | 235 |
| Fatigue limit ($R = -1$, MPa) | 124.0 | 303.0 |
| Threshold stress intensity factor (MPa$\sqrt{m}$) | 3.52 | 6.93 |

these metals are given in Table 1 and Table 2, respectively. Both materials were tested in the as-received condition.

Flat plate specimens, with the geometry and dimensions illustrated in Fig. 1, were machined from rolled plates of the materials such that the loading axis was parallel to the final direction of rolling. Circular notches were then drilled at the centre of the plate specimens. Notch diameters of 3 mm, 1 mm, 0.5 mm, and 0.24 mm were used. The surface of each specimen was hand polished to remove any sharp edges surrounding the notch. The geometry and dimensions of each notch was then checked by means of a travelling microscope with a resolution of 0.025 mm.

All tests were performed in a laboratory environment at room temperature (23°C) using a closed-loop servo-controlled electrohydraulic testing system. The loading frame was equipped with a liquid metal gripping system which ensured proper alignment of the specimen. Previous investigations using this type of specimen and gripping system **(23)(24)** have shown that the amount of bending induced by high compressive loading is negligible.

The specimens were subjected to a constant amplitude sinusoidal loading waveform (load control) until fracture occurred. The typical test frequencies ranged between 1 Hz and 60 Hz depending on the applied load amplitude. The long life tests on the aluminium alloy were performed at frequencies between 100 Hz and 120 Hz. Unnotched specimen stress–life data was obtained from previous investigations **(25)–(27)**.




| DIMENSION, mm | |
|---|---|
| L | 116.8 |
| W | 44.6 |
| E | 22.9 |
| D | 22.2 |
| thickness | 2.5 |
| ϕ | 3.0 |
| | 1.0 |
| | 0.5 |
| | 0.24 |

**Fig 1** Notched plate specimen geometry and dimensions (not to scale)

## Results

The observed stress–life data for the notched specimens are given in Fig. 2(a) for the 2024-T351 aluminum alloy and in Fig. 2(b) for the SAE 1045 steel. The unnotched specimen curves are also shown in those figures. In general, the fatigue strength is observed to decrease as the notch radius is increased.

The fatigue notch factors, $K_f$, for the 2024-T351 aluminum alloy were computed using the fatigue strengths at $2 \times 10^7$ cycles since the notched specimens of this material did not exhibit a distinct endurance limit. The endurance limit stresses were used to compute the $K_f$ values for the SAE 1045



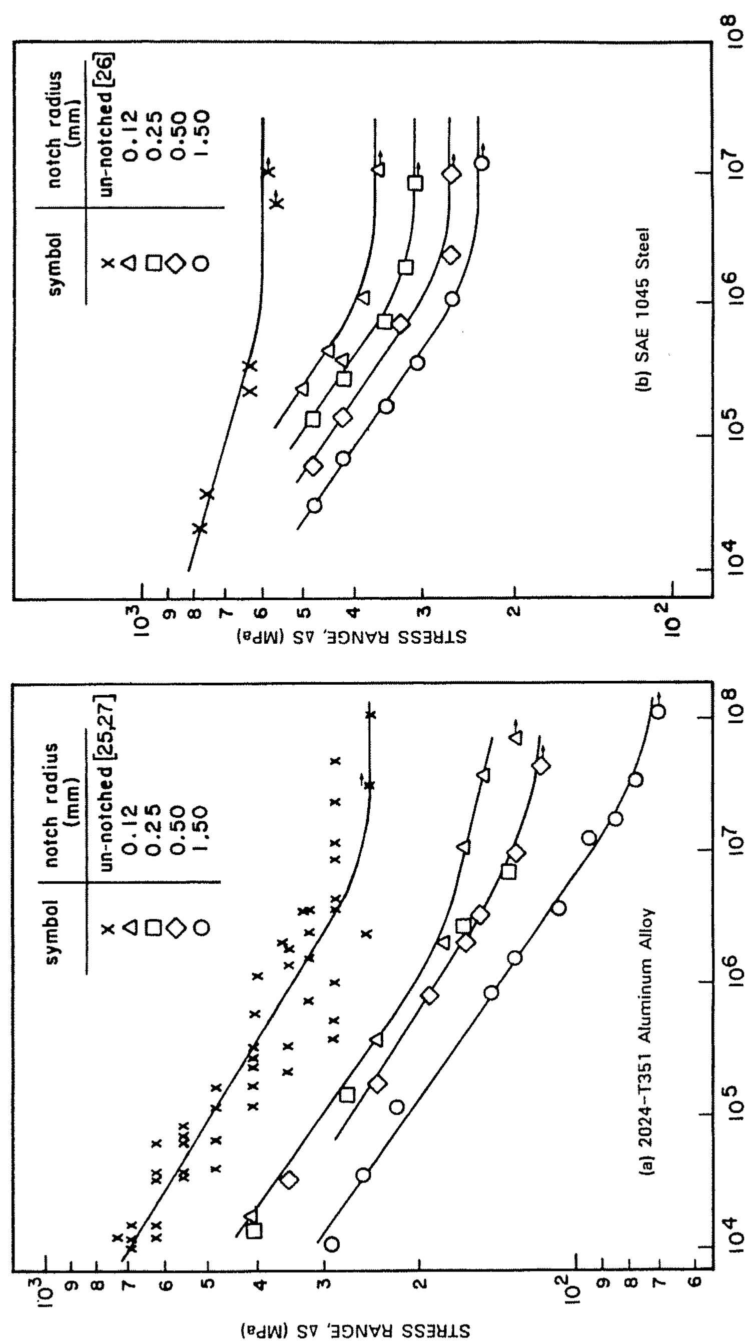



**Fig 2** Fully reversed constant amplitude stress-life curves

**Table 3 Theoretical and observed stress concentration factors**

| *2024-T351 aluminum alloy* | | | |
|---|---|---|---|
| Notch radius (mm) | $\Delta S_{fat}$ (MPa) | $K_t$ | $K_f$ |
| Unnotched | 248 | 1.00 | 1.00 |
| 0.12 | 160 | 3.00 | 1.55 |
| 0.25 | 124 | 2.96 | 2.00 |
| 0.50 | 124 | 2.94 | 2.00 |
| 1.50 | 90 | 2.82 | 2.76 |
| *SAE 1045 steel* | | | |
| Notch radius (mm) | $\Delta S_{fat}$ (MPa) | $K_t$ | $K_f$ |
| Unnotched | 608 | 1.00 | 1.00 |
| 0.12 | 360 | 3.00 | 1.69 |
| 0.25 | 310 | 2.96 | 1.96 |
| 0.50 | 276 | 2.94 | 2.17 |
| 1.50 | 248 | 2.82 | 2.45 |

steel. The observed fatigue notch factors and the theoretical elastic stress concentration factor for each notch size, corrected to account for the effects of finite specimen width (**2**), are given in Table 3.

The results for the 2024-T351 aluminum alloy (Table 3) show that the fatigue notch factor decreases from a value of 2.76 for a notch radius of 1.5 mm to a value of 2.00 for notch radii of 0.50 mm and 0.25 mm, and decreases further to a value of 1.55 for a notch radius of 0.12 mm. The data in Fig. 2(a), obtained for the 0.50 mm and 0.25 mm radii notches, are not significantly different, and consequently these notch radii have the same value for $K_f$. Larger $K_f$ values would be obtained for this material if the fatigue limit was defined at a life greater than $2 \times 10^7$ cycles. The blunt notch ($\rho = 1.50$ mm) has an observed $K_f$ of about 3.2 at a life of $10^8$ cycles, which is greater than the theoretical maximum value ($K_t = 2.82$) for this notch.

The results for the SAE 1045 steel given in Table 3 show that the fatigue notch factor decreases from a value of 2.45 for a notch radius of 1.5 mm to values of 2.17 and 1.96 for notch radii of 0.50 mm and 0.25 mm, respectively. The value of $K_f$ decreases further to a value of 1.69 for a notch radius of 0.12 mm.

## Discussion

### *Short crack fracture mechanics*

Topper *et al.* and El Haddad *et al.* (**17**)–(**19**) postulated the following relationship for the stress intensity factor, $\Delta K$, for short cracks in notches when the material behaviour is nominally elastic



$$\Delta K = K' \, \Delta S \sqrt{\{\pi(l_o + l)\}} \tag{5}$$

where $K'$ is the elastic stress concentration at the crack tip and is a function of the crack length (**28**), $\Delta S$ is the nominal stress range in the component, $l$ is the length of the crack, and $l_o$ is a length parameter given by the expression

$$l_o = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_e}\right)^2 \tag{6}$$

where $\Delta K_{th}$ is the threshold stress intensity factor range. At any crack length the condition for continued propagation is that the local stress intensity factor exceeds the threshold value. This condition also implies that the local stress range at the crack tip exceeds the threshold stress range

$$K' \Delta S > \Delta \sigma_{th} \tag{7}$$

The threshold stress range is given in terms of local parameters as:

$$\Delta \sigma_{th} = \frac{\Delta K_{th}}{F\sqrt{\{\pi(l_o + l)\}}} \tag{8}$$

In the case of a centrally notched plate specimen subjected to constant amplitude fully reversed stressing in the elastic range, the crack propagates through a diminishing stress field depicted in Fig. 3. At every increment of length while the crack is short the magnitude of both the maximum and minimum stresses at the crack tip are decreasing. Recent work by Topper, Au, and Yu(**23**)–(**25**) has shown that the threshold stress intensity factor, $\Delta K_{th}$, increases as the compressive portion of the stress cycle, $\sigma_{min}$, decreases. In addition, closure gradually builds up behind the crack tip as the crack length increases. When the crack has just initiated ($l \approx 0$) no closure exists because there is no material behind the crack tip. As the crack propagates to a few grain diameters in length, the material behind the tip becomes plastically deformed. However, the presence of a locally high cyclic compressive stress tends to retard the build up of closure (**29**). Steady-state conditions will prevail when the crack tip is remote form the notch.

It is apparent that the length parameter, $l_o$, defined by equation (6), is a function of the stress range and the minimum stress, and is not a constant for a given material. This length parameter may be thought of as a measure of the rate at which closure builds up and $\Delta K_{th}$ increases to its steady-state value.

Figure 4 illustrates the variation of the threshold stress, $\Delta \sigma_{th}$, with crack length, $l$, for a crack in a notch. The threshold stress varies between the bounds given by the material fatigue limit, $\Delta \sigma_e$, and the local threshold stress intensity factor, $\Delta K_{th}$. The local threshold stress intensity factor varies from a lower bound, $\Delta K_{th}$ at initiation, which represents the 'no-closure' condition, to an upper bound, $\Delta K_{th}$ steady-state, which represents the 'full-closure' condition. The actual path followed by the threshold stress may not be identical to the path given by equation (8) using only the steady-state value of $l_o$. However, if the

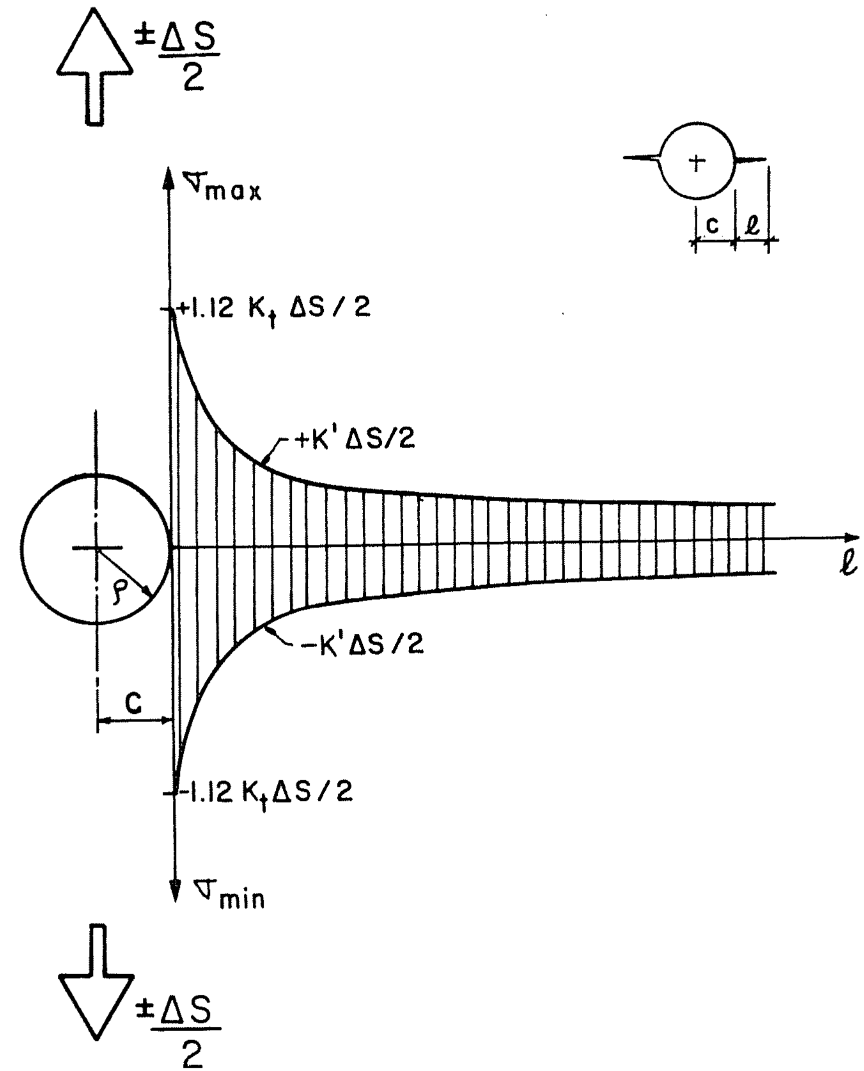



Fig 3 The diminishing stress field at the tip of a crack in a notch

distance over which closure builds up is not appreciably different on either of these paths, then the steady-state value of $l_o$ should provide a suitable approximation for characterizing short crack behaviour in a notch. The region where non-propagating cracks are possible is indicated by the hatched area in Fig. 4. A crack will become non-propagating at any crack length for which the inequality (7) is not satisfied.



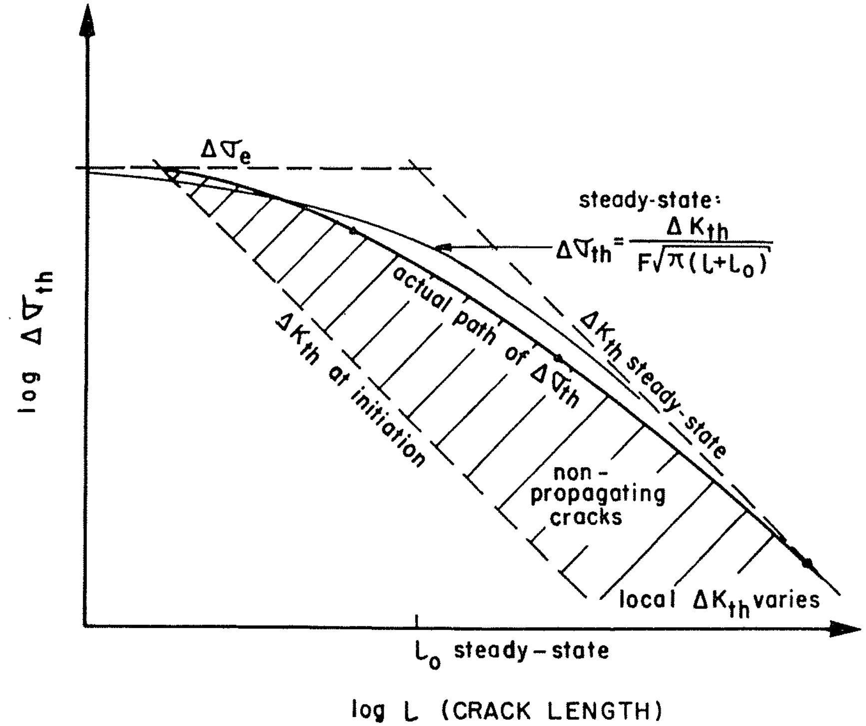



Fig 4 The variation of the threshold stress with crack length

*Prediction of the fatigue notch factor*

Equation (4) may be modified for central circular notches by assuming the notch to be small compared with the specimen width, and using an approximate value of 1.12 for the geometric factor, F. Hence

$$K_f = 0.89\{1 + \sqrt{(\rho/l_o)}\} \qquad (9)$$
$$(1.0 \leq K_f \leq K_t)$$

The critical radius, $\rho_{cr}$, which divides sharp and blunt notches can be approximated, using the value of F given above, as

$$\rho_{cr} \approx 5l_o \qquad (10)$$

Topper and El Haddad (**17**) produced satisfactory results by using the steady-state value of $l_o$ for notched mild steel specimens. In this investigation, the steady-state value of $l_o$ were also used to predict the variation of $K_f$ with $\rho$ for the 2024-T351 aluminum alloy and the SAE 1045 steel.

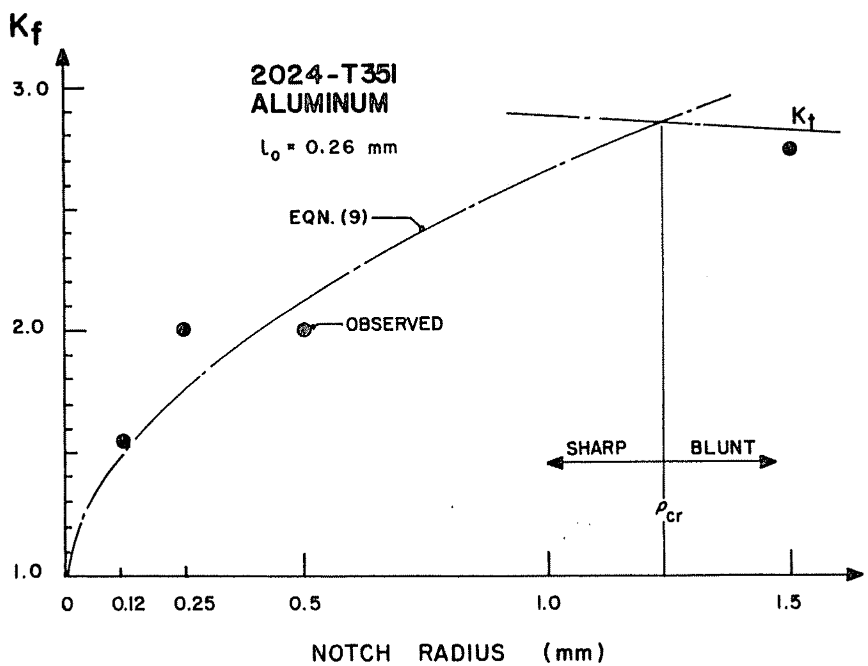



**Fig 5 Predicted and observed variation of $K_f$ with notch radius for 2024-T351 aluminum alloy**

The predicted and the observed data are shown together in Figs 5 and 6 for the two materials. These figures show a reasonable agreement between the predicted and the observed values of $K_f$ for the sharp notches. The results suggest that the steady-state $l_o$ is an appropriate parameter for use in short crack fracture mechanics applied to constant amplitude stressing at notches. The difference between the observed $K_f$ and the predicted $K_t$ for the 1.5 mm blunt notch in the SAE 1045 steel is due to cyclic plasticity which is exhibited by this material at the high stresses present at the notch root.

The above results show that short crack fracture mechanics may be useful for analysing 'notch-size' effects in fatigue. This is significant from a practical engineering viewpoint, despite the fact that no physical interpretation of the parameter $l_o$ has yet been determined. The method is relatively simple to apply and provides reasonable predictions of notch strength reductions for a range of sizes, including very small flaws, which occur in real components.

In variable amplitude loading the analysis becomes more complex. Both $\Delta K_{th}$ and the closure level will be varied by the load history and $l_o$ may not be a useful parameter. However, some finite length term, perhaps a grain diameter, is needed in order for an engineer to apply the short crack fracture mechanics analysis. In this case, it may be useful to think of $l_o$ as a fictitious length term



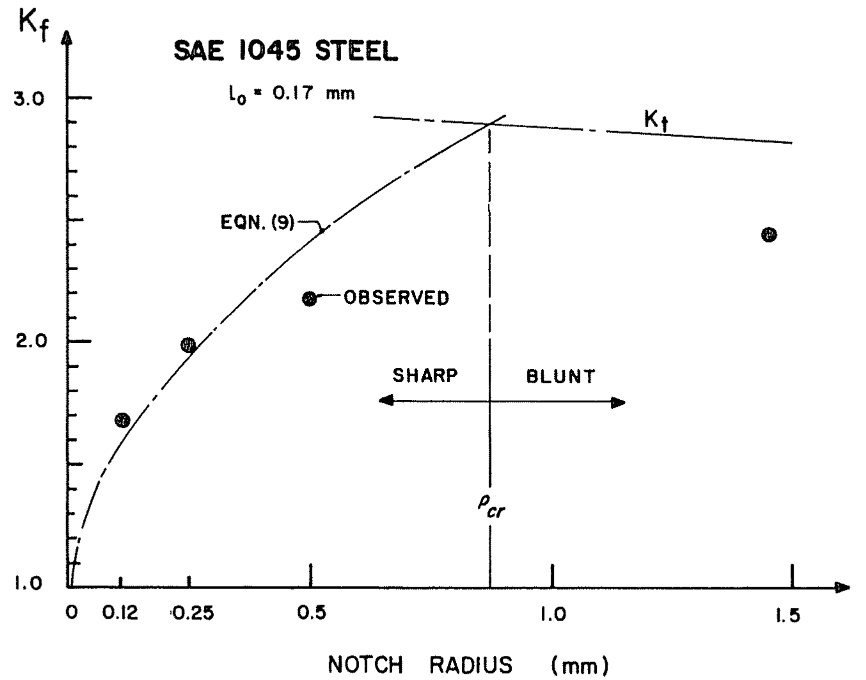



**Fig 6 Predicted and observed variation of $K_f$ with notch radius for SAE 1045 steel**

used to give a smooth transition from traditional crack initiation conditions to fracture mechanics growth calculations.

## Conclusions

(1) The fatigue notch factor, calculated at a fatigue limit of $2 \times 10^7$ cycles, decreases from close to its theoretical maximum value of 3 to values approaching its minimum of 1 as the notch root radius is decreased.

(2) Short crack fracture mechanics gives reasonable predictions of the observed values of the fatigue notch factor for the sharp circular notches.

(3) The steady-state value of the length parameter, $l_o$, is appropriate for use in the equations of short crack fracture mechanics applied to constant amplitude loading of thin notched plates.

## Acknowledgements

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