V. M. Radhakrishnan and Y. Mutoh†*

On Fatigue Crack Growth in Stage I

REFERENCE Radhakrishnan, V. M. and Mutoh, Y., **On Fatigue Crack Growth in Stage I**, *The Behaviour of Short Fatigue Cracks*, EGF Pub. 1 (Edited by K. J. Miller and E. R. de los Rios) 1986, Mechanical Engineering Publications, London, pp. 87–99.

ABSTRACT Investigations have been carried out to study the effect of grain size on stage I crack growth of two types of cracks; cracks starting from smooth surfaces and cracks starting from pre-existing notches. It has been observed that increase in grain size increases the threshold stress intensity factor but reduces the fatigue strength. Analysis indicates that in both types of cracks, the crack driving parameter is $\sigma\sqrt{a}$ and the resistance parameter is $\sigma_{ys}\sqrt{d}$. Based on this approach the non-propagation of both types of cracks is discussed.

Introduction

Fatigue crack growth in stage I is dependent on the grain size and the yield strength of the material both for cracks starting from smooth surfaces and cracks starting from pre-existing notches. In the case of smooth-surfaced materials it is well known that a fine-grained material will give a better fatigue resistance than a coarse-grained one. However, it has been observed by many researchers **(1)**–**(3)** that, in general, the threshold stress intensity factor, ΔK_{th}, increases with increase in grain size. This implies that materials of larger grain size will be better suited to resist crack initiation and subsequent slow growth if a pre-existing crack or a sharp notch happens to be present in the structure. The practical implications of these opposing microstructural requirements for good resistance to fatigue crack propagation in smooth specimens (high fatigue limit) and a good resistance to fatigue crack propagation in previously fatigue cracked specimens (high ΔK_{th}) must be clearly understood in selecting a material for a given application **(4)(5)**. In between these two extreme cases of a smooth specimen and a specimen having a sharp long crack, actual structures will have blunt notches or very small cracks – mechanical or metallurgical – and, in such cases, fatigue crack nucleation and further propagation will very much depend on the notch field and the microstructural properties of the material **(6)**.

Experimental

The materials investigated are (a) type 304 stainless steel, and (b) a low carbon steel. The chemical compositions are given in Table 1. The heat-treatment and the corresponding grain size and yield strength are given in Table 2. Two types of specimen, as shown in Fig. 1, were used for the stainless steel, a CT specimen of thickness 12.7 mm for the study of crack propagation and a round bar

* Metallurgy Department, IIT, Madras-36, India.
† Mechanical Engineering Department, Technological University of Nagaoka, Nagaoka, Japan.

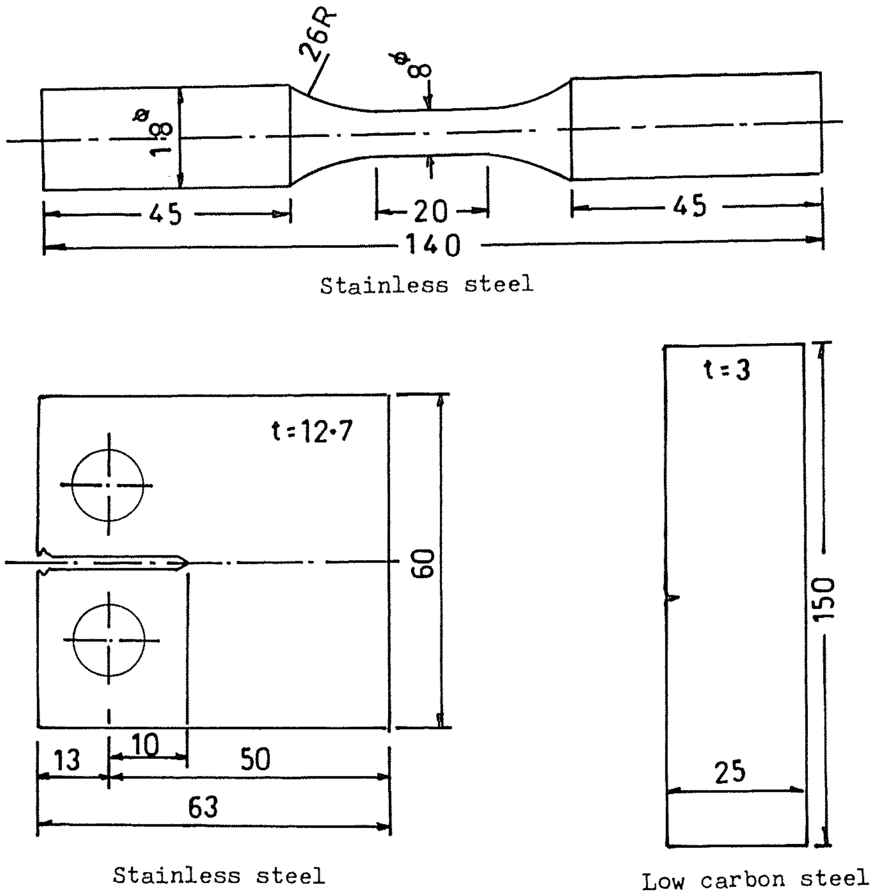

Fig 1 Stainless steel and low carbon steel specimens (dimensions in mm)

bar specimen for the determination of fatigue strength. In the case of the low carbon steel a plate specimen of thickness 3 mm with an edge notch (SEN type) was used. A servo-hydraulic MTS testing machine operating at a frequency of 40 Hz was used to estabish the fatigue threshold at a stress ratio $R = 0.05$ and the fatigue limit at a stress ratio $R = -1$ of the stainless steel. A Vibrophore fatigue testing machine was used to establish both the fatigue threshold and the fatigue limit of the low carbon steel at $R = 0.1$. For both materials an optical travelling microscope (×30) was used to measure the crack length. The accuracy of measurement was 0.01 mm. In the establishment of ΔK_{th} a load shedding method was employed and the crack was allowed to grow a distance corresponding to two to three times the plane stress maximum plastic zone size of the previous loading in order to avoid the residual stress effect due to load

Table 1 Chemical composition (wt%)

	C	*Si*	*Mn*	*P*	*S*	*Ni*	*Cr*
Stainless steel	0.06	0.74	1.24	0.027	0.005	8.45	18.10
Low carbon steel	0.09	0.12	0.2	0.03	0.03	—	—

Table 2 Mechanical properties

Material	*Heat treatment*	*Grain size* (μm)	σ_{ys} (*MPa*)
Stainless steel	S1 1223 K (½ hr)	34	399
	S2 1323 K	74	308
	S3 1473 K	86	288
	S4 1523 K	110	258
Low carbon steel	LC1 1173 K	18	240
	LC2 1223 K	28	210
	LC3 1273 K	44	190

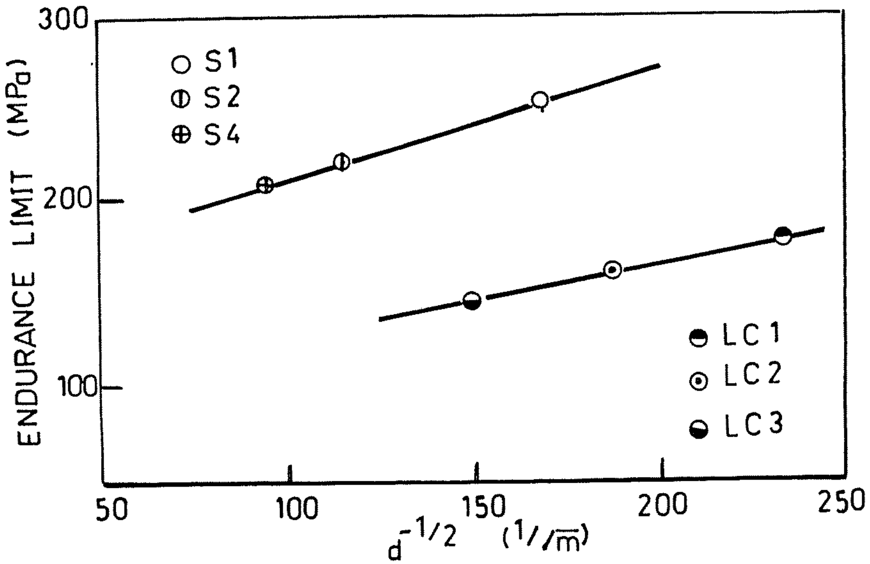

Fig 2 Relation between endurance limit and grain size (S = Stainless steel; LC = Low carbon steel)

reduction (7). At least two specimens were tested under similar loading conditions.

Results and discussion

Figures 2 and 3 show the relation between the endurance limit ($\Delta\sigma_e$) and the grain size, and that of the threshold stress intensity factor (ΔK_{th}) and the grain size (d), respectively, according to the relations

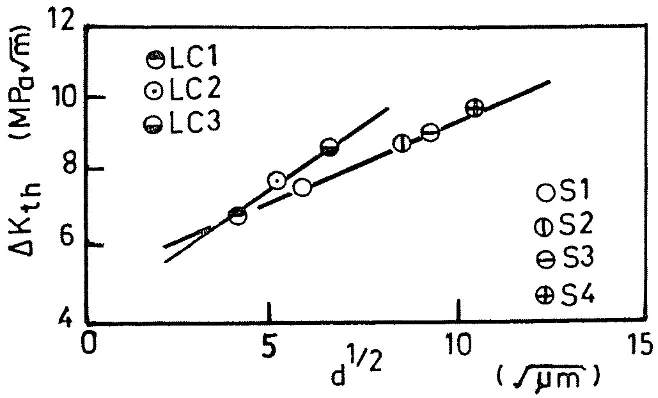

Fig 3 **Relation between threshold stress intensity factor and grain size (S = Stainless steel; LC = Low carbon steel)**

$$\Delta\sigma_e = 167 + 0.4/\sqrt{d} \quad \text{Stainless steel}$$
$$= 102 + 0.31/\sqrt{d} \quad \text{Low carbon steel} \qquad (1)$$

and

$$\Delta K_{th} = 6.2 + 320\sqrt{d} \quad \text{Stainless steel}$$
$$= 4.1 + 667\sqrt{d} \quad \text{Low carbon steel} \qquad (2)$$

where $\Delta\sigma_e$ is in MPa, d in metres, and ΔK_{th} in MPa$\sqrt{\text{m}}$. Within the range of grain size investigated (where it has been found that the Hall–Petch relation is valid as shown in Fig. 4) the fatigue strength decreases and the threshold stress intensity factor increases with increase in grain size. Figure 5 shows the relation between the endurance limit $\Delta\sigma_e$ and the yield strength σ_{ys} given as

$$\Delta\sigma_e = 0.5\sigma_{ys} + 51 \qquad (3)$$

with both $\Delta\sigma_e$ and σ_{ys} in MPa.

Figures 6(a) and (b) show the relation between σ_{max} and the crack length, a, for different threshold levels of stainless steel and low carbon steel, respectively. It can be observed that below the value of crack length a_I the σ_{max} for crack initiation remains more or less constant at $\Delta\sigma_e$. The value of a_I is dependent on the grain size and is given by

$$a_I = nd \qquad (4)$$

where $n = 15$ in the case of low carbon steel and 4 in the case of stainless steel.

In some of the low carbon steel specimens, sharp notches with a depth equal to 2 mm and a root radius in the range 0.1–0.2 mm were introduced and the crack growth from the root of the notch was studied as explained earlier. The stress intensity factor for the crack at the notch tip was calculated using the relation (**8**)

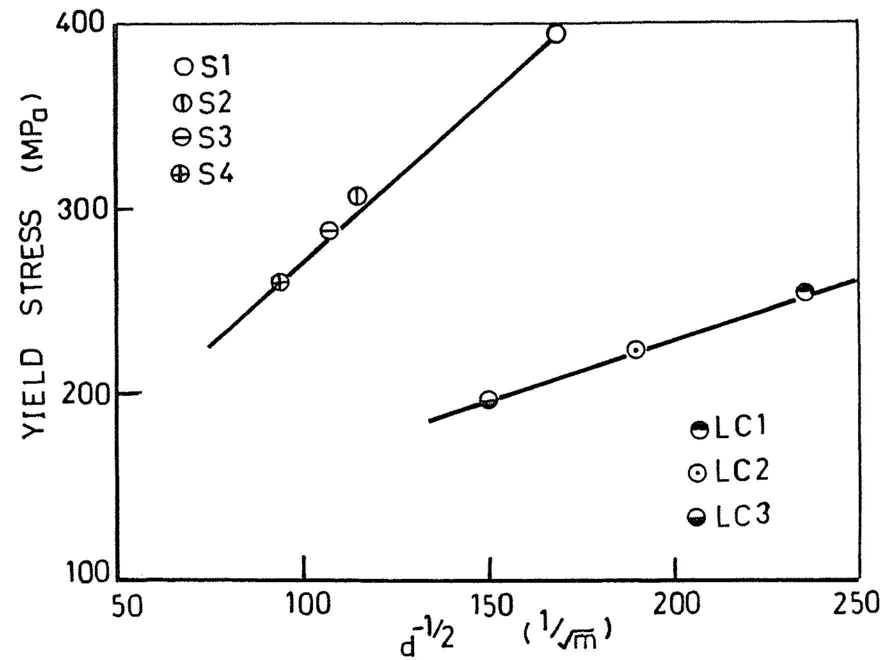

Fig 4 **Hall–Petch relation (S = Stainless steel; LC = Low carbon steel**

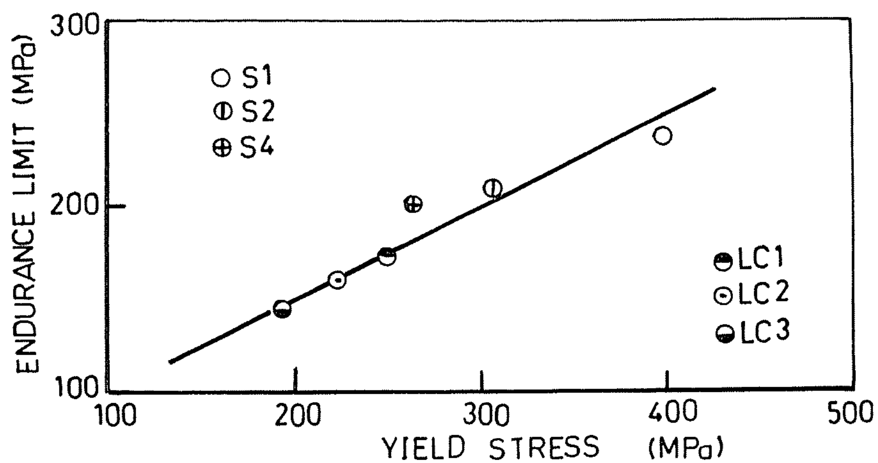

Fig 5 **Dependence of endurance limit on the yield strength (S = Stainless steel; LC = Low carbon steel)**

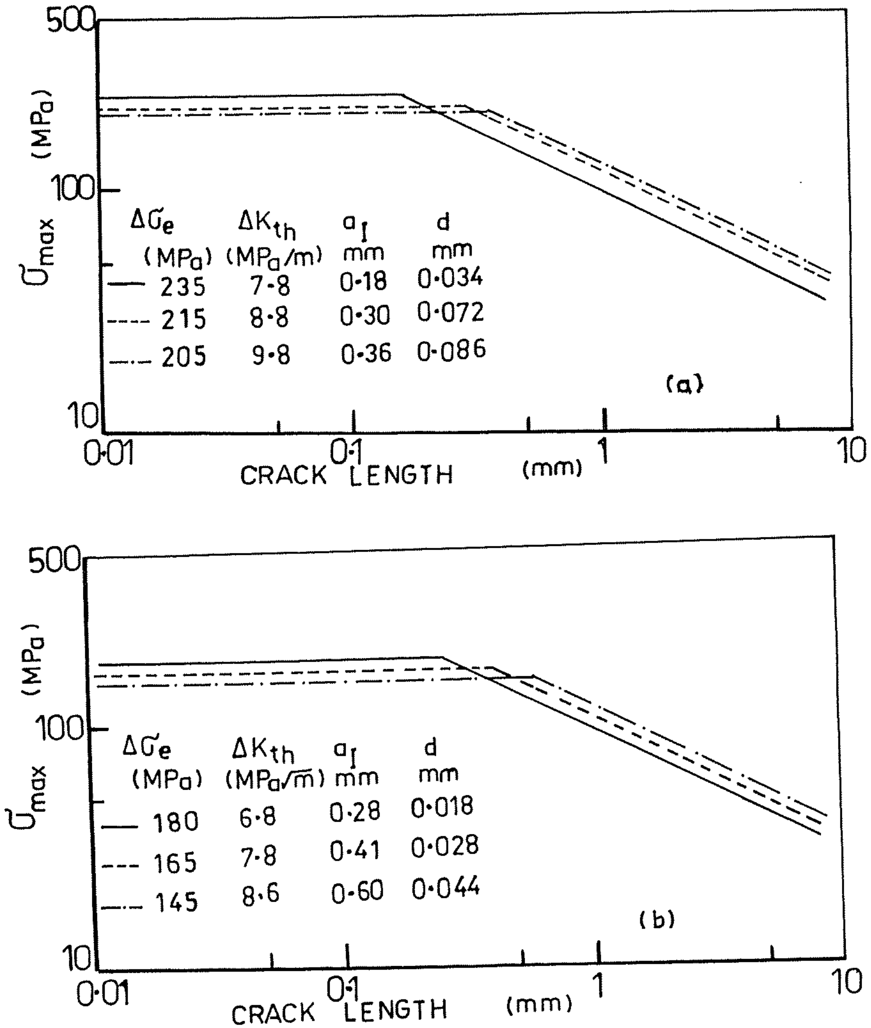

Fig 6 **Kitagawa–Takahasi (16) diagrams**
(a) Stainless steel type 304
(b) Low carbon steel

$$K = \{1 + 4.762\sqrt{(D/\rho)}\}^{1/2} K_{SEN} \qquad (5)$$

where D is the depth of the notch and ρ is the root radius, and K_{SEN} is

$$K_{SEN} = \sigma\sqrt{(\pi a)}(1.12 - 0.23\alpha + 10.55\alpha^2 - 21.72\alpha^3 + 30.39\alpha^4) \qquad (6)$$

where $\alpha = a/W$ and a is the fatigue crack length. Figure 7 shows the crack growth from notches of different included angles β and also from a sharp

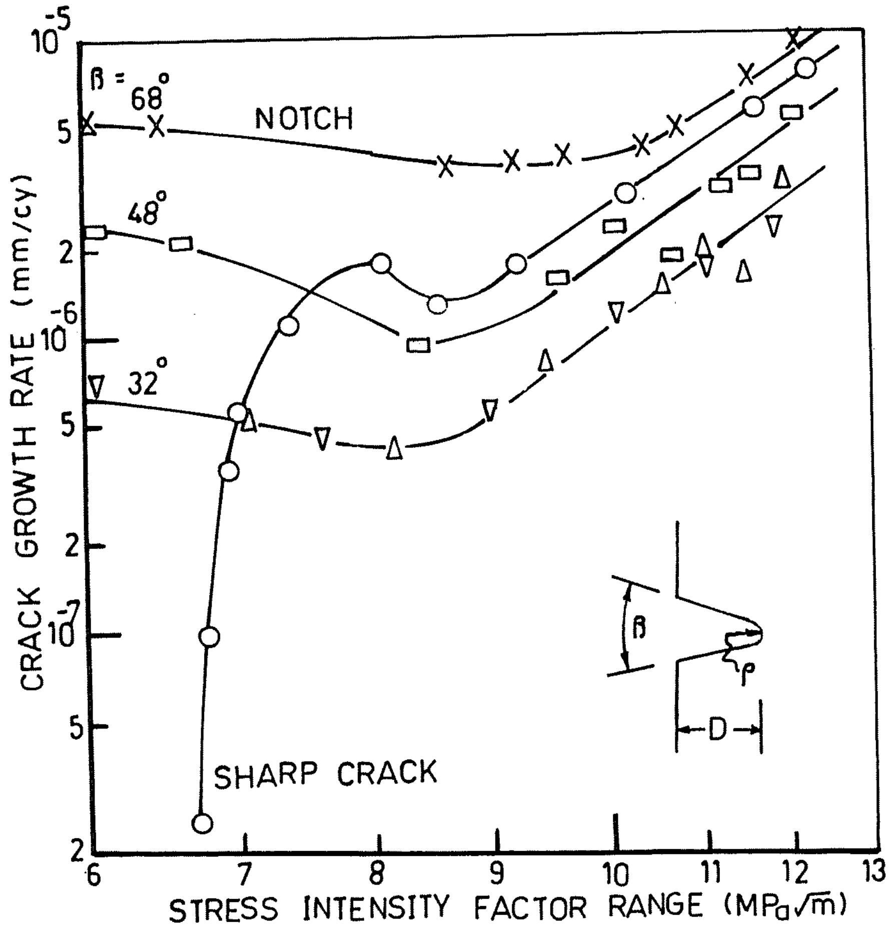

Fig 7 **Crack growth from notches of different included angles. The solid symbols indicate crack growth from a pre-existing crack. The material is low carbon steel**

pre-existing crack. It can be seen that in the case of comparatively sharp notches the crack propagation rate is somewhat lower than that in the case of blunt notches. On the limit, when the notch is so sharp that it can be simulated to a crack, the crack growth rate becomes zero until the stress intensity factor is increased to ΔK_{th}. However, when the cracks start propagating and become stage II cracks, all the curves tend to merge together.

A model for crack nucleation

A model for crack nucleation at the endurance limit from a smooth surface has been proposed (**9**), the essence of which is as follows. Consider a smooth specimen subjected to a stress range τ_{max}–zero. When the maximum shear

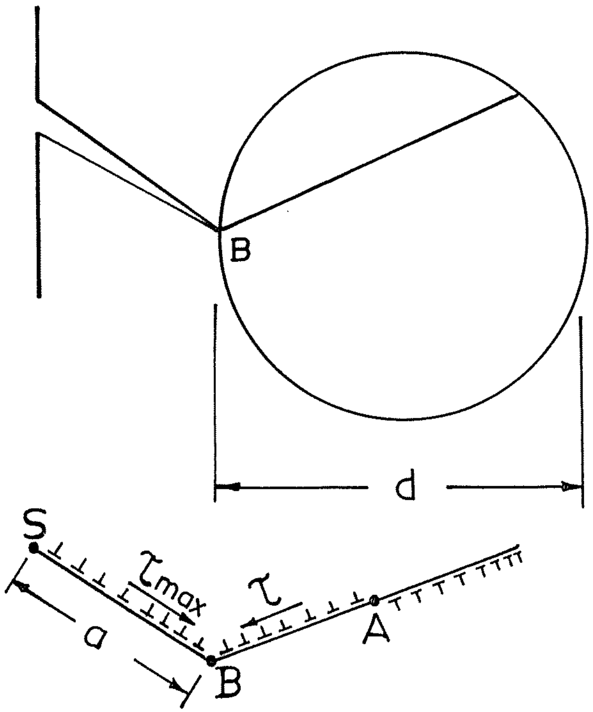

Fig 8 A model for crack development on a smooth surface

stress τ_{max} is equal to or greater than the yield stress in shear τ_{ys} of any grain oriented for easy slip, persistent slip bands will form which will transform themselves into a small crack. This small crack will encounter the next grain as shown in Fig. 8. The small surface crack can be treated as a row of dislocations emitted from an F-R source, S, and blocked by a grain boundary at B. If the stress concentration is of sufficient magnitude, it will activate the source at A in the next grain. A–B is the plane over which these activated dislocations will move. Assuming that the dislocation source in the next grain is activated when the shear stress at the distance r^* from the boundary B reaches a critical value τ^*, one obtains

$$\tau^* = (\tau_{max} - \tau_i)\left(\frac{a}{r^*}\right)^{1/2} \tag{7}$$

where τ_i is the internal stress and a the length of SB. On the other hand, from a similar assumption the yield shear stress τ_{ys} is given as

$$\tau^* = (\tau_{ys} - \tau_i)\left(\frac{d/2}{r^*}\right)^{1/2} \tag{8}$$

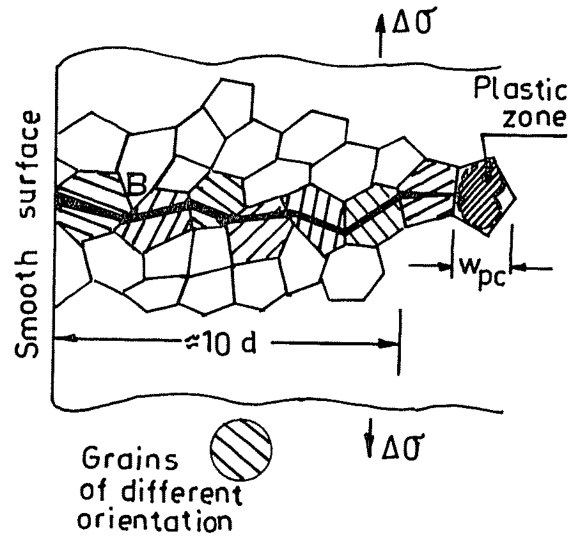

Fig 9 Crack growth in stage I and stage II. The crack encounters grains of different orientation on its path

where d is the grain diameter. Combining equations (7) and (8) we get

$$(\tau_{max} - \tau_i)\sqrt{a} = \frac{1}{\sqrt{2}}(\tau_{ys} - \tau_i)\sqrt{d} \tag{9}$$

Assuming τ_{max} to correspond to the endurance limit, surface cracks of depth approximately equal to the grain size can form at the endurance stress which may or may not propagate further. The yield stress τ_{ys} of that grain which is oriented for easy slip represents a minimum value for the grain that will lie on the crack path. The above relation also shows that in general, if the bulk yield strength of the material increases, the endurance limit will also increase, as has been shown in the present investigation and indicated in Fig. 5 and equation (3) of this paper.

If the conditions are favourable the crack, which is of an initial length equal to the grain size, will grow further. In stage I the orientation of the crack will be in the direction of maximum shear. But after a certain growth, the mode will change to mode I and the direction will be normal to the applied stress. A schematic representation of this situation is shown in Fig. 9. At the point where the crack enters mode I, corresponding to stage II of the crack propagation, it is postulated that the plastic zone will be of the order of the grain size. Further increase in crack length will enlarge the plastic zone to encompass several grains and as a result the bulk yield strength will control the size of the enclave. Thus at the beginning of stage II we can write the cyclic plastic zone size w_{pc} as

$$w_{pc} = d \tag{10}$$

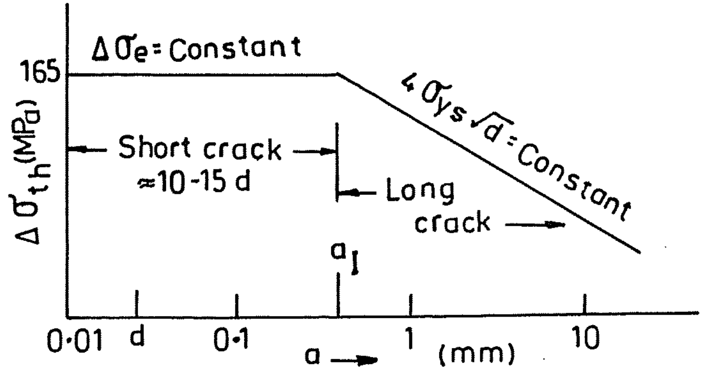

Fig 10 Schematic representation of regions of short and long cracks

Even for a crack emanating from a very sharp notch a similar assumption has been made by Yoder *et al.* (**3**) when the crack moves from stage I to stage II, so that

$$\Delta K_T = 5.5\sigma_{ys}\sqrt{d} \quad (11)$$

where ΔK_T corresponds to the SIF range at the transition.

The threshold stress intensity factor ΔK_{th} can be taken to be of the same order as ΔK_T . Liu and Liu (**10**) have shown that ΔK_{th} is of the order of $0.7\,\Delta K_T$. The value of a_I up to which the threshold stress will be approximately equal to the endurance limit (Fig. 10) can be given as

$$\begin{aligned}\Delta\sigma_e\sqrt{(\pi a_I)} &= 0.7\,\Delta K_T \\ &= 0.7 \times 5.5\sigma_{ys}\sqrt{d} \quad (12)\end{aligned}$$

Taking $\Delta\sigma_e \approx (1/\sqrt{2})\sigma_{ys}$, as indicated in equation (9), we get

$$a_I \approx 10d \quad (13)$$

It has been noticed by Taylor and Knott (**11**) that a_I is structure dependent and is of the order indicated by the above relation.

Non-propagating cracks

It has been observed that for both types of cracks, the crack driving parameter is $\Delta\sigma\sqrt{a}$ and the resistance parameter is $\sigma_{ys}\sqrt{d}$, as given in equations (9) and (11). In stage I, as the crack propagates, these two parameters just balance each other and the important stresses encountered are the applied shear stress τ and the yield stress in shear τ_{ys} of the grain just ahead of the crack tip. Hence the crack growth is structure sensitive and will be governed by the orientation of the grain which it will encounter on its path. As schematically shown in Fig. 9, when the surface crack starts growing from position *B* the yield stress τ_{ys} of each grain will control the resistance parameter $\tau_{ys}\sqrt{d}$. If $\sqrt{(a/d)} > \tau_{ys}/\sqrt{2}\,\tau_{max}$ the crack

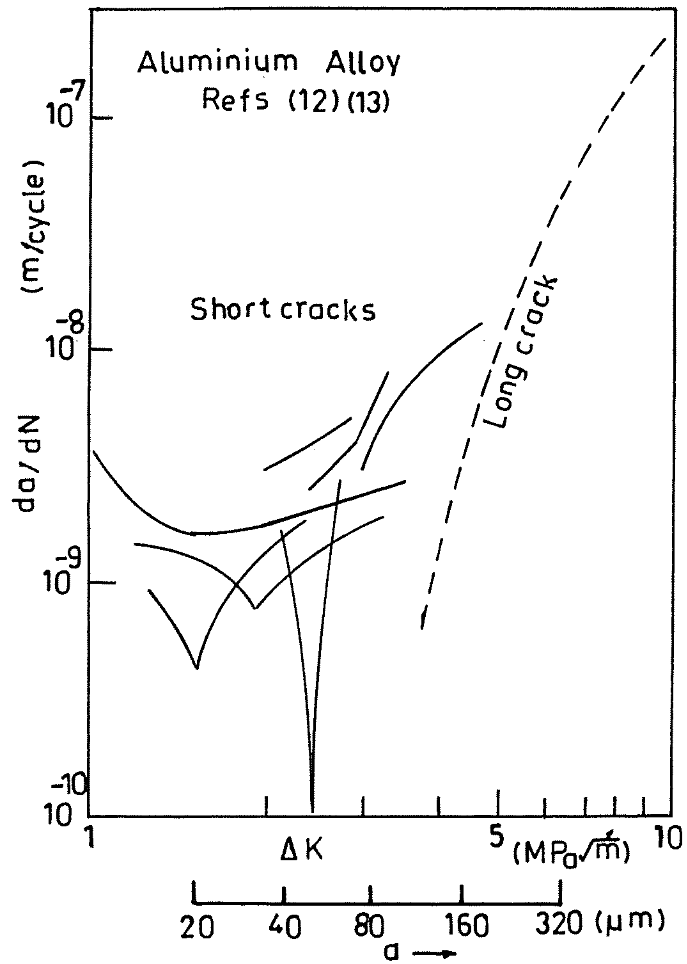

Fig 11 Crack growth behaviour of short and long cracks

will grow: otherwise there will be a retardation and sometimes non-propagation of the crack. Thus crack growth may be smoothly increasing (if all grains are favourably oriented) or zig-zag in character, with partial or complete arrest in some cases. Such behaviour is illustrated in Fig. 11, for an aluminium alloy; this data is taken from references (**12**) and (**13**). Thus the condition for non-propagation is that the resistance parameter is higher than the crack driving parameter and the length of such non-propagating cracks cannot be greater than a_I. This fact is also borne out in Fig. 11 where the crack lengths encountered are not more than 300 μm. Similarly an examination of the data reported in (**14**) indicated that the length of non-propagating cracks ranged from 50 μm to 250 μm.

For cracks starting from pre-existing notches or cracks the growth behaviour is very much determined by the stress and strain distribution ahead of the notch. A critical analysis on the prediction of non-propagating cracks for this condition, based on notch stresses, is given by El Haddad *et al.* (**15**). The plastic zone, w_{pn}, that forms ahead of the notch of length, $2D$, is a function of the stress concentration factor, K_t, and the root radius, ρ, of the notch and can be given by

$$w_{pn} = \rho f\left(\frac{K_t \sigma}{\sigma_{ys}}\right) \tag{14}$$

Whilst the crack is within the notch zone, it will propagate. Once it develops beyond the influence of the notch, its propagation depends on whether the crack driving parameter $\Delta\sigma\sqrt{\{\pi(D+a)\}}$ is greater than $4\sigma_{ys}\sqrt{d}$, i.e., the plastic zone size, w_{pc}, due to the total crack length $(D + a)$ needs to be larger than (or at least equal to) the grain size, d, for continued crack growth.

The influence of grain size on smooth surface cracks appears to be straightforward in that refining the grain size will, in general, increase the yield strength and the fatigue strength. However, in the case of notch generated cracks, as well as increasing the yield stress, the effect of grain refinement will also be to decrease the notch zone w_{pn} for a given notch, as indicated by equation (14). In this case, for a fine grain size to be advantageous requires the decrease in w_{pn} to be more than that in d, so that a crack cannot initiate from the tip of the notch.

Equation (14) also indicates that sharper notches will develop relatively smaller plastic zones. As a result it can be expected that under a given stress intensity condition the growth rate of cracks from sharp notches will be slower than that from relatively blunt notches. This trend can be seen from the experimental results, shown in Fig. 7. If the notch is very sharp and approaches a crack in shape, then the growth rate will be zero till the stress intensity is raised to the ΔK_{th} value. However, the growth rate curves for stage II cracks, whether from the notch or from the pre-existing crack, tend to merge, as can be seen in Fig. 7.

Concluding remarks

From the experimental investigations it is clear that for single phase materials and others for which a Hall–Petch type dependence on grain size is appropriate, increasing the yield stress will increase the endurance limit and decrease the threshold stress intensity factor. For a stage I crack generated from a smooth surface the resistance which the crack encounters as it grows can be given by the resistance parameter $\tau_{ys}\sqrt{d}$, where τ_{ys} is the yield strength of individual grains on the path of the crack. Since τ_{ys} of each grain depends on its orientation, the resistance will also change, as a result of which the crack growth may be smooth, or zig-zag, with partial or complete arrest in some cases. The maximum length of a non-propagating crack may not exceed more than 10–15 grain diameters. Increase in bulk yield strength will in turn increase the yield strength

of the individual grains and, hence, the resistance to stage I crack growth will also increase.

In the case of cracks starting from notches, the growth rate will be relatively high when the crack lies inside the notch zone. Increase in yield stress will reduce the notch zone size and the crack growth rate inside the notch zone. Once the crack leaves the notch effect, its further propagation depends on the plastic zone size of the total crack under the applied stress range. Increasing the yield strength of the material thus appears to be beneficial to both types of stage I cracks.

Acknowledgement

The authors are thankful to the university authorities at the Technological University of Nagaoka, Japan, and to the Director, IIT, Madras, India, for their kind permission to publish this paper.

References

(1) MASOUNAVE, J. and BAILON, J. P. (1976) Effect of grain size on the threshold stress intensity factor in fatigue of a ferrite steel, *Scripta Met*, **10**, 165–170.
(2) USAMI, S. and SHIDA, S. (1979) Elastic–plastic analysis of fatigue limit for a material with small flaws, *Fatigue Engng Mater. Structures*, **1**, 471–481.
(3) YODER, G. R., COOLEY, L. A., and CROOKER, T. W. (1981) A critical analysis of grain size and yield strength dependence of near threshold fatigue crack growth in steels, NRC Memorandum, Report 4576, NRC, Washington DC.
(4) BENSON, J. P. and EDMONDS, D. V. (1978) Effect of microstructure on fatigue in threshold region in low alloy steel, *Metal Sci.*, **12**, 223–232.
(5) RITCHIE, R. O. (1977) Influence of microstructure on near threshold fatigue crack propagation in ultra high strength steel, *Metal Sci.*, **11**, 368–381.
(6) MILLER, K. J. (1982) The short crack problem, *Fatigue Engng Mater. Structures*, **5**, 223–232.
(7) MUTOH, Y. and RADHAKRISHNAN, V. M. (1986) Effect of yield stress and grain size on threshold and fatigue limit, *J. Engng Mater. Technol.*, in press.
(8) CAMERON, A. D. and SMITH, R. A. (1981) Upper and lower bounds for the lengths of non-propagating cracks, *Int. J. Fatigue*, **3**, 9–15.
(9) MUTOH, Y. AND RADHAKRISHNAN, V. M. (1981) An analysis of grain size and yield stress effects on stress at fatigue limit and threshold stress intensity factor, *J. Engng mater. Technol.*, **103**, 229–233.
(10) LIU, H. and LIU, D. (1984) A quantitative analysis of structure sensitive fatigue crack growth in steels, *Scripta Met.*, **18**, 7–12.
(11) TAYLOR, D. and KNOTT, J. F. (1981) Fatigue crack propagation behaviour of short cracks: The effect of microstructure, *Fatigue Engng mater. Structure*, **4**, 147–155.
(12) CHAN, K. S. and LANKFORD, J. (1983) A crack tip strain model for the growth of small cracks, *Scripta Met.*, **17**, 529–532.
(13) LANKFORD, J. (1982) The growth of small fatigue cracks in 7075-T6, *Fatigue Engng Mater. Structures*, **5**, 233–248.
(14) KUNIO, T. and YAMADA, K. (1979) Microstructural aspects of the threshold condition for non-propagating cracks in martensitic–ferritic structures, *ASTM STP 675*, pp. 342–360.
(15) EL HADDAD, M. H., TOPPER, T. H., and SMITH, K. N. (1979) Prediction of non-propagating cracks, *Engng Fracture Mech.*, **11**, 573–584.
(16) KITAGAWA, H. and TAKAHASHI, S. (1976) Applicability of fracture mechanics to very small cracks in the early stage, *Proc. ICM 2*, Boston, pp. 627–631.