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Fatigue Short Crack Propagation and Plasticity-Induced Crack Closure at the Toe of a Fillet Welded Joint

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ABSTRACT During fatigue tests performed at constant load amplitude on stress-relieved fillet welded joints, the crack initiated from the weld toe stress concentrator, which is similar to a V-notched specimen in plane strain.

A highly sensitive crack monitoring system was developed for these tests and short cracks as small as 10–20 μm could be measured, as well as crack opening load levels. Under fully reversed loading, the fatigue fracture could be subdivided into three successive periods.

(1) No crack length greater than 20 μm was detected during the initial portion of life.
(2) A rapid crack growth was recorded, together with an absence of crack closure. The crack growth rate decreased and remained nearly constant, during which time the crack opening level increased and then tended to a stabilized positive value.
(3) When the crack opening level was stabilized, the crack growth rate invariably increased until final fracture; this corresponding to typical long crack behaviour.

Short crack propagation behaviour (period 2) is well correlated with long crack behaviour on the basis of the effective stress intensity factor range. Crack lengths, after stabilization of the crack opening level, increase with nominal applied stress, and are well correlated with notch cyclic plastic zone sizes computed by finite element analysis. Reasons why short crack behaviour is exclusively controlled by notch plasticity are discussed.

Under zero-to-tension loading the short crack effect is barely observed because of a smaller crack closure variation.

Nomenclature

a	Crack length
a_{pz}	Notch plastic zone extent
da/dN	Crack growth rate
h	Height of gauge centre from fracture plane
K	Stress intensity factor
K_{min}, K_{max}	Minimum, maximum value of K during a loading cycle
R	S_{min}/S_{max}
S	Nominal stress
S_{min}, S_{max}	Minimum, maximum value of S during a loading cycle

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S_{op}	Crack opening stress level
α	Singularity of a V-notch stress field
ΔK	Stress intensity factor range
ΔK_{th}	Conventional ΔK threshold
$\Delta K_{eff,th}$	Effective ΔK threshold
θ	Weld toe angle
σ_{yc}	Cyclic yield stress

Introduction

Many experimental investigations have clearly demonstrated that linear elastic fracture mechanics (LEFM), i.e., the ΔK parameter, cannot account for the behaviour of short fatigue cracks. Whatever the initiation site geometry (notch root or smooth surface), the following 'anomalies' of short cracks are often reported: (i) faster growth than long cracks at the same ΔK value; (ii) growth below the conventional long crack threshold, ΔK_{th}; and (iii) short cracks can grow at a decreasing rate, and possibly arrest; so-called non-propagating cracks are observed near the endurance limit level.

This non-LEFM propagation behaviour was first studied for cracks emanating from notches, and it was proposed that the notch plasticity is responsible **(1)(2)**, since a newborn crack, which is completely surrounded by the notch plastic zone, exceeds the conditions of LEFM analyses and such a crack can be defined as a 'mechanically short crack' **(3)**. The notch plasticity influence was well substantiated by Leis **(4)**; he found, for a variety of notches, materials and nominal stress levels, a 'one-to-one correspondence' (in log–log coordinates) between the plastic zone extent and the crack length at the transition to LEFM behaviour. However, the comparison, which covered the range 50 μm–10 mm, revealed about half a decade of scatter. Moreover, experimental data were limited to fully reversed loading ($R = -1$) and little data is available for $R \neq -1$ when monotonic and cyclic plastic zones are not of the same size **(5)**.

Ohji *et al.* **(6)** and, later, Newman **(7)** performed numerical analyses in order to simulate the plasticity-induced closure of short cracks growing at notches under fully reversed loading (a 60 degree V-notch and a circular notch, respectively). Both analyses revealed an initial transient variation of the crack opening level: S_{op} starts from $S_{min} = -S_{max}$ at very short crack lengths, rapidly increases as the crack tip moves away from the notch root, and finally tends to a stabilized slightly positive value given by $S_{op}/S_{max} \approx 0.10$–$0.20$. Such variation explains qualitatively the short crack propagation behaviour **(5)**. For example, although K_{max} increases with crack length, the effective crack driving force, ΔK_{eff} decreases because of the rapid and important increase of S_{op}; which causes short cracks to arrest at low nominal stresses when ΔK_{eff} decreases below an effective threshold $\Delta K_{eff,th}$ **(6)**. The transient variation was reported to be less pronounced for a larger radius at the V-notch root for which case ΔK_{eff} continuously increased. This is also in general agreement with experiments

which show that the growth rate only decreases if the geometry is severe enough **(2)**.

Although previous analyses show that plasticity-induced crack closure can explain the short crack propagation behaviour, they do not show if it is due to notch plasticity. Moreover, crack closure values have not yet been experimentally calibrated or verified since conventional techniques for monitoring crack opening (compliance, potential drop) are inappropriate to short cracks for which high resolution is needed **(8)**.

A number of studies performed within the last years have shown that short crack growth at smooth surfaces presents the same anomalies as mentioned above. However, a fundamental difference is that the crack growth rate decreases, and possible arrests, are due to microstructural barriers (e.g., grain boundaries) during the initial crystallographic cracking stage, or at a transition to non-crystallographic cracking **(9)**. This 'microstructurally short crack' behaviour cannot be explained by continuum mechanics, especially since it appears to be characterized by transient retardation(s) and re-acceleration(s) from a 'mean', and possibly a continuously increasing growth rate. El Haddad *et al.* have reported that such a 'less anomalous' short crack growth could be successfully described by a mechanical parameter **(2)**. Although multiple microstructural interactions are sometimes reported, in most cases such retardation(s) take place at the very first grain boundary, while transition to LEFM behaviour often occurs at a larger scale **(9)(10)**.

Several arguments have been put forward to explain the existence of a mechanical component in short crack behaviour at smooth surfaces; for example: (i) the loss of the inverse square root singularity **(11)**; (ii) closure-based arguments which includes an initial absence of Elber's mechanism in the crack wake **(12)**, or of rugosity-induced crack closure **(8)** which can be considered as a microstructural potential effect; (iii) an initial absence of a surrounding elastic medium in which the crack tip plastic zone is fully contained and constrained **(11)**.

Regarding the specific problem of short cracks growing at notches, an exclusive control by notch plasticity appears somewhat doubtful when considering the above microstructural and mechanical effects. Furthermore, the experimental and theoretical information previously mentioned does not allow an exact evaluation of notch plasticity effects on short crack propagation and closure. However, the present investigation shows that short crack behaviour is exclusively controlled by notch plasticity when some conditions are fulfilled.

Experimental conditions

This paper presents some results of a project of the fatigue of automatic welded joints **(13)**. A cruciform welded joint configuration was selected (Fig. 1). Fatigue tests were performed at constant amplitude loading in the X direction ($R = -1$ and $R = 0$). The crack propagated from one weld toe in the Y–Z

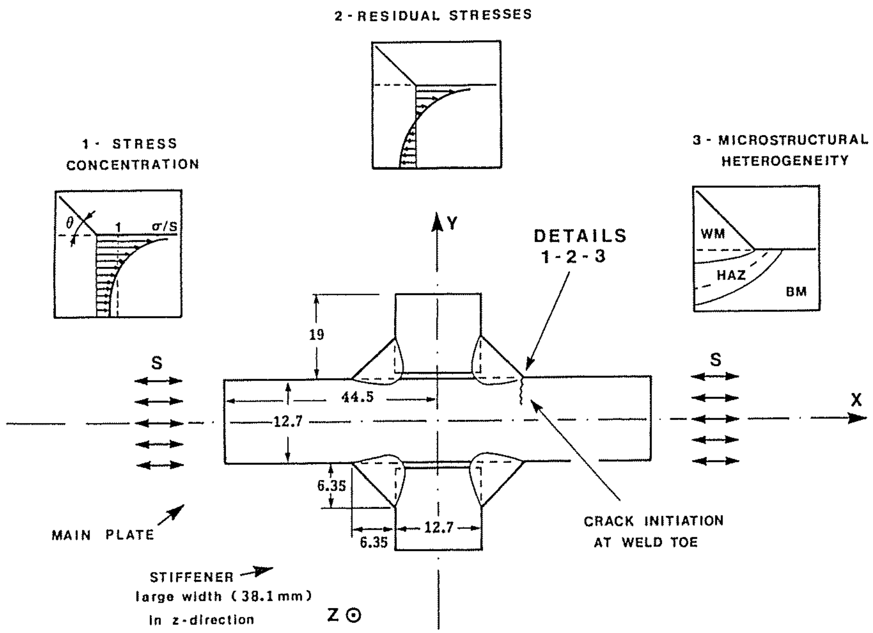

Fig 1 The cruciform welded joint (dimensions in mm)

plane as shown in Fig. 1; the three other weld toes were machined to give a smooth geometry. The major specimen dimensions were; main plate and stiffener thickness, 12.7 mm and specimen width, 38.1 mm.

The base metal was ASTM A36 steel and the weld metal AWS-CSA E70S-7 steel. Welding was performed by an automatic MIG process with gravity in the *Y* direction. This gave the weld toe an ideal V-shaped geometry, as schematically drawn in Fig. 1, and a straight weld bead in the *Z* direction. The weld toe stress concentration fulfils three conditions: (i) severity – the curvature radius at the weld toe apex is about 50 μm (or less); (ii) triaxiality, due to severity and large dimensions in the *Z* direction which impose a plane strain state; (iii) uniformity in the *Z* direction.

Fatigue at the weld toe is also related to the residual stress field and the heterogeneous microstructure due to welding. The residual stress influence **(13)(14)** is beyond the scope of this paper, which is only concerned with the stress-relieved state of the welded joint obtained after an appropriate heat treatment. The microstructure grain size of the heat affected zone is generally fine ($\simeq 5$ μm), while the unaffected base metal (Fe and Fe_3C), which extends from about 1.5 mm below the weld toe, has elongated grains (10–50 μm). Finally the coarse-grained HAZ (which extends to 0.3–0.5 mm in depth) prior austenite grain size is also about 10–50 μm.

Uniform through-cracks have been systematically observed, even at very small crack depths and low nominal stresses. This is mainly due to weld toe unformity in the *Z* direction, but notch macro-plasticity also bears some responsibility. The overall result is that nearly 90 per cent of the crack propagation life is consumed within the first millimeter of crack growth, and so short crack growth represents a large portion of the fatigue life for those automatic welds. This led us to develop an efficient crack monitoring system.

Crack length and opening level monitoring system

The main features of the system are schematically presented in Fig. 2. When the crack initiates, the response of a strain gauge installed on the main plate near the weld toe is such that there is a progressive deflection of the upper part of the recording. This is due to the fact that the stress flow lines bypass the tip of the propagating crack when it opens.

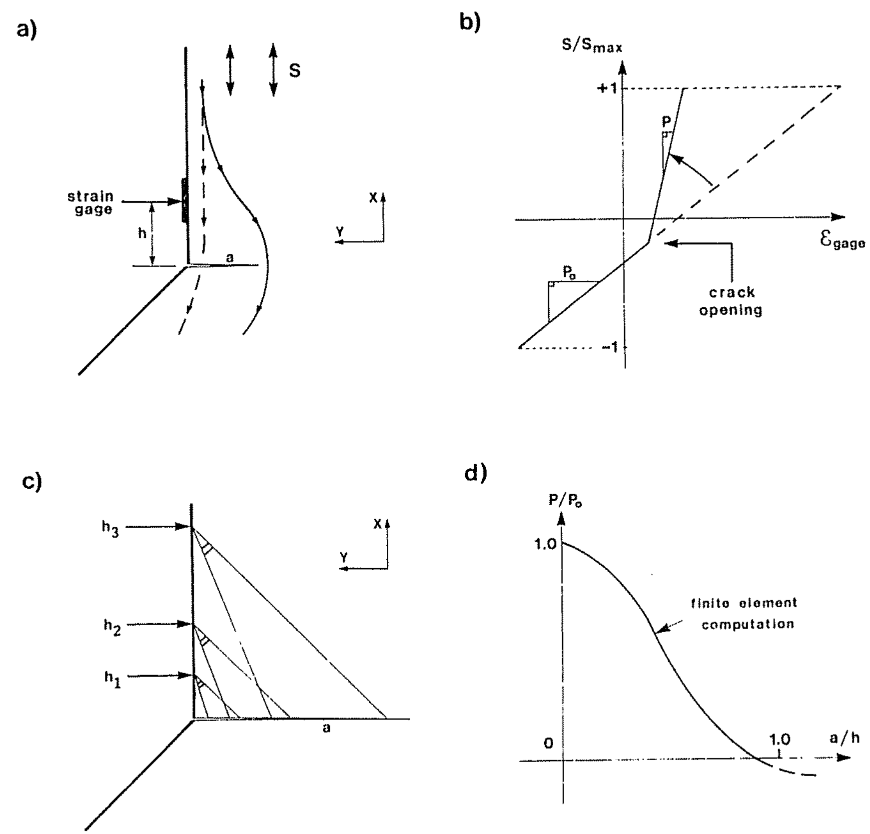

Fig 2 The crack length and opening level monitoring system

The gauge response ratio, P/P_0 is calibrated against crack length a by finite element computation (Fig. 2(d)) which depends on the height, h, of the gauge centre from the fracture plane. The response saturates when $a \simeq h$. The first gauge is progressively replaced by others (Fig. 2(c)), which are less sensitive but which cover longer crack lengths.

This system has many advantages. For example it is simple, very sensitive, and accurate. The first gauge, which is installed as close as possible to the fracture plane, easily detects crack lengths of 10–20 μm, and the theoretical calibration, which includes an integration over the 350 μm gauge length, has been verified by 14 marks, obtained via overload or china ink techniques, which were distributed among seven specimens submitted to different loading conditions. The relative error is within 5 per cent over the calibrated crack lengths which ranged from 70 μm to 5 mm. Secondly the ratio P/P_0 is independent of many parameters, such as strain gauge calibration, Young's modulus, Poisson's ratio, and the angle θ at the weld toe. Thirdly the system allows monitoring of the crack opening level from a $\simeq$10–20 μm with an absolute error within $\pm$6 MPa.

From the results obtained in this investigation, the system, which can be used in many other situations, would appear to be suitable for studying the propagation and closure behaviour of short fatigue cracks.

Elasto-plastic finite element analysis

Some theoretical features are briefly mentioned here because they allow a better physical interpretation of the experimental results. Further details are presented in references (**5**) and (**13**).

Except for a very small zone, 6 μm in depth, which is influenced by the 50 μm radius at the weld toe apex, the stress distribution is linear on a log–log plot (Fig. 3), and the slope is identical to that of a V-notch with the same θ angle (**15**). In other words, the stress field is singular, and the singularity, α (defined as the absolute value of the slope of Fig. 3), varies from 0.5 in the case of a crack (θ = 180 degrees), to 0 in the case of a smooth surface (θ = 0 degrees). However, α decreases only slightly when θ decreases from 180 degrees: α = 0.407 for θ = 90 degrees (**5**), and α = 0.333 for θ = 45 degrees (Fig. 3). As a consequence, there is a quasi-similitude between the stress field ahead of a V-notch and the one ahead of a crack tip, i.e., ahead of the most severe notch.

Figure 4 shows the finite element mesh (straight lines) and the contours of the notch cyclic plastic zones which are obtained under fully reversed loading for different nominal stresses, ranging, approximately, from the endurance limit stress to general plasticity; the nominal (cyclic) yield stress chosen for the computations is well representative of the experimental behaviour. Such a directional macro-plasticity, which is more pronounced for θ = 90 degrees is not surprising when considering the quasi-similitude with a crack (in plane strain). This indicates that the severity, plus the triaxiality ahead of the notch, can be sufficient to initially force a newborn crack to propagate along the plane

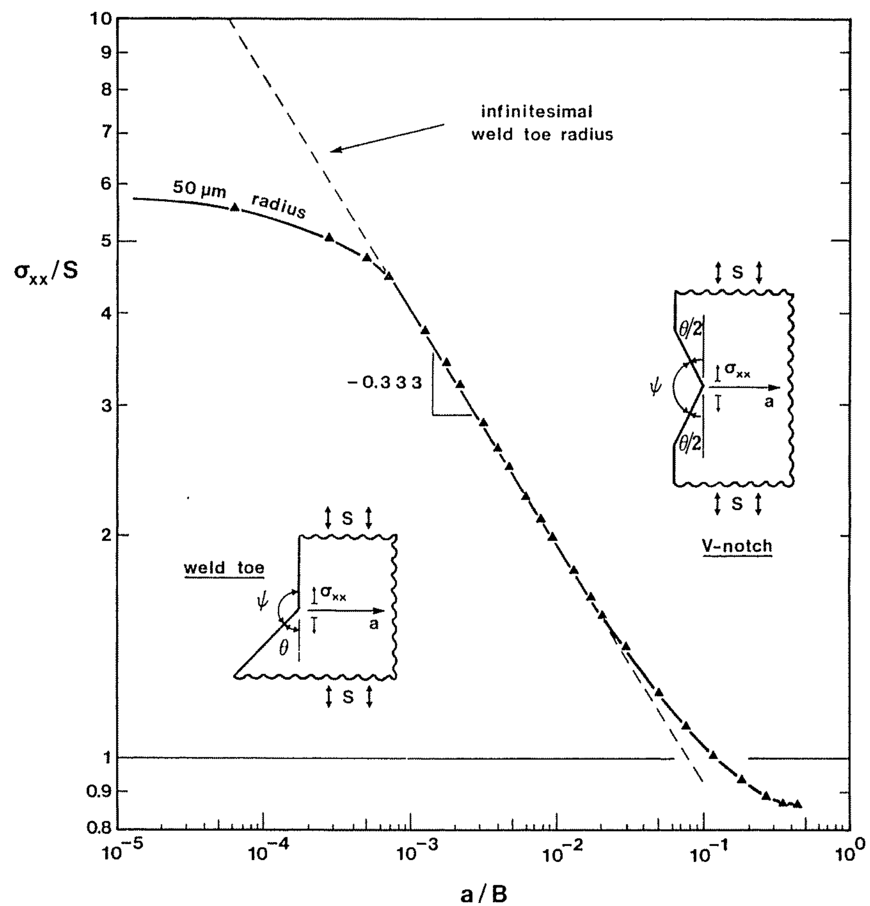

Fig 3 The elastic stress distribution at the weld toe (θ = 45 degrees; α = 0.333; B = 12.7 mm = plate thickness)

of maximum tensile stress. Hence one can expect there is neither an initiation stage nor a microstructurally short crack propagation stage. This was experimentally verified since the crack initiation life (as detected by the first gauge) was always short even at the endurance limit level and striations were found very close to the edge of the fracture surface.

The elasto-plastic finite element analysis determines the notch plastic zone extent a_{pz} (Fig. 4) as a function of the nominal stress. The computations show that one can expect an important effect of notch plasticity on short crack propagation behaviour since the newborn crack is completely surrounded by a plastic zone which is still about 100 μm in depth and 500 μm in width at the endurance limit level. This is due to the fact that, even in plane strain, the small root radius is insufficient to moderate the V-notch severity.

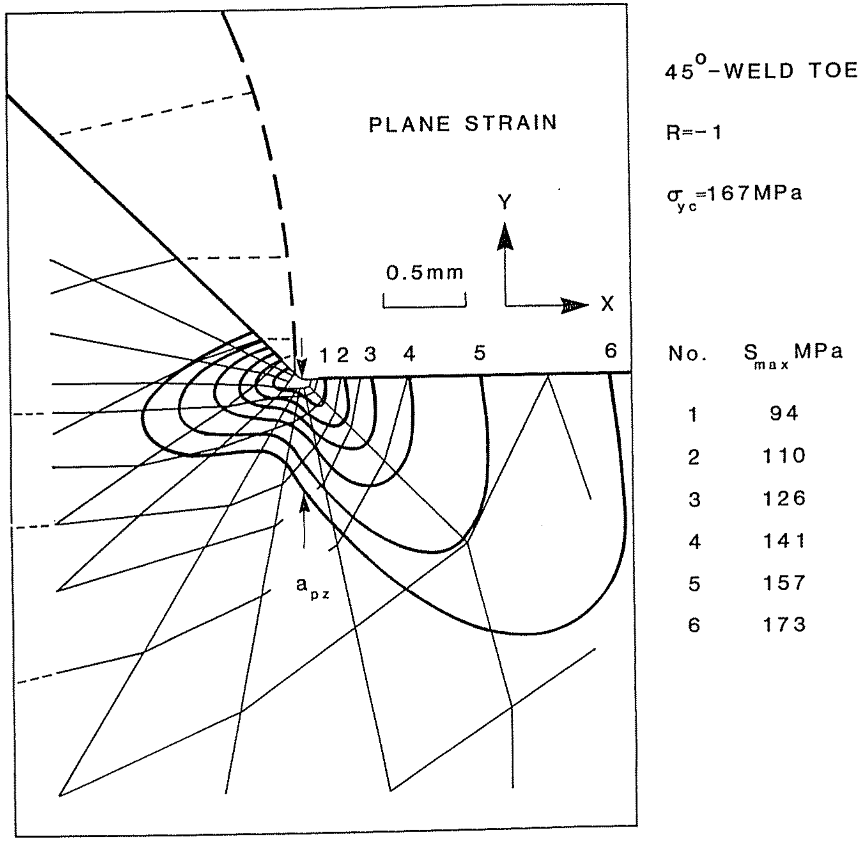

Fig 4 Cyclic plastic zones at the weld toe (kinematic hardening)

Experimental results

Figure 5(a) represents a typical evolution of crack length vs cycle life recorded under fully reversed loading below the nominal cyclic yield stress (σ_{yc} = 167 MPa). The fatigue fracture can be divided into three successive periods.

(1) The first gauge does not detect any variation greater than 20 μm, but life at this detection level never exceeds 10 per cent of total life.

(2) A rapid crack growth is recorded (2a), then the crack growth rate decreases by one order of magnitude and remains nearly constant (2b); LEFM cannot account for this short crack behaviour since the factor K increases with crack length.

(3) The crack growth rate invariably increases until final fracture and is now compatible with LEFM.

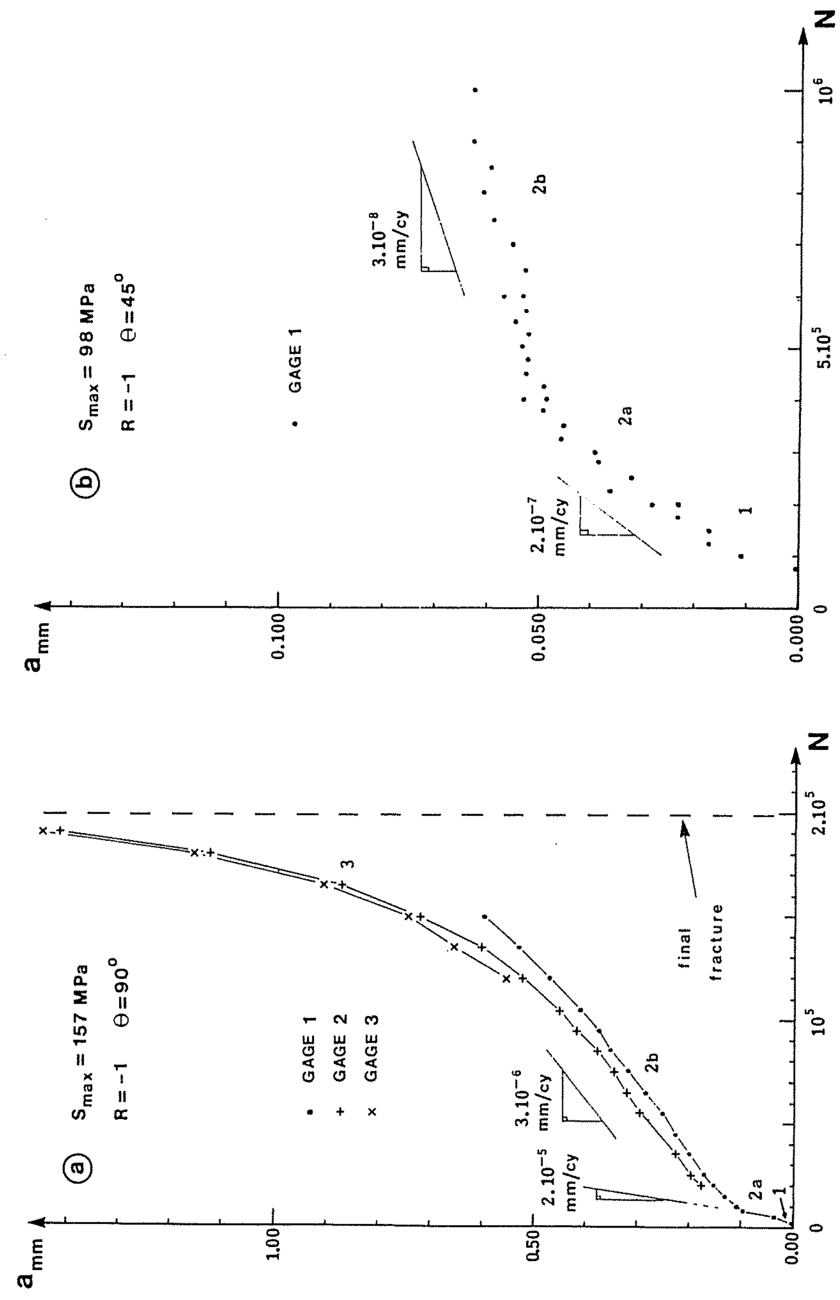

Fig 5 Crack length versus cycle life: (a) below nominal yield stress; (b) at endurance limit level

A similar behaviour is still observed at the endurance limit level (Fig. 5(b)), though crack growth rates differ from Fig. 5(a) by two orders of magnitude. Period 3 is not apparent here because of a power failure after 10^6 cycles. The rate of change of crack length in sub-period 2(b) is very small and may be due to a drift in the electronics with time. Hence, this crack may become non-propagating.

Crack growth data from five specimens subjected to different nominal stress levels are reported in Fig. 6 on a conventional da/dN versus K_{max} plot. As expected, crack growth rates corresponding to part 3 (open symbols in Fig. 6) are well correlated with K_{max} (dashed curve), while short crack growth rates (solid symbols) exhibit important discrepancies from this long crack trend. Behaviour above the nominal yield stress (square symbols of Fig. 6) is not considered here. The major discrepancy occurs at the endurance limit level where cracks grow below the conventional long crack threshold, and can become non-propagating (S_{max} = 98 MPa; Fig. 6).

Two factors concerning the effect of notch plasticity on short crack behaviour are: (i) crack length of transition to LEFM behaviour increases with nominal stress and is comparable in size with the extent of the notch plastic zone computed by finite element analysis. This length cannot be correlated with any microstructural parameter such as grain size; (ii) the short crack growth rate systematically decreases except above the nominal yield stress. Indeed the 'initial crack driving force' produced by the V-notch is likely to decrease rapidly because of a severe plastic strain gradient. the fact that the same short crack behaviour is still observed at the endurance limit level can be attributed to the V-notch severity. However, in this case, the notch plasticity effect can vanish before the long crack threshold is reached (Fig. 6), i.e., before the crack can grow by its own plasticity in the LEFM regime.

Generally speaking, crack growth data show an important influence of notch plasticity on short crack behaviour whatever the nominal stress level. Notch plasticity not only promotes immediate crack propagation along the plane of maximum tensile stress, but also has an effect over a crack length up to 500 μm at high nominal stresses (Fig. 5(a)). However, comparison with theory can only be semi-quantitative. For example, the crack length at transition to LEFM behaviour cannot be clearly determined because the crack growth rate is nearly constant over an important distance (Fig. 5(a)). In other words crack growth data alone do not allow strict quantitative evaluation of the effect of notch plasticity and other parameters related to short crack behaviour.

The results relative to the variation of the crack opening level with crack length (Fig. 7) are in good agreement with computations from Ohji and Newman (6)(7). Moreover, they show that the initial transient variation is related to notch plasticity. Note that in this figure $2U = 1 - S_{op}/S_{max}$ and that even at the endurance limit, the slope da/dU of the initial transient variation is proportional to the computed plastic zone extent. Further, that the crack length at which the opening level stabilizes is approximately equal to this extent. This was confirmed by two other tests with a weld toe angle θ of 90 degrees.

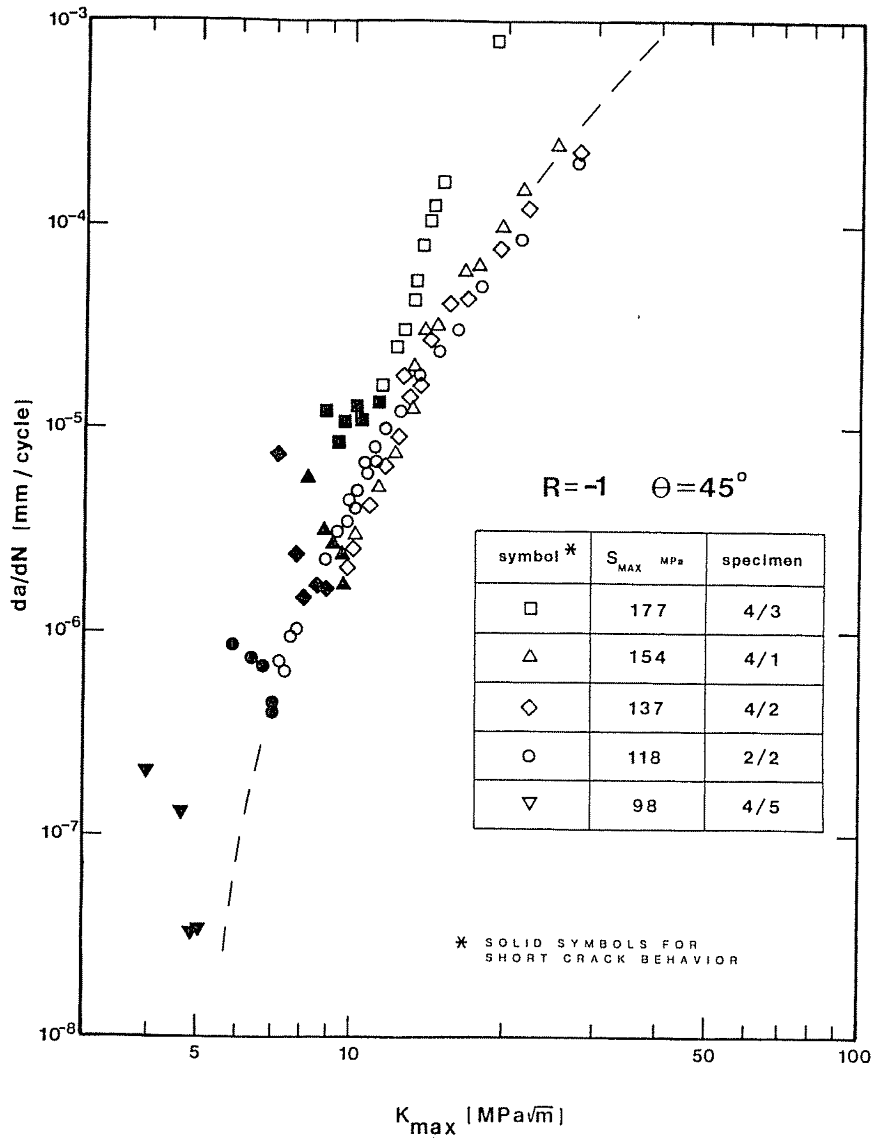

Fig 6 Crack growth rate versus K_{max}

As may be expected, correction of raw crack growth data with corresponding opening levels (Fig. 7) leads to a much stronger relation as witnessed on a da/dN versus ΔK_{eff} plot (Fig. 8). More specifically, the short crack propagation behaviour is quantitatively rationalized by opening level transient variations inside the notch plastic zone. One will note that the knee at low growth rates seen on Fig. 6 disappears on the da/dN vs ΔK_{eff} plot.

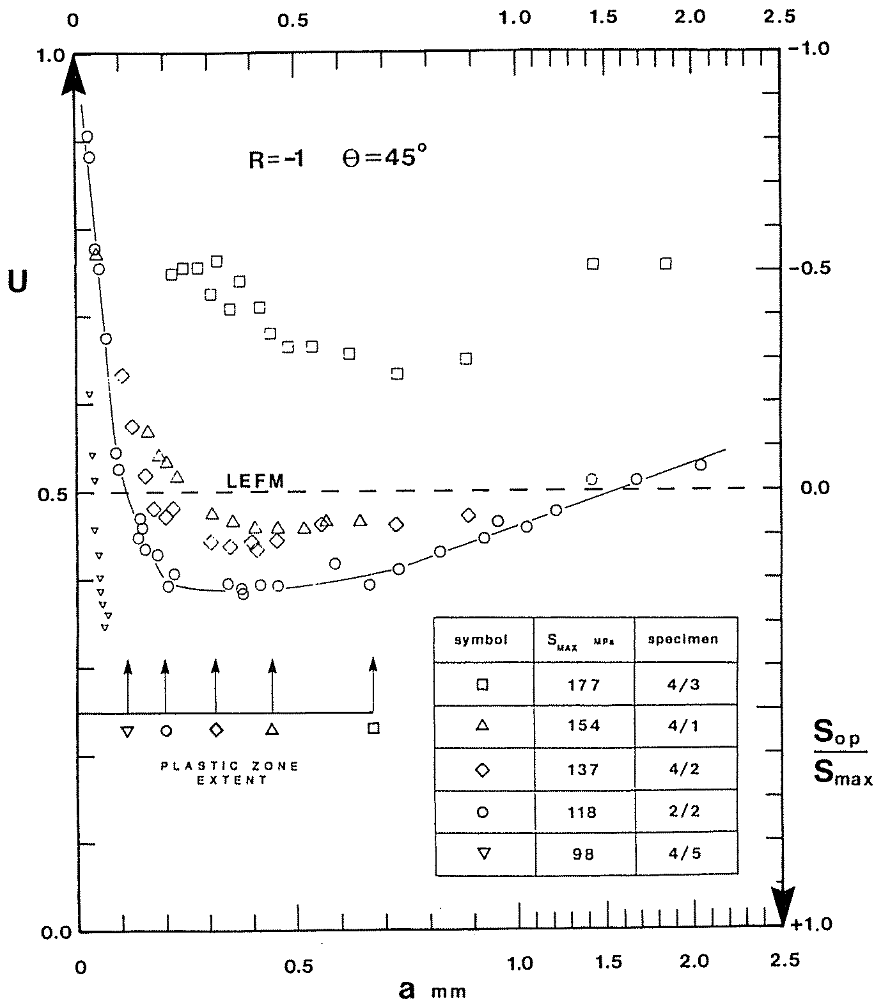

Fig 7 **Crack opening level versus crack length. (Note that the crack length scale is linear, but is reduced above 1 mm)**

Discussion

The fact that short and long crack propagation data are both correlated by ΔK_{eff} is a further confirmation that the short crack behaviour observed in this investigation is purely 'mechanical'. In addition, the systematic correspondence between the crack length at which the opening level becomes stabilized and the computed plastic zone extent (Fig. 7) shows that notch plasticity is totally responsible for the short crack (closure and propagation) behaviour reported here.

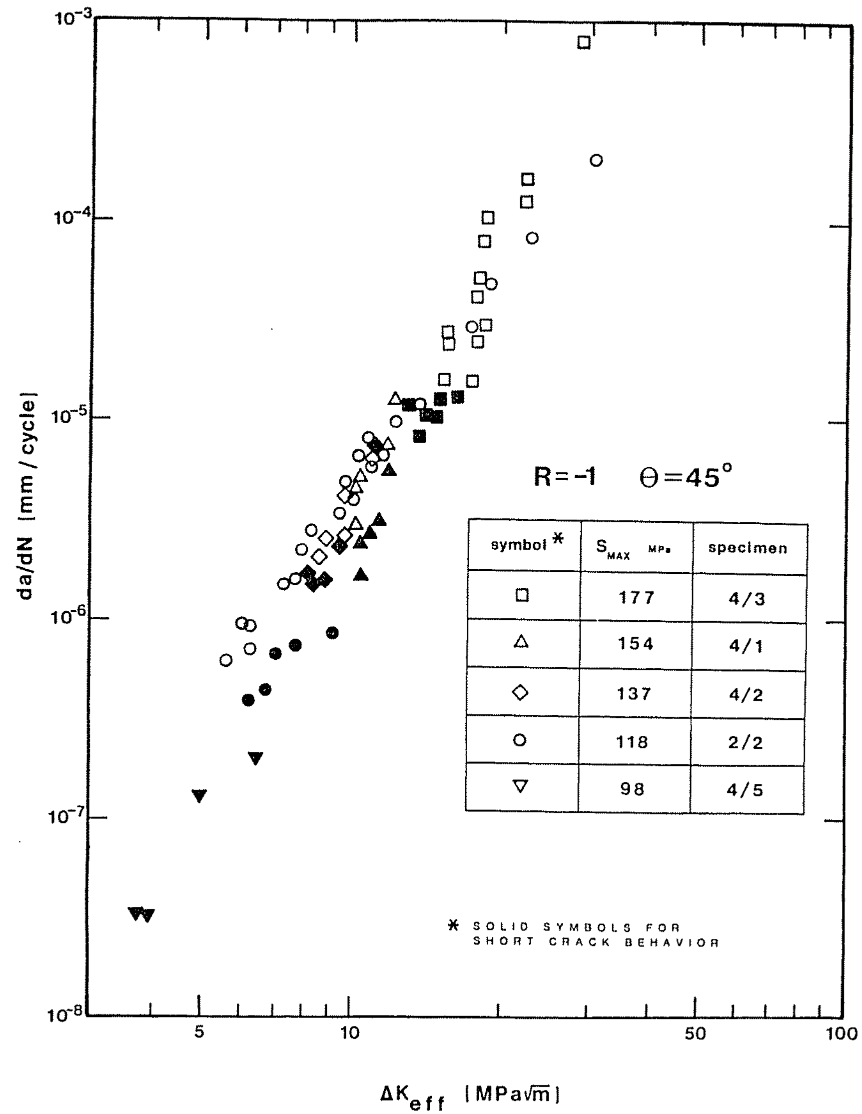

Fig 8 **Crack growth rate versus ΔK_{eff}**

The absence of closure ($S_{op} < 0$; Fig. 7) is a direct consequence of the notch plasticity which is ahead of the crack tip; note that an initial absence of Elber's mechanism in the crack wake cannot explain negative S_{op} values. The total absence of closure when the crack is starting (Fig. 7) is in agreement with the computations of Ohji and Newman. Although the decrease of U is slightly convex, it is sufficiently large and abrupt for ΔK_{eff}, and thus the crack growth

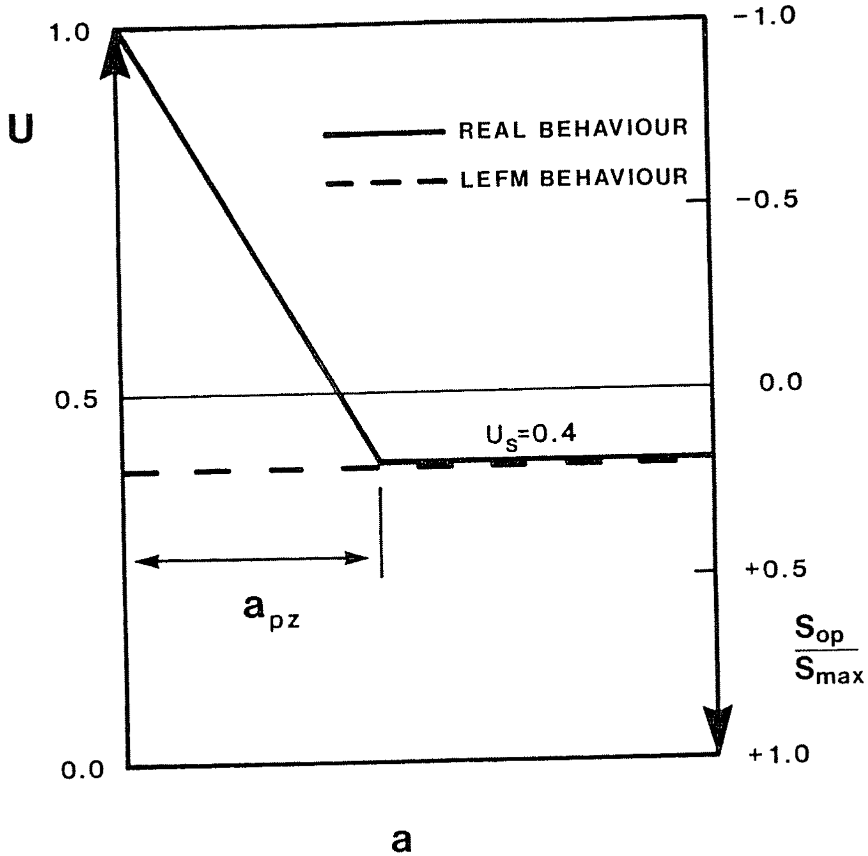

Fig 9 Idealization of the crack opening level variation at $R = -1$

rate, to decrease, and the possibility of a non-propagating crack at low nominal stresses is explained if the existence of an effective threshold $\Delta K_{eff,th}$ is assumed.

Variations of S_{op} are much smaller in the 'quasi-LEFM' regime outside the notch plastic zone (Fig. 7). The fact that the stabilized value of S_{op}/S_{max} slightly increases when S_{max} decreases is in agreement with other computations by Newman (**16**), which show that Elber's mechanism in the crack wake has a maximum effect at low nominal stresses. This increase corresponds to a less effective stress intensity factor range, which explains the disappearance of the knee in lower part of the da/dN versus ΔK_{eff} plot. Reappearance of negative S_{op} values when approaching general plasticity (Fig. 7) is also in agreement with Newman's computations: plasticity ahead of the crack tip overcomes plasticity behind the crack tip, and removes closure. Note that general plasticity has already an important effect on short crack behaviour when nominal stress

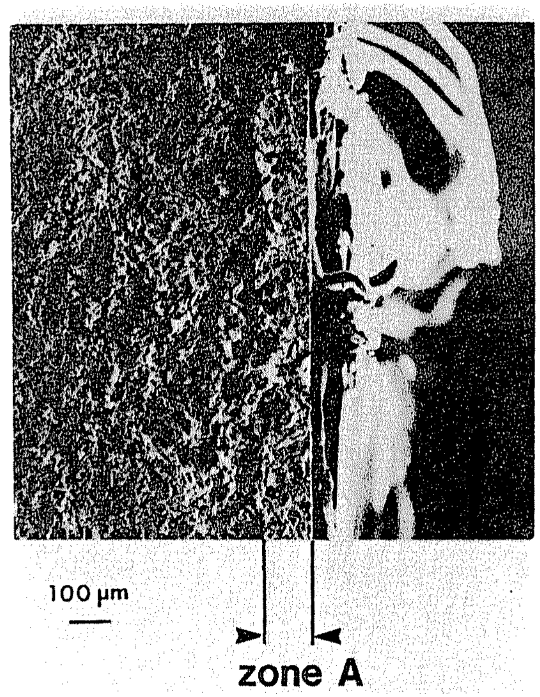

Fig 10 Straightness of the almost non-propagating crack front (da/dN = $3 \cdot 10^{-8}$ mm/cycle; Fig. 5(b))

exceeds yield stress (square symbols; Fig. 7), and in this particular case, although validity of the ΔK_{eff} parameter is somewhat doubtful, the much slower transient variation of the opening level inside the notch plastic zone explains why the short crack growth rate does not decrease (Fig. 6).

From a practical point of view, the variations of the crack opening level in the 'quasi-LEFM' regime outside the notch plastic zone and below the nominal yield stress are either not very significant with respect to scatter in the da/dN versus ΔK_{eff} plot or they concern a very small portion of the total fatigue life; note that crack length scale is reduced above 1 mm in Fig. 7. As a consequence, one can idealize the dependence of U on the crack length by a linear decrease inside the notch plastic zone, followed by a single stabilized value outside it (Fig. 9). This confirms our assumptions (**5**) proposed to estimate the resulting errors in fatigue life prediction when the notch plasticity effect is not taken into account. Consequently a computerized life prediction model using the ΔK_{eff} parameter has been developed on the basis of such an idealization of real behaviour (**14**).

The fundamental reasons which allow such an idealization whatever the nominal stress (except for general plasticity) are believed to be as follows.

(1) Newman's computations (**16**) show that the opening level of long cracks strongly depends on the plastic constraint factor, especially at low R ratios. Absence of triaxiality (e.g., plane stress) amplifies the competition between plasticity behind the crack tip (involving closure by Elber's mechanism) and plasticity ahead of the crack tip (involving absence of closure). As a consequence U rapidly increases with S_{max}. However, in plane strain, which is the present situation, triaxiality is sufficient to induce a quasi-stabilized value of U whatever the nominal stress (Fig. 7).

(2) Near the endurance limit level, it is well known that growing short cracks have a natural tendency to present a semi-elliptical crack front shape (more rigorously, they should be called 'small cracks' (**9**)). This was not the case in the present investigation where, even at the endurance limit level, uniform through-cracks have been systematically observed all along the weld bead. Figure 10 shows the fracture surface of the specimen where the crack was non-propagating (zone A): the crack is 100 μm in depth while it is 38.1 mm in width. This extreme situation must be attributed to the V-notch severity. As previously shown, notch plasticity is at a macroscopic scale, and the high degree of triaxiality tends to activate numerous slip systems everywhere along the crack width; this also contributes to the regulation of the crack front, and probably explains why short crack (propagation and closure) behaviour is similar to the one observed at higher stresses.

Another practical consequence follows from this investigation. The V-notch macro-geometry is such that crack propagation occurs very early even at the endurance limit level (Fig. 5). As a consequence, small pre-existing crack-like defects at the weld toe (which are seldom observed here) would not represent a critical situation. Their influence on total fatigue life should almost equate to the cycle life necessary for producing an initial crack of the same length. Crack growth curves in Fig. 5 show that this life is negligible up to a = 100–150 μm at a high nominal stress (Fig. 5(a)), and at least a = 50 μm at the endurance limit level (Fig. 5(b)). In this latter case, the presence of an initial defect of 50 μm should not influence the endurance limit since a crack of the same length becomes non-propagating (Fig. 5(b)).

Finally, other finite element computations and fatigue tests have been performed under zero-to-tension loading (R = 0) and these can be summarized as follows. (a) For the same maximum nominal stress, S_{max}, the notch monotonic plastic zone has the same size as under fully reversed loading; however the cyclic zone is smaller by a factor of from 5 to 8 depending on the weld toe angle θ. (b) Although reversed yielding is less pronounced, notch severity is such that crack initiation life is still short when compared to total life; however, anomalous (non-LEFM) crack growth rates are barely observed. (c) The variations of crack opening level with crack length are smaller than under fully reversed loading; this confirms the above observations on crack propagation behaviour; S_{op}/S_{max} varies from 0 to nearly 0.35 at a crack length corresponding to the cyclic plastic zone extent. It then decreases to nearly 0.2 at a crack length corresponding to the monotonic plastic zone extent. Subsequent crack growth occurs with an opening level stabilized at the last value, except when approaching general plasticity. (d) From a practical point of view, one can assume a LEFM behaviour with S_{op}/S_{max} equal to 0.2 whatever the crack length.

Conclusions

Fatigue fractures of V-notched members in plane strain have the following characteristics under fully reversed loading, even at the endurance limit level.

(1) The crack initiation stage is very short and can be neglected and the pre-existence of small crack like defects is not a critical condition.
(2) The microstructurally short crack propagation stage appears to be non-existent.
(3) Short crack behaviour is exclusively controlled by notch plasticity (excepting general plasticity).
(4) Short crack growth rates are well correlated with long crack rates when using the effective stress intensity factor range.
(5) Short crack closure behaviour is characterized by an initial transient variation within the notch plastic zone.
(6) A high degree of notch severity and triaxiality allows a simple idealization for fatigue life prediction.

Under zero-to-tension loading, the same conclusions hold; however, the short crack effect is barely pronounced because of a smaller crack closure variation.

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