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Fatigue of Short Cracks: the Limitations of Fracture Mechanics

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ABSTRACT Cracks less than some critical length, defined as l_2, show anomalous growth behaviour which cannot be quantified by LEFM. At present there are no reliable methods for predicting the growth rates and thresholds of these cracks. This paper presents a simple approach which has had some success in predicting the value of l_2 for various materials. This parameter is shown to depend on the material properties yield strength and grain size. The prediction of l_2 for a given material enables the designer to make conservative predictions of the fatigue life of components in situations where short crack behaviour dominates.

Introduction

In recent years there have been many investigations into the behaviour of short fatigue cracks; this body of work is well summarized by three recent reviews(**1**)–(**3**). Possibly the most important point to emerge is that, where short cracks are concerned, present design procedures, whether based on S/N type data or on fatigue crack propagation data, are in danger of being non-conservative. This can be illustrated using the now-common method of displaying short crack data, i.e., the plot of threshold stress range as a function of crack length. Figure 1 shows schematically the type of results obtained; the open-circle data points represent the region in which use of either the fatigue limit or the long-crack threshold value gives too high a value for the threshold stress and is thus non-conservative.

In a paper concerned with the fatigue behaviour of an aluminium bronze alloy (**4**) the author defined the terms l_1 and l_2 as the values of crack length at the limits of this non-conservative region. The parameter l_1 is possibly only of academic interest, since there are few practical situations for which defect sizes as small as l_1 are important. However, the parameter l_2 has considerable importance in design and failure analysis work; l_2 values may be greater than 1 mm in some materials, so in components for which there is good control of surface condition and defect size, the initial defect sizes may be below l_2.

Given the practical difficulties of measuring l_2 values, some method of estimating l_2 from other material parameters is needed. The author himself has had to deal with failure analysis problems which have required the estimation of an l_2 value, for example, the failure of cast ships' propellers (**5**). If the practical situation is such that inherent cracks larger than l_2 exist, than a normal

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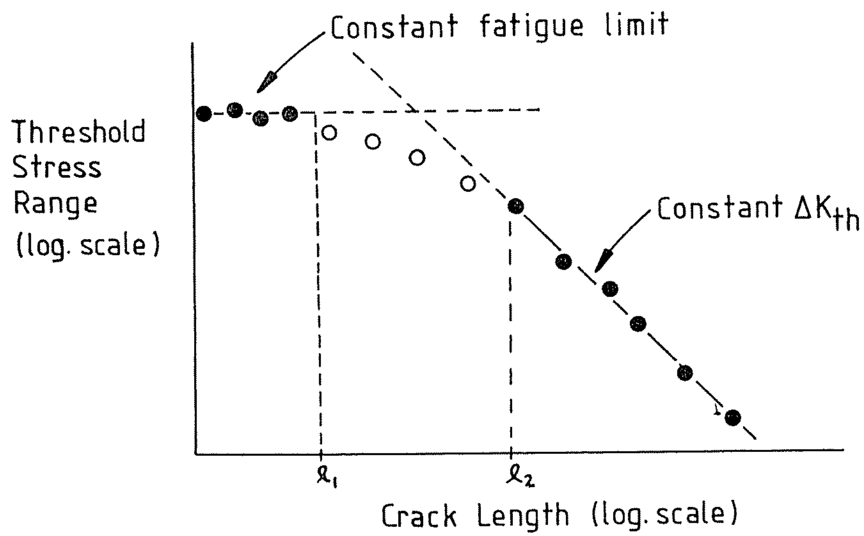

Fig 1 Typical short crack fatigue results; the open circles indicate the region of non-conservative behaviour

conservative approach has to be adopted; the simplest conservative approach in such cases is to take the defect size as l_2.

This paper outlines a simple method which aims for an approximate and conservative estimate of l_2 for any given metallic material. After outlining the form of the prediction and showing that it agrees with known data on short cracks, a detailed justification of the approach follows.

The prediction of l_2

In two previous publications **(4)(6)** a microstructural argument was used to suggest that l_2 could be approximated by

$$l_2 \approx 10d \tag{1}$$

Here d is a 'characteristic microstructural dimension' which is usually equal to the grain size but becomes carbide lath width or precipitate spacing in microstructures dominated by fine precipitates. Thus d is similar to the 'effective grain size', $\bar{l}$, defined by Yoder *et al.* **(7)** in their work on long crack thresholds.

It was shown **(4)(6)** that quite good correlation existed between l_2 and $10d$ for available data on a range of materials. However, the microstructural argument is clearly not a complete one. A number of other hypotheses have been advanced to account for short crack behaviour, not all of which are concerned with microstructural details; these include the effects of crack closure **(1)**, crack-tip stress distribution **(8)**, crack deflection **(9)**, and constraint **(1)**. These various models will be discussed in more detail below, where it will be shown that they can be divided into two types: those based on microstructure, for which the parameter d is important, and those based on plasticity for which the appropriate parameter is the plastic zone size. The extent of the reversed plastic zone r_p in fatigue can be estimated as $0.04(\Delta K/\sigma_{y,c})^2$, where $\sigma_{y,c}$ is the cyclic yield strength.

In what follows it will be shown that the plasticity based arguments can be reduced to the condition that, for LEFM to apply, the crack length must be greater than $10r_p$; hence a second estimate of l_2 is

$$l_2 \approx 10r_p \tag{2}$$

Equations (1) and (2) represents two independent conditions on the value of l_2; thus, for any given material, logic dictates that l_2 will be given by whichever is the larger of $10d$ and $10r_p$.

There is a fundamental point which should be borne in mind which makes this simple analysis of l_2 possible; there are many different factors which affect the behaviour of a short crack, including the five listed above, and this makes the prediction of the growth rate and threshold of a short crack very difficult. However, each of these different effects can be thought of as having an l_2 value associated with it, so the observed value of l_2 for the material will be whichever is the largest of these, assuming only that the various effects act independently. For instance, if microstructural considerations lead to a prediction of, say, $l_2 = 10\ \mu m$, but a closure argument predicts $l_2 = 100\ \mu m$ for the same material, then 100 μm will be the relevant value and microstructural effects can be discounted as far as l_2 is concerned.

Comparison of predictions and results

Figure 2 shows measured values of l_2 for various materials **(3)(4)(10)–(17) (19)(34)** plotted against $10d$ and $10r_p$. The materials covered include mild steels, high strength steels, titanium, copper, and aluminium alloys; the open circles are the $10d$ values and the solid circles are the $10r_p$ values. The letters alongside the points refer to the original publications, as shown in Table 1.

The accuracy of the determined values of l_2 is estimated to be within 20 per cent unless the error bars on Fig. 2 indicate to the contrary, likewise the accuracy of the $10d$ and $10r_p$ determinations is estimated at 20 per cent unless otherwise indicated.

Figure 2 shows that there is good predictive capacity with this model over the whole range of l_2 values and different materials. Taking the largest of $10d$ and $10r_p$ (when both are available) then of the 17 different results presented, 15 of the predictions are either correct or slightly conservative. For the 11 cases where both d and r_p values were available, there are two cases in which $d > r_p$, three cases in which $r_p > d$ and six cases where d and r_p are equal within the errors of estimation.

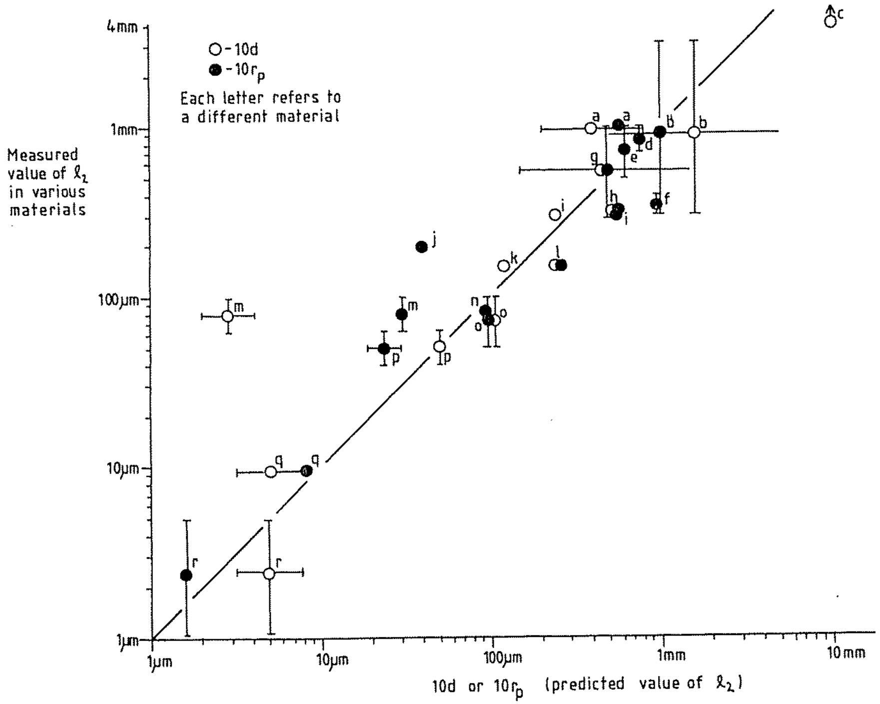

Fig 2 Comparison of measured and predicted values of l_2. See Table 1 for source references

Table 1 Sources of data on Figure 2

Reference letter on Fig. 2	*Publication*
a	(11)
b	(13)
c	(37)
d	(16)
e	(16)
f	(16)
g	(14)
h	(4)(10)
i	(13)
j	(17)
k	(15)
l	(13)
m	(3)
n	(19)
o	(3)
p	(3)
q	(12)
r	(13)

Another important point to note is that the values of d and r_p are always similar in magnitude here; in only one case (3) do they differ by more than a factor of three, though the data range over three orders of magnitude. This arises because we are considering near-threshold behaviour; it has been shown (7)(20)–(22) that d and r_p are approximately equal near the long-crack threshold, ΔK_{th}. The implication of these results is that, if the value of l_2 is controlled by r_p rather than by d, then the value of l_2 might be expected to increase with increasing applied load. In the present paper, only near-threshold results are used, and l_2 is defined only at the long-crack threshold; few results exist at present to show how anomalous short-crack behaviour varies with applied load, though clearly this point should be pursued in the future.

Hypotheses relating to short crack behaviour

The above section has shown that the simple hypothesis described by equations (1) and (2) is capable of giving good predictions for l_2 values. The following section examines various proposed models of short crack behaviour in order to justify this hypothesis in the light of the various mechanisms advanced by other workers. Five distinct reasons for anomalous short-crack behaviour have been advanced:

(1) microstructure (**4**);
(2) closure (**1**);
(3) K-estimation errors (**8**);
(4) crack deflection (**9**);
(5) constraint (**1**).

These have already received much attention and discussion elsewhere (**1**)–(**3**). Here it will be shown that the microstructure and deflection arguments can be related to the $10d$ prediction and that the other three arguments can be related to the $10r_p$ prediction.

(1) Microstructure

Observations by the present author and others (**4**)(**5**)(**15**)(**17**) showed that anomalous growth rates in short cracks frequently occurred when cracks were growing in a single grain or a small number of grains. It was noted that linear elastic fracture mechanics demands a homogeneous continuum, which is unlikely to be the case until the crack front length becomes considerably greater than the grain size. Considering a typical surface crack, semi-circular in shape, a crack length of $10d$ would give a crack front length of about $31d$, which would satisfy a homogeneity requirement such as the one outlined in section (4) below. Thus equation (1); i.e., $l_2 \approx 10d$ can be thought of as a sufficient condition on the value of l_2 in this case, and therefore a conservative prediction. It must be remarked, however, that d is the mean of a grain-size distribution; thus, some cracks of length $10d$ will pass through fewer than 31 grains along their fronts. It can be shown (**23**) that there is only a very small probability of the crack front passing through less than 10 grains at this length, given the normal form of the grain-size distribution, and assuming no grain-shape texture effects, such as are common in wrought alloys.

(2) Crack closure

Since crack closure is caused by residual stresses in the crack wake, it has been proposed (e.g., (**19**)) that a short crack will experience less closure because there is insufficient length behind the crack tip for the wake field to fully develop. This seems to be a very powerful argument, and it has certainly been shown that the closure characteristics of long and short cracks differ considerably. The picture is confused by the difficulty of accurately measuring short-crack closure; only very small changes in compliance are detected, and by the effect of microstructural features such as grain boundaries (**15**)(**24**).

It has been proposed (**1**) that for a homogeneous continuum the crack length should be greater than the reversed plastic zone size, r_p, in order to establish the full wake field and therefore ensure normal closure behaviour. This seems a rather short length; as yet there is no reliable evidence to settle the question, but in that case the condition of equation (2), i.e., $l_2 \approx 10r_p$ will express a sufficient condition on l_2.

(3) K estimation errors

Perhaps not enough attention has been given to the fact that the normally-used equation for stress distribution at a sharp crack

$$\sigma = \sigma_o \sqrt{(a/2r)} \tag{3}$$

where

σ = stress at a distance, r, from the crack tip

and

σ_o = applied nominal stress

is valid only for r very much less than a and should be replaced in other cases by the more complete form of the Westergaard equation

$$\sigma = \frac{\sigma_o\{1 + (r/a)\}}{\{2(r/a) + (r/a)^2\}^{1/2}} \tag{4}$$

Sinclair and Allen (**6**) have shown that for short cracks which show anomalous behaviour, the difference between equations (3) and (4) is significant. However it is difficult to define a K value from equation (4) because of the lack of a simple r singularity. Using a method based on comparison of plastic zone size, Sinclair and Allen define an effective K value, K_{eff}, as

$$K_{eff} = K(1 + r_p/a)^{1/2} \tag{5}$$

Equation (5) implies that for any value of r_p, K_{eff} will be greater than K. However, if r_p/a is small, K_{eff} will approach K in magnitude. If K_{eff}/K is arbitrarily set to 1.05 (i.e., taking a 5 per cent difference between K_{eff} and K), this will lead to a value for R_p/a of approximately 0.1, which would amount to a restatement of the condition in equation (2). The choice of the figure of 5 per cent is arbitrary, but represents the amount of scatter usually observed on threshold data measurement. The use of 10 or 2 per cent would lead to estimates of l_2 which would be of the same order of magnitude.

(4) Crack deflection

Figure 3 shows schematically the difference between an idealized straight crack and a real crack. The crooked, deflected crack path tends to lower the effective K value, as has been considered very thoroughly by Suresh (**9**) and Kitagawa *et al.* (**25**). Suresh also showed qualitatively that a short crack, because it has only a few deflections in it, would be expected to have a K value which varied from one crack to another, whereas in a long crack these effects would average out, giving a consistent value for K. Suresh was unable to use this deflection argument to explain the increased growth rates of short cracks, and indeed this does not seem to be possible unless one postulates a very strong dependence on the K_{II} value.

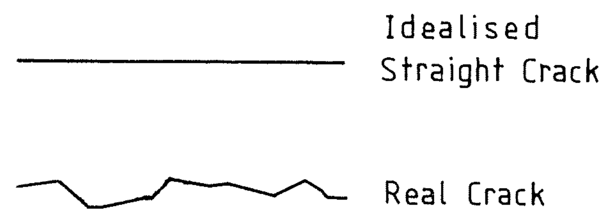

Fig 3 Schematic of idealized and real cracks

Considering the definition of l_2, this deflection condition amounts to another microstructural effect, since it has been shown that for cracks growing under near-threshold conditions a faceted or 'structure sensitive' growth mode occurs (**21**) in which the facets are equal to the grain size.

Two effects can be considered here: the effect of crack length and the effect of crack front length. Considering crack front length, a deflected crack will be modelled here as shown in Fig. 4, i.e., a crack with a straight section of length a and an end portion of length d deflected through an angle θ. Using (**27**) it is possible to estimate the reduction in effective K value resulting from deflecting the end portion. For example, if θ = 45 degrees the k value is reduced from its nominal value by about 20 per cent. This figure is only slightly dependent on the values of a and d, because an increase in the ratio d/a tends to decrease K_I but to increase K_{II} for a given θ.

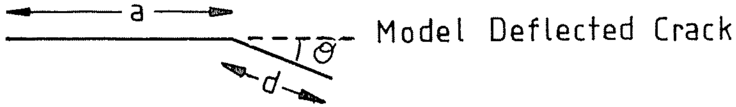

Fig 4 Model deflected crack to be used in analysis

Taking a crack front only one grain in length, and allowing θ to vary at random from +45 degrees to −45 degrees gives a distribution of possible K values; expressed as the ratio $K/K_{nominal}$ these vary from 0.99 to 0.87 if one takes two standard deviations from the mean of the distribution. Thus a very small crack front such as this would be expected to show about 13 per cent variation in its K value if a large number of cracks were examined. If the crack front length is now increased, using a model crack front, as shown in Fig. 5, then the distribution of possible K values tends to narrow, as the effects from separate grains tend to cancel each other out. Figure 5 shows the results of a simple computer analysis of this model; here $K/K_{nominal}$ is plotted as a function of crack front length, expressed as a number of grain diameters. It can be seen that for a crack front longer than $10d$ the distribution becomes narrow and roughly constant at about 2 per cent of the mean.

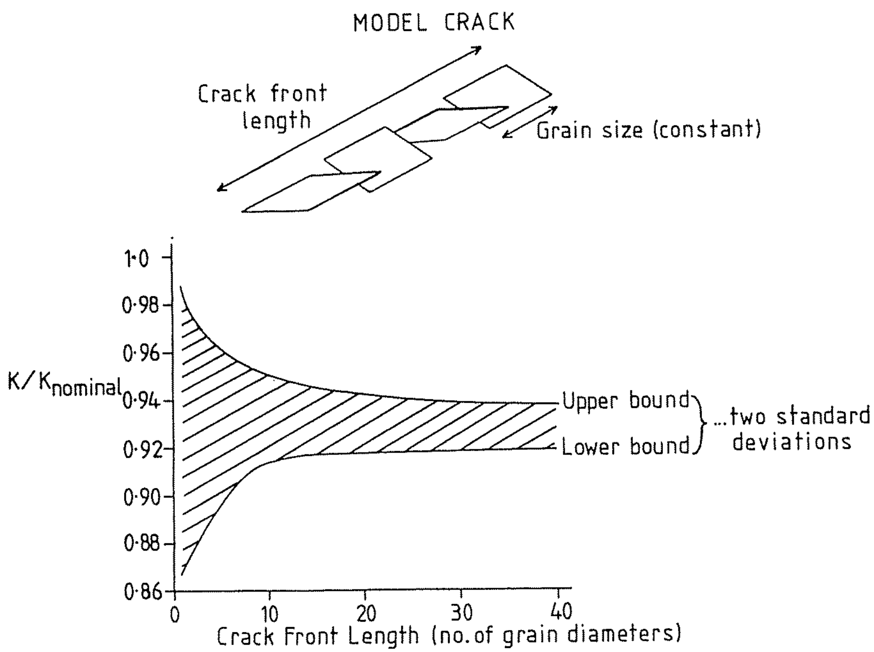

Fig 5 Results of analysis on the effect of crack front length on K. The scatter bands are placed at two standard deviations from the mean of the data

This analysis implies that, as far as this condition is concerned, a crack with a short front may grow faster or slower than a crack with a long front. Taking again the case of a semi-circular surface crack, if the crack length is $10d$ then the crack front length will be $31d$, so the condition $l_2 \approx 10d$ will again express a sufficient condition for l_2.

The effect of deflections along the crack length is more difficult to analyse. Suresh (**9**) considered the singly and doubly deflected cracks and developed a method to deal with a long crack containing many deflections, but as yet the critical range of lengths around $a = 10d$ cannot be treated.

The other postulated effect of crack deflection, as advanced by Beevers and others, is that it induces premature closure, the roughened crack surfaces coming into mutual contact at a higher applied K value in the cycle than expected. If we note again that the degree of surface roughness is proportional to the grain size at near-threshold growth, a similar microstructural condition, i.e., equation (1), would be expected.

(5) *Constraint*

It has been pointed out (**1**) that a small crack is essentially contained in the surface plane stress field until it penetrates some distance into the body of the material, where part of the crack front begins to experience plane strain conditions. Unfortunately it is by no means clear whether a crack in a plane stress field will grow faster or slower than a plane strain crack; experimental results are conflicting (e.g., compare (**26**) and (**27**)).

Following Knott (**28**), it can be assumed that the plane stress field extends into the material from the surface for a distance approximately equal to the plastic zone size. This is shown schematically in Fig. 6 for a semi-circular crack.

Applying the condition of equation (2), i.e., putting $a = 10r_p$, simple geometry shows that approximately 94 per cent of the crack front will be out of the plane stress region; 6 per cent experiencing plane stress conditions. This

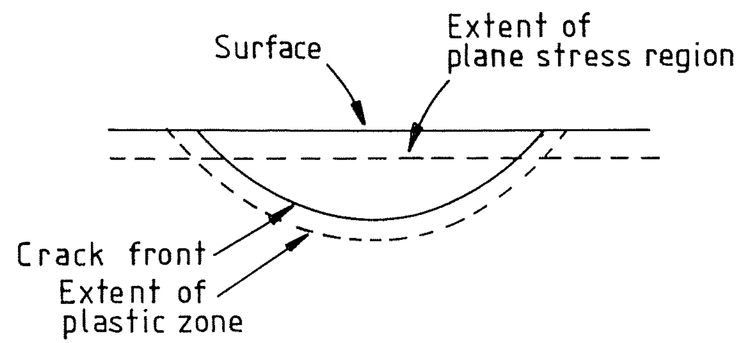

Fig 6 Schematic illustrating the amount of the crack front length which is contained within the surface plane stress region

would seem enough to ensure that the plane stress region was unimportant; hence, equation (2) is again shown to be a sufficient condition on the value of l_2.

Summary of the above discussion

It has been possible to show that the initial hypothesis used in this paper, that l_2 can be estimated as the larger of $10d$ and $10r_p$, constitutes a sufficient condition for each of the five effects considered. Since we are seeking a conservative prediction, a sufficient condition is all that is required. The experimental results shown in Fig. 2, however, suggest that the condition used here is generally either correct or slightly conservative only, so it seems that no severe overestimate of l_2 is likely.

Methods of production of short cracks

When examining experimental data on this subject, it is important to consider the method used by the experimenters to produce the short cracks, as this may affect their behaviour.

In most experimental studies of short cracks, the cracks are produced by initiation from plain specimens, the threshold values being achieved either by load shedding or by load increase after stress relief heat treatment. The process of stress relief presents some problems **(29)** but the two methods seem to give roughly comparable results.

However, some workers (e.g., **(18)(30)**–**(32)**) produce short cracks by machining material from specimens containing long through-cracks. James and Knott **(30)** point out the possible nature of stress history effects and show that, in an alloy steel, even if stress relief is attempted, a much higher value of l_2 would be deduced from these short through-cracks than from surface cracks of more common form. Incomplete stress relief, leaving residual closure stresses, may be involved. Similar results were obtained by Breat *et al.* **(18)** and in an aluminium alloy by Zeghloul and Petit **(32)**.

A conservative design approach

The emphasis throughout this paper has been to formulate and justify a simple method of predicting an appoximate and conservative value of l_2. It is believed that this addresses a growing need for designers and failure analysts attempting to use fracture mechanics in fatigue situations.

It is to be hoped that in the near future, analytical methods will be developed to predict the growth behaviour of cracks shorter than l_2; some progress has been made in that direction using a statistical approach **(37)**, thus extending our predictive capacity into this difficult area. Until this has been achieved, a conservative approach must be used, such as setting the defect size to l_2 in the case where the real defect size is smaller than l_2.

Finally, it should be recognized that the number of situations in practice

where small crack behaviour dominates is small. The most widely quoted example of the short crack problem is the jet engine turbine blade, a situation in which the extreme demands placed on the material require a very small inherent defect size. Indeed this is probably the only commonly-occurring case of a crack of length less than l_2 being subjected to stresses well in excess of its propagation threshold. However a number of other cases arise which involve short cracks stressed close to their thresholds; one example would be precision machine parts such as engine pistons, which require very good surface finish and may be made from materials with quite large l_2 values. It is unlikely that NDT crack inspection techniques can be used to help monitor short cracks in service, as the necessary detection accuracy cannot be achieved by any NDT methods presently in use.

Conclusions

(1) Cracks less than some critical length, denoted l_2, show anomalous growth behaviour which cannot be described by linear elastic fracture mechanics.
(2) Difficulties in the experimental measurement of l_2 call for some method of predicting this parameter for any required material. The predicted value of l_2 should be conservative, i.e., an overestimation, in order to be useful for design purposes.
(3) It is proposed that l_2 can be estimated to be the larger of $10d$ and $10r_p$, d being the effective grain size and r_p the cyclic plastic zone size.
(4) It is shown by comparison with experimental data that this hypothesis gives a reasonable prediction of l_2 for many different materials.
(5) The use of this simple predictive model is justified in detail by examining the various current theories on short crack behaviour.

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