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A Small-Crack Growth Law and its Application to the Evaluation of Fatigue Life

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ABSTRACT In fatigue tests on carbon steels, the life of a plain specimen is occupied mainly by the life in which a crack propagates from an initial size up to about 1 mm. This means that the growth law of a small crack must be known in order to evaluate the fatigue life of plain members.

The growth rate of a small crack cannot be predicted usually by linear long-crack fracture mechanics, but is determined uniquely by the term $\sigma_a^n l$ where σ_a is the stress amplitude, l is the crack length, and n is a constant. However, if the condition of small scale yielding is satisfied during the propagation of such a small crack, it is possible to quantify the behaviour by linear fracture mechanics.

In this paper, a unifying treatment for small-crack growth is presented, and a method for determining fatigue life is suggested.

Introduction

The fatigue life of a plain specimen is controlled mainly by the life in which a crack propagates from a certain initial size up to about 1 mm. This means that we must know the growth law of small cracks if we want to predict the fatigue life of the specimen. Linear fracture mechanics cannot be applied usually to such a small crack. The fatigue life of a structure may also be closely related to the growth of small cracks.

Many investigations of fatigue crack propagation have been carried out and many growth laws have been reported (**1**). All of these growth laws have succeeded in describing crack growth behaviour over a limited experimental programme, though some equations formally contradict each other.

It seems natural to use the crack tip opening displacement (CTOD) as a measure of control in fatigue crack propagation studies (**2**)(**3**). However, the definition of CTOD is not clear, and is somewhat obscure and difficult to measure in polycrystalline metals; therefore, using an assumption that crack growth rate dl/dN is proportional to the reversible plastic zone size (r_{pr}) (**4**), since r_{pr} seems to be substantially similar to CTOD, a unified explanation of fatigue crack growth laws was made by Nisitani (**5**). He suggested two crack growth laws, that is, $dl/dN = C\Delta K^m$ for low nominal stresses and $dl/dN = B\sigma_a^n l$ (**6**)–(**10**) for high nominal stresses.

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In this paper, fatigue tests under axial loading on plain steel specimens are reported, and the unifying treatment for fatigue crack growth under low and high nominal stresses is shown from data obtained on small cracks on the specimen surface. Moreover, a method of fatigue life prediction based on the fatigue crack growth laws is suggested.

Material, specimen, and experimental procedures

The material used in a rolled round bar (about 18 mm in diameter) of 0.45% C steel. The specimens were machined from the bar after annealing for 60 minutes at 845°C. The chemical composition (wt%) was 0.45C, 0.25Si, 0.79Mn, 0.01P, 0.01S, 0.09Cu, 0.03Ni, 0.18Cr, remainder Ferrite and mechanical properties are 364 MPa lower yield stress, 631 MPa tensile strength, 1156 MPa true breaking stress, and 45.8 per cent reduction of area. The shape and dimensions of the specimen are shown in Fig. 1(a). Although the specimen has a fine shallow partial notch, see Fig. 1(b) its strength reduction factor is only about 1.1 and can be considered as a plain specimen. Before testing, all the specimens were re-annealed in vacuum at 600°C for 60 minutes in order to remove residual stresses, and were then electro-polished to remove about 20

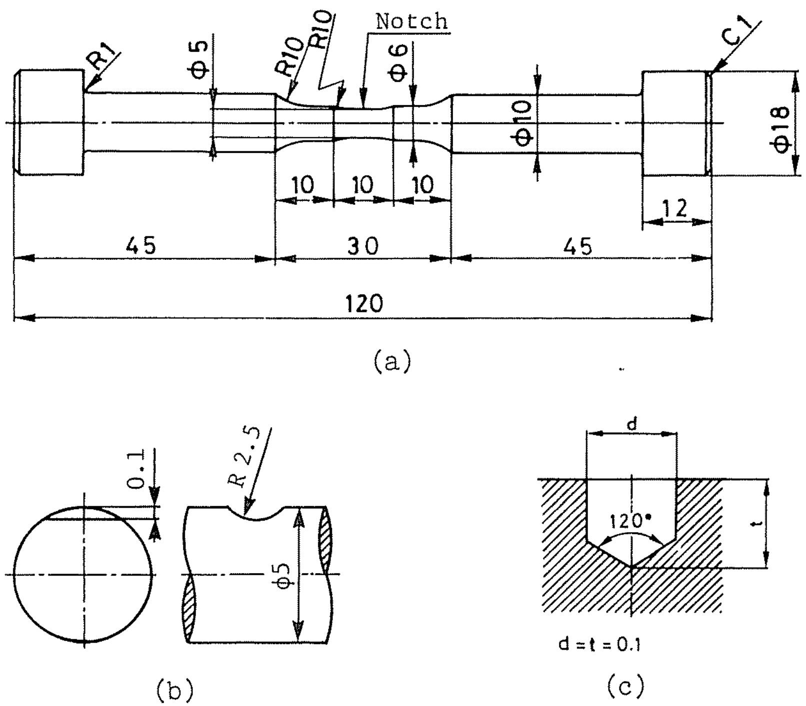


Fig 1 Dimensions (in mm) of: (a) the specimen, (b) the transverse notch, (c) the hole

μm from the surface layer. In some specimens, a small hole was drilled on the central part of the shallow notch after electro-polishing. The shape and dimensions of the hole are shown in Fig. 1(c). Both the diameter and depth of the hole are 0.1 mm.

All the tests were carried out under constant stress using an electrohydraulic control tension–compression fatigue machine at a frequency of 5–32 Hz. The observations of fatigue damage on the specimen surface and the measurement of crack length were made via plastic replicas using an optical microscope (×400). The crack length means the length along the circumferential direction on the specimen surface and when appropriate includes the 0.1 mm diameter of the hole.

The value of stress in this paper means the nominal stress σ_a at the minimum cross section (5 mm in diameter) calculated by neglecting the existence of the shallow notch and the small hole. The stress range $\Delta\sigma$ is given by $2\sigma_a$.

Experimental results and discussion

Crack initiation and propagation behaviour in plain specimens

Figure 2 shows the change in surface state. Until the initiation of a crack, fatigue damage is accumulated gradually at the same region whose dimension is closely related to the grain size, and then this region turns into a crack. After initiation, the crack propagates by the concentration of stress and strain near the crack tip and then final fracture occurs. It can be concluded that the crack initiation process is essentially different from the crack propagation process (**11**).

Figure 3(a) shows the relation between crack length and the relative number of cycles N/N_f. It is found that a crack initiates at $N/N_f \cong 0.2$ and the fatigue life of a plain specimen is controlled mainly by the life in which a crack propagates from an initial size (≅10 μm in this material) to about 1 mm; this life is about $0.7N_f$. This means that we must known the growth law of small cracks if we want to predict the fatigue life of plain members. In Fig. 3(a), the marks A, B, C, D, E, F, and G correspond to the marks in Fig. 2.

Figure 3(b) shows the relation between the crack growth rate and the stress intensity factor range ΔK. Here ΔK is the effective parameter for the propagation of large cracks in which the condition of small scale yielding is satisfied. However, dl/dN of a small crack under a high stress is not determined uniquely by ΔK, as shown in this figure.

Figure 3(c) shows the relation between dl/dN and the term $\sigma_a^n l$. The growth rate of a small crack is determined uniquely by $\sigma_a^n l$ (**6**)–(**10**) and this can be seen more clearly in later figures.

Figure 4(a) shows the dependency of dl/dN on crack length l. For every constant stress range, a straight line can be drawn approximately for crack

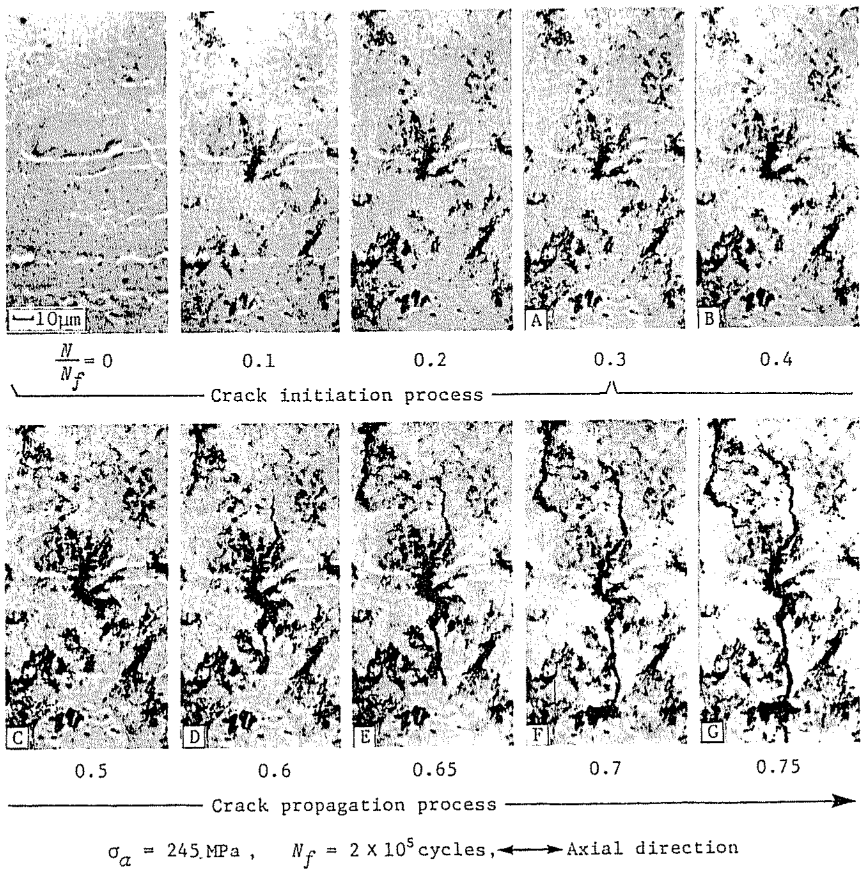


Fig 2 Change in surface states of a plain specimen. (Marks A, B, C, D, E, F, and G correspond to those in Fig. 3(a))

lengths smaller than 1 mm. The slope is about unity. Accordingly, dl/dN is proportional to l for plain specimens.

Figure 4(b) shows the dependency of dl/dN on stress amplitude σ_a. In the high stress levels above the fatigue limit, the dependency is expressed by the relation $dl/dN \propto \sigma_a^n$. The value of n is constant and about 8 in this case. The dotted lines show the results obtained from drilled specimens and will be mentioned in the following section.

Putting the results of Fig. 4(a) and Fig. 4(b) together, we can obtain the growth law of a small crack, i.e.

$$\frac{dl}{dN} = B\sigma_a^n l \tag{1}$$

where n is approximately 8.

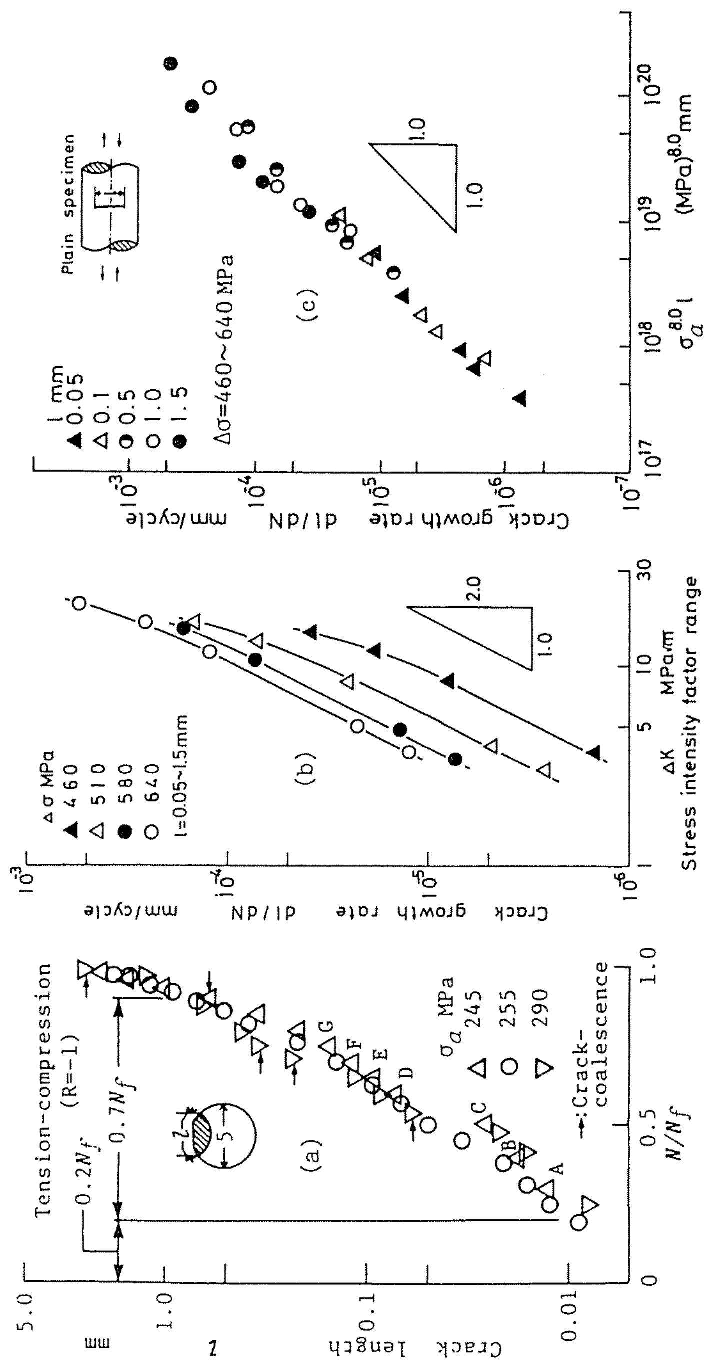


Fig 3 Crack initiation and propagation in a 0.45% C steel plain specimen.
(a) A crack initiates at $N/N_f = 0.2$. The value of l is mainly controlled by N/N_f alone
(b) dl/dN is not determined uniquely by ΔK in the case of $\sigma_a > 0.6\sigma_y$
(c) dl/dN is not determined by $\sigma_a^8 l$ in the case of $\sigma_a > 0.6\sigma_y$

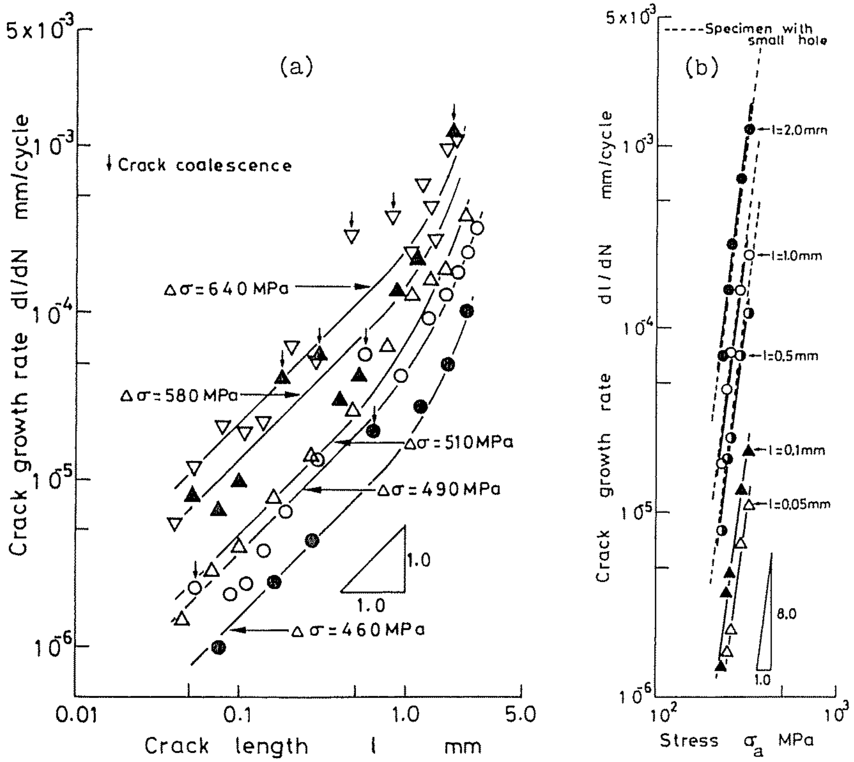


Fig 4 Propagation behaviour of small cracks.
(a) d*l*/d*N* versus *l* relations in plain specimens
(b) d*l*/d*N* versus σ_a relations in plain specimens and specimens with a small hole

Crack propagation behaviour in drilled specimens

The fatigue tests on drilled specimens and 0.5 mm pre-cracked specimens were also carried out to measure the crack growth rate at stress levels below the fatigue limit of plain specimens.

Figure 5 shows the dependency of dl/dN on l. For every constant stress range, a straight line can be drawn approximately for cracks smaller than 1 mm. The slope of the straight lines in the range of $\Delta\sigma > 420$ MPa (approx.) is about unity. The slope increases with decrease in $\Delta\sigma$ and is about 2 in the range of $\Delta\sigma < 380$ MPa (approx.). Thus the dependency of dl/dN on l is influenced by the stress level.

Figure 6 shows the results obtained from all of the specimens (plain specimens, drilled specimens, and pre-cracked specimens). Figure 6(a) shows the dl/dN versus ΔK relation and if the stress is relatively low ($\sigma_a < 0.5\sigma_y$ (approx.),

where σ_y is the yield stress), dl/dN is determined by ΔK alone and the following relation holds

$$\frac{dl}{dN} = C\Delta K^{m} \tag{2}$$

with $m = 4$ (approx.).

When the stress is relatively high however, dl/dN is not determined by ΔK. In this case, the parameter $\sigma_a^n l$ is effective. Figure 6(b) shows the dl/dN versus $\sigma_a^n l$ relation at high stress levels ($\sigma_a > 0.6\sigma_y$ (approx.)).

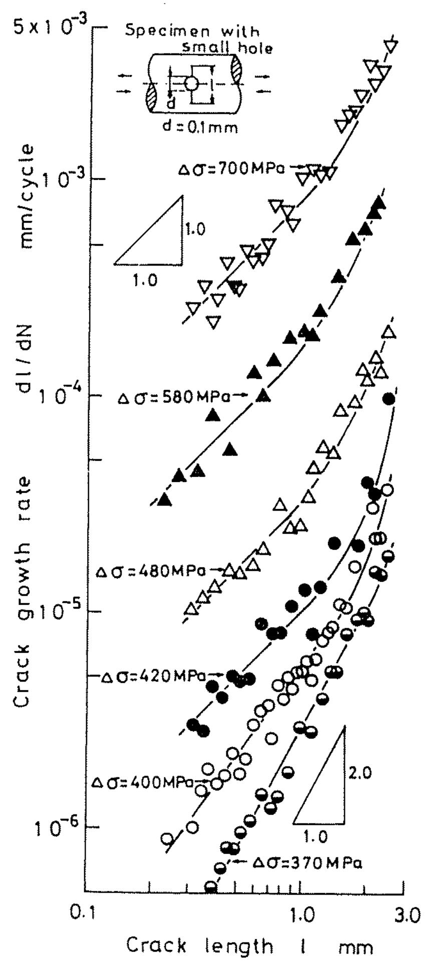


Fig 5 Crack growth rate versus crack length relation in specimens with a small hole

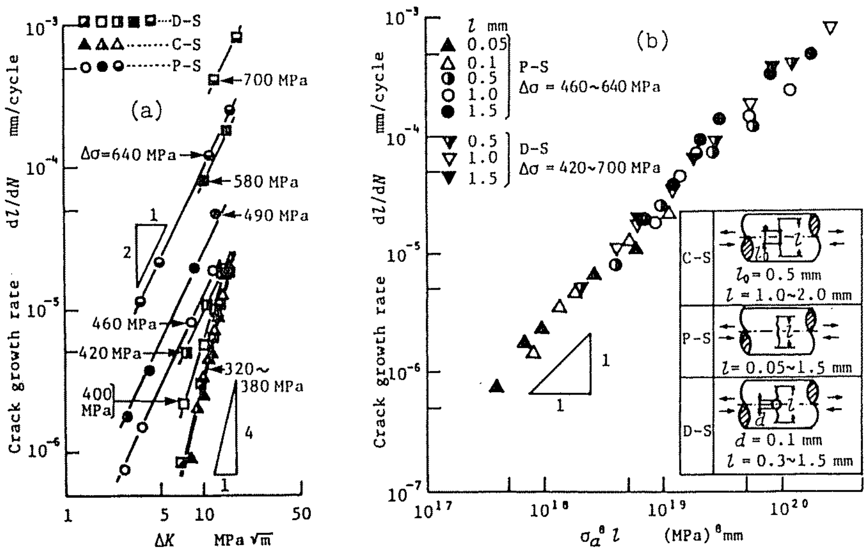


Fig 6 Crack growth data for a 0.45% C steel.
(a) dl/dN is determined by ΔK alone in the case of $\sigma_a \leq 0.5\sigma_y$ approximately ($\Delta\sigma \leq 380$ MPa approximately). dl/dN is not determined by ΔK in the case of $\sigma_a \geq 0.6\sigma_y$ approximately
(b) dl/dN is determined by $\sigma_a^8 l$ alone in the case of $\sigma_a \geq 0.6\sigma_y$ approximately ($\Delta\sigma \geq 420$ MPa approximately)

A unifying treatment for large-crack growth and small-crack growth

The propagation of a fatigue crack occurs by the accumulation of irreversible deformation at the crack tip. Therefore, the cyclic crack tip opening displacement ($CTOD_r$) seems to be appropriate as a measure of crack propagation. However, it is probably more advantageous to use the cyclic plastic zone size r_{pr} which is closely related to $CTOD_r$, because the measurement of $CTOD_r$ is generally difficult. In this section, using the assumption shown in the following relation, a unifying explanation of fatigue crack growth was made **(4)(5)**

$$\frac{dl}{dN} \propto r_{pr} \tag{3}$$

It has been shown experimentally that the relation $dl/dN \propto r_{pr}$ holds for a Fe–3% Si alloy **(12)**.

(1) A crack growth law for large cracks. In general equation (2) gives good results in the case when a large crack propagates by a small nominal stress. Kikukawa *et al.* **(13)** have measured systematically the value of $U(<1)$, the opening ratio equal to $\Delta K_{eff}/\Delta K$ **(14)**, for cracks in many materials. Their results indicate that U is nearly proportional to ΔK in a limited range of ΔK. This indicates that the effective stress intensity factor range ΔK_{eff} is proportional to ΔK^2, and r_{pr} is considered to be proportional to ΔK_{eff}^2. From these considerations, we can obtain the crack growth law given by equation (2), namely $dl/dN = C\Delta K^4$.

(2) A crack growth law for small cracks. A small crack does not propagate unless the nominal stress is high enough. Accordingly, when a sufficiently small crack propagates with a finite growth rate (for example, $10^{-6} \sim 10^{-3}$ mm/cycle), the condition of small scale yielding is not satisfied. In this case, equation (1) is valid.

It is natural to assume that U is nearly constant when the nominal stress is high and the condition of small scale yielding does not hold. In this case, the dependency of r_{pr} on σ and l can be estimated from the dependency of r_p, the monotonic plastic zone size, on σ and l under unidirectional loading. In the Dugdale model **(15)** for unidirectional loading, the relation $r_p/l \propto (\sigma/\sigma_y)^n$ holds. The index n is 2 for $\sigma/\sigma_y \ll 1$ and as σ/σ_y tends to unity, the value of n becomes much larger than 2. The plastic zone size r_p is proportional to crack length l under constant stress. In this case, equation (1) holds and is explained on the assumption based on equation (3).

As mentioned above, although equations (1) and (2) contradict each other, these two growth laws are explained consistently from the same physical background, based on an assumption that the crack growth rate is proportional to r_{pr}. The schematic explanation of the above statements is shown in Fig. 7.

The effect of material properties on the small-crack growth law

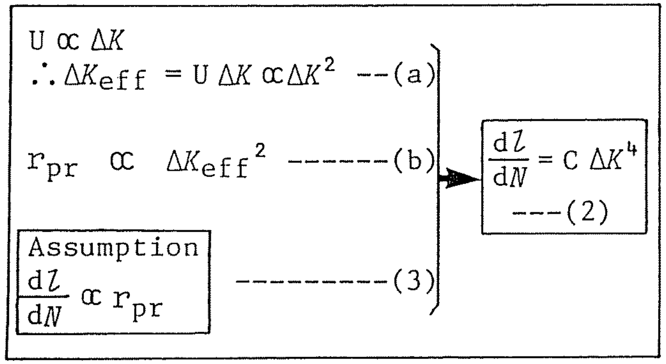
From the results of the previous section, it is found that the two crack growth laws given by equations (1) and (2) must be used according to the magnitude of stress amplitude. Until now, only equation (2) has been studied for a wide range of materials. In this section, the relation between equation (1) and material properties is investigated.

Equation (1) can express the growth rate of a small crack in a material, but is not convenient for a comparison between different materials. Therefore, we propose the following crack growth law in which the effect of material properties is partly considered

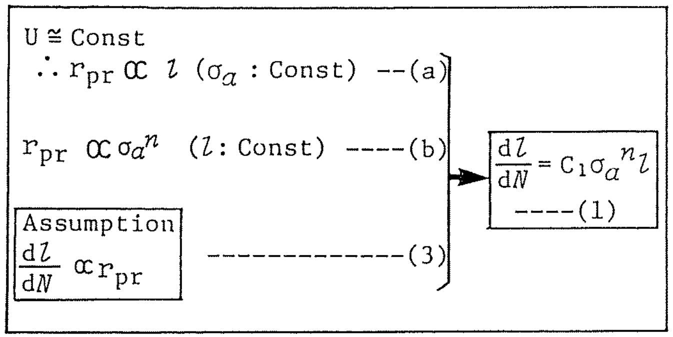
$$\frac{dl}{dN} = D\left(\frac{\sigma_a}{\sigma_y}\right)^n l \tag{4}$$

Table 1 compares the constants B and D for various carbon steels. In the comparison of experimental results under the different conditions, it is necessary to know the effects of specimen size and shape of a specimen on the growth law. However, it is found that these effects are small (see Appendix 1).

The table shows that D is a stable constant for materials with similar mechanical properties and is suitable for the comparison of different materials. The constant D represents the resistance for crack propagation in each



(a)



(b)

Fig 7 Schematic explanation of crack growth laws.
(a) Large cracks ($\sigma_a \leqslant 0.5\sigma_y$ approximately)
(b) Small cracks ($\sigma_a \geqslant 0.6\sigma_y$ approximately)

material. The dl/dN versus $(\sigma_a/\sigma_y)^n l$ type relations of Table 1 are shown in Figs. 8(a) and (b), respectively.

Though the experiments No. 1 and No. 2 in Table 1 are of the same material, the values of n are different ($n = 6.5$ for No. 1 and $n = 8.5$ for No. 2). This difference of n is clearly based on a change of material property due to macroscopic yielding (see Appendix 2). However, even in such a case the value of D is stable.

Application of the small-crack growth law in evaluating fatigue life

It is reasonable to predict the fatigue life of plain members based on the small-crack growth law, because the fatigue life of a plain specimen is dominated by the life in which a crack propagates from a certain initial size up to about 1 mm; see Fig. 3(a).

Table 1 Values of B and D and n in equations (1) and (3) (16)

No.	Material	Shape of cross section	σ_y (MPa)	σ_B (MPa)	Range of σ_a (MPa)	Range of σ_a/σ_y	n	B ($\times 10^{-23}$)	D ($\times 10^{-4}$)	Range of l (mm)	Type of loads
(a) for the case of annealed steels (1 to 11)											
1	S 10 C	D = 10 mm	203	373	196 ~ 265	0.95 ~ 1.29	6.5	4.4×10^{2}	4.4×10^{-2}	0.5 ~ 3.0	R–B
2	S 10 C	5 mm	206	373	190 ~ 220	0.87 ~ 1.07	8.5	6.4×10^{-3}	3.0×10^{-2}	0.3 ~ 1.0	
3	S 35 C	6 mm	331	592	196 ~ 314	0.59 ~ 0.95	8.4	1.2×10^{-2}	1.8	0.5 ~ 2.0	
4	S 45 C	5 mm	364	631	235 ~ 300	0.73 ~ 0.93	7.5	1.0	1.6	0.3 ~ 1.5	
5	S 50 C	10 mm	347	674	279 ~ 373	0.86 ~ 1.09	7.5	1.1	1.2	0.5 ~ 3.0	
6	S 20 C	5 mm	276	469	215 ~ 287	0.78 ~ 1.04	7.5	1.2	2.4×10^{-1}	0.05 ~ 1.0	
7	S 35 C	5 mm	331	592	235 ~ 358	0.71 ~ 1.08	7.3	6.7	1.7	0.05 ~ 1.5	
8	S 45 C	5 mm	364	631	235 ~ 372	0.65 ~ 1.02	7.5	1.1	1.8	0.05 ~ 1.5	
9	S 45 C	5 mm	364	631	210 ~ 350	0.58 ~ 0.96	8.0	0.3	8.3	0.05 ~ 1.0	T–C
10	S 45 C	5 mm	364	631	235 ~ 300	0.65 ~ 0.83	8.0	0.2	7.3	0.3 ~ 1.0	
11	S 45 C	8 mm	284	543	294 ~ 543	1.04 ~ 1.87	6.7	5.1×10^{2}	1.4	0.3 ~ 1.0	

No.	Material	Shape of cross section	$\sigma_{0.2}$ (MPa)	σ_B (MPa)	Range of σ_a (MPa)	Range of $\sigma_a/\sigma_{0.2}$	n	B ($\times 10^{-13}$)	D ($\times 10^{-4}$)	Range of l (mm)	Type of loads
(b) for the case of heat-treated steels (12 to 16)											
12	SCM 4-35	D = 5 mm	748	853	400 ~ 470	0.53 ~ 0.63	4.5	2.9×10^{-4}	2.5	0.5 ~ 1.5	R–B
13	S 45 C	5 mm	871	981	400 ~ 470	0.46 ~ 0.54	4.0	6.3×10^{-3}	3.6	0.5 ~ 1.5	
14	S 45 C	5 mm	1376	1510	800 ~ 900	0.58 ~ 0.65	3.5	6.2×10^{-2}	6.0	0.05 ~ 0.5	
15	S 50 C	10 mm	1133	1247	364 ~ 736	0.32 ~ 0.65	3.0	3.5	5.1	0.5 ~ 3.0	
16	S 50 C	10 mm	1133	1247	343 ~ 540	0.30 ~ 0.48	3.5	0.17	8.1	0.5 ~ 2.0	T–C

R–B: Rotating–bending, T–C: Tension–compression
$\sigma_{0.2}$: Proof stress, σ_B: Ultimate tensile strength

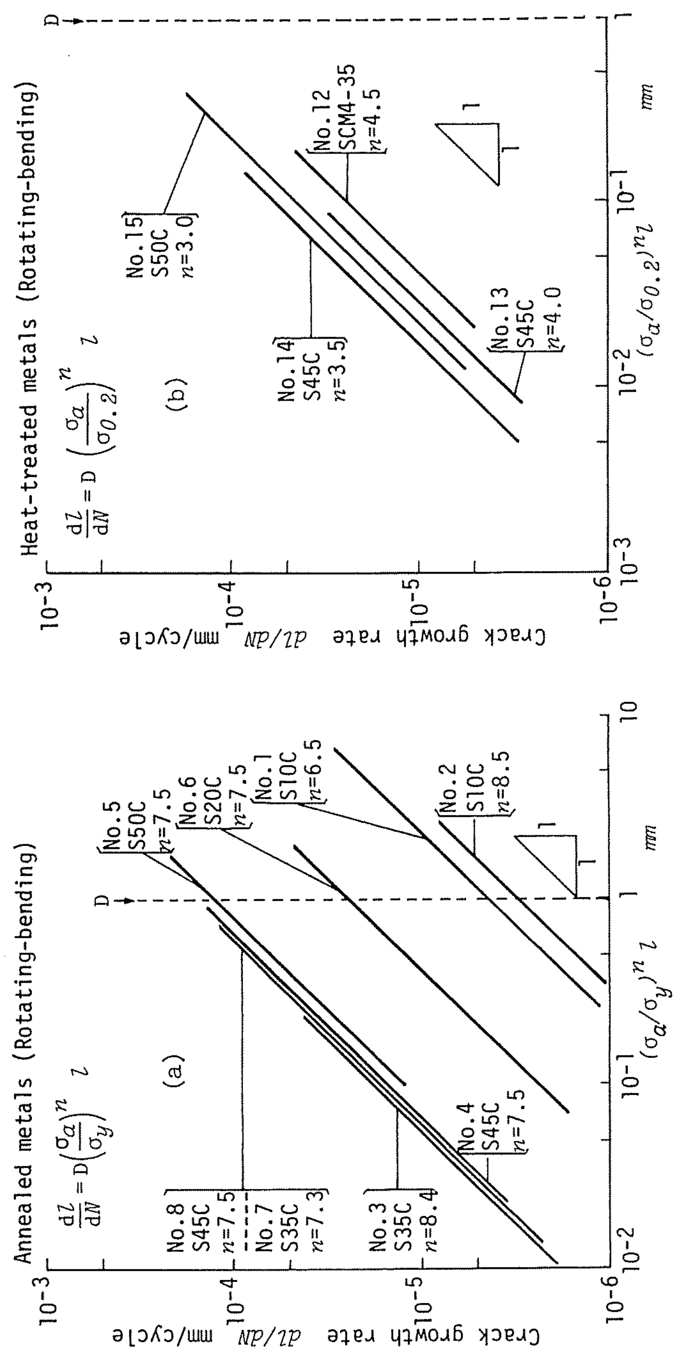


Fig 8 The crack growth relation for (a) annealed carbon steels, and (b) heat treated carbon steels

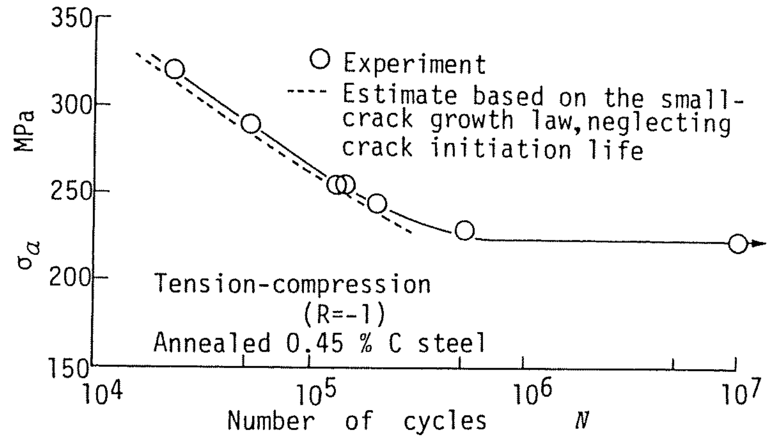


Fig 9 **Evaluation of fatigue life based on the growth law of small cracks**

Figure 9 shows the comparison of fatigue life between experiment and the estimate based on the behaviour of cracks smaller than 1 mm, neglecting the crack initiation life. The prediction is in good agreement with experiment.

In the cases where we predict the life of large machine members, the prediction based on the behaviour of cracks smaller than 1 mm is also meaningful. For example consider three crack sizes: l_1, an initial size; l_2, an arbitrary size in the propagating period; and l_3, the final size corresponding to fracture. The number of cycles to failure, N_f, is approximately given by

$$N_f \cong \frac{1}{D}\left(\frac{\sigma_y}{\sigma_a}\right)^n\left(\ln\frac{l_2}{l_1} + \ln\frac{l_3}{l_2}\right) \tag{5}$$

If we take $l_1 = 0.01$ mm, $l_2 = 1$ mm, and $l_3 = 100$ mm, the life from l_1 to l_2 is equal to the life from l_2 to l_3. All members having a crack of length 100 mm soon break, and so the real fatigue life in this case is approximately twice the life predicted from the behaviour of a crack smaller than 1 mm. This means that the behaviour of a crack smaller than 1 mm is dominant in the estimation of the fatigue life of machines and structures.

Conclusions

The following conclusions were reached with regard to small crack propagation of plain carbon steel specimens under constant stress amplitudes.

(1) In annealed 0.45% C steel plain specimens subjected to axial loading, the propagation life of a crack smaller than 1 mm occupies about 70 per cent of the total fatigue life.

(2) The crack growth rate of a plain specimen is not determined by the stress intensity factor range ΔK. The crack growth rate increases with increase in the applied stress range $\Delta\sigma$, for a given value of ΔK and the growth rate of a plain specimen is determined uniquely by the term $\sigma_a^n l$ where σ_a is stress amplitude, l is crack length, and n is a constant.

(3) The following crack growth laws are obtained from experiments on specimens with a small hole:

(a) When a crack is propagating under high nominal stress ($\sigma_a \gtrsim 0.6\sigma_y$, σ_y: the lower yield stress),

$$\frac{dl}{dN} = B\sigma_a^n l$$

(b) When a crack is propagating under low nominal stress ($\sigma_a \lesssim 0.5\sigma_y$).

$$\frac{dl}{dN} = C\Delta K^m$$

Both these crack growth laws can be explained consistently from the same physical background, based on the assumption that crack growth rate is proportional to the reversible plastic zone size r_{pr}.

(4) At high stress levels the growth rate of a small crack in one material cannot compare with that of a different material. Therefore the following small crack growth law is proposed

$$\frac{dl}{dN} = D\left(\frac{\sigma_a}{\sigma_y}\right)^n l$$

The effect of material properties is therefore partly considered. The constant D represents the resistance to crack propagation in each material and is a stable constant for those materials with similar mechanical properties.

(5) The fatigue life of 0.45% C steel plain specimens can be evaluated based on the behaviour of cracks smaller than 1 mm. The prediction is in good agreement with experiments.

Appendix 1

Fatigue tests on geometrically similar specimens and specimens having various cross sections were carried out to examine the effectiveness of the relation $dl/dN \propto \sigma_a^n l$ when evaluating fatigue life. Figure 10(a) shows this relation for the geometrically similar specimens having square sections. It is found that the difference in specimen size does not affect the small-crack growth law.

Similar experiments were carried out on plain specimens having various cross sections, i.e., the square section, the circular section and the circular section having a fine shallow notch. Again the difference in propagation behaviour of cracks is hardly observable, see Fig. 10(b).

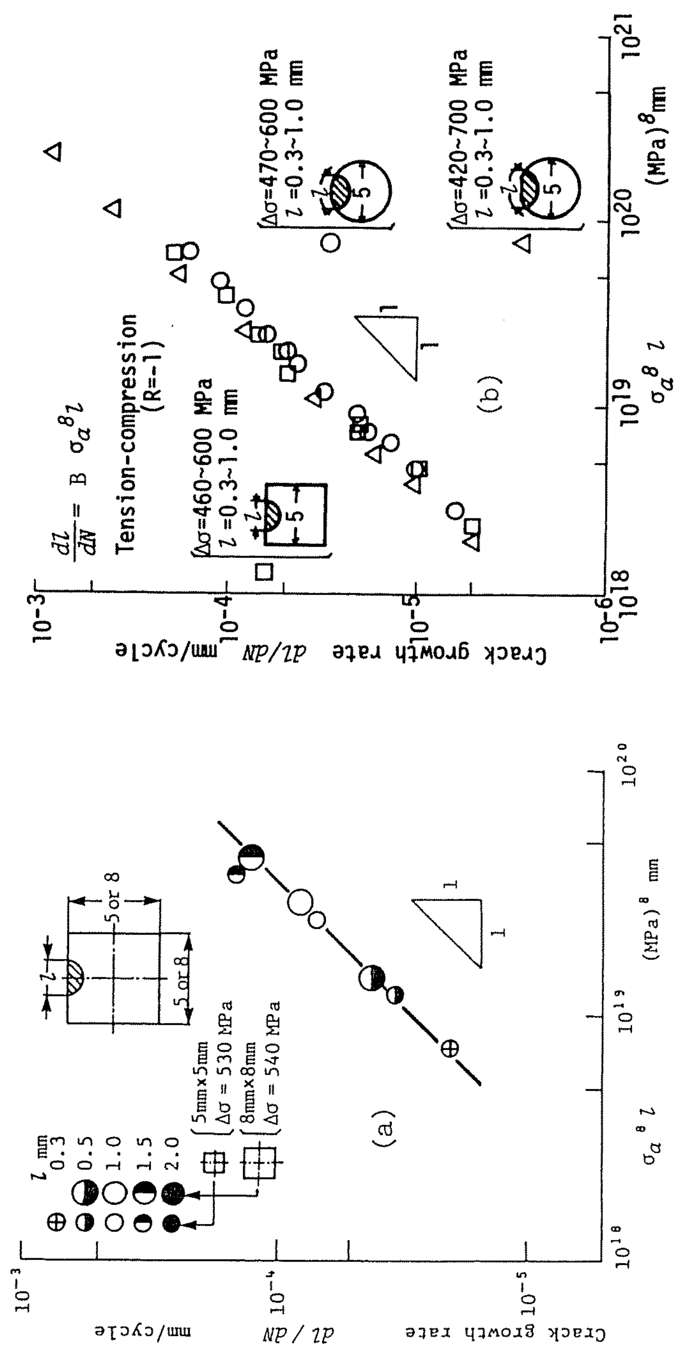


Fig 10 Relation between propagation behaviour of small surface cracks and specimen shape.
(a) dl/dN versus $\sigma_a^8 l$ relation in geometrically similar specimens
(b) Crack growth behaviour in specimens having various cross sections

From these results on the same material, it is found that the propagation behaviour of surface cracks is not affected by the specimen shape. This suggests that the behaviour of small surface cracks in actual machines and structures can be predicted from specimens such as that shown in Fig. 1.

Appendix 2

The behaviour of small cracks in annealed and pre-strained 0.45% carbon steel specimens was investigated in order to clarify the effect of macroscopic yielding on the growth of small cracks (**17**).

Figure 11 shows the loading cycle for both materials. In the case of annealed material Fig. 11(a) the growth law is shown in Fig. 6. That is, the growth rate of small cracks at high nominal stress is given by $\sigma_a^8 l$. In the case of pre-strained material, see Fig. 11(b), the experimental results are shown in Fig. 12. From Fig. 12(a) and (b), it is clear that the dl/dN versus $\sigma_a^{6.4} l$ relation holds for high nominal stress ($\Delta\sigma \gtrsim 630$ MPa) and the dl/dN versus ΔK relation holds for low nominal stress ($\Delta\sigma \lesssim 570$ MPa).

The value of n is a material property. That is, n has a value 7.5–8.5 for annealed carbon steels and 4–5 for heat-treated carbon steels. The difference of n between annealed specimen ($n \cong 8$) and pre-strained specimen ($n \cong 6.4$) is explained by material property changes caused by macroscopic yielding from pre-straining. This means that the value of n depends on the magnitude of stress (above or below yield stress) despite the material being the same; see No. 1 and No. 2 in Table 1.

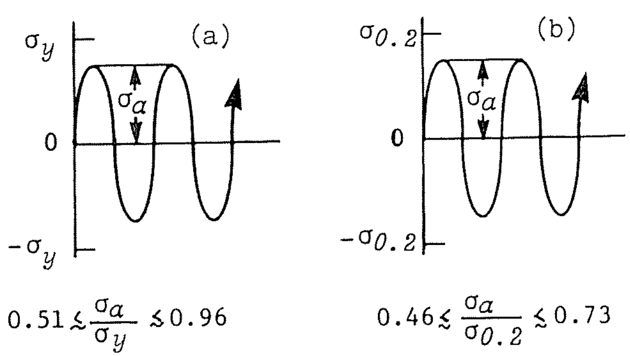


Fig 11 Condition of loads in (a) annealed, and (b) pre-strained specimens

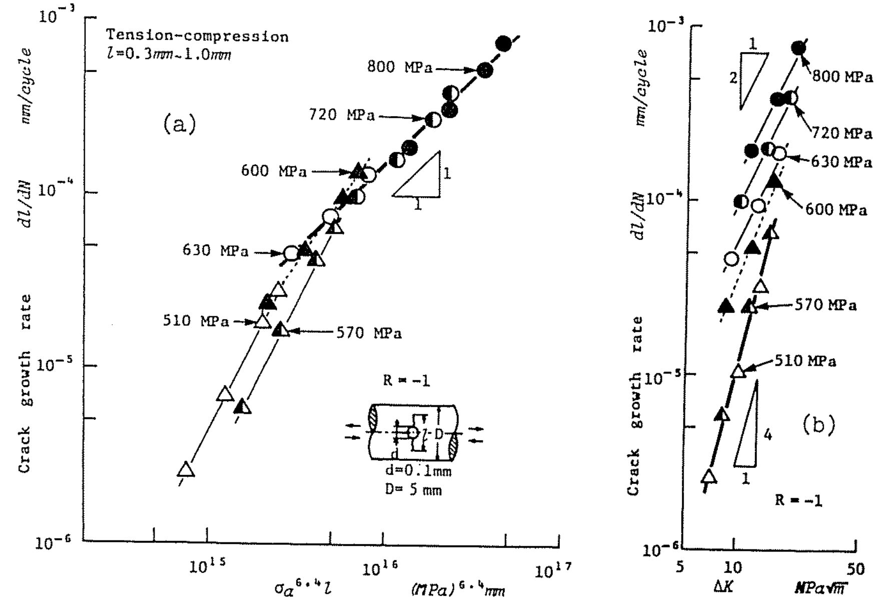


Fig 12 Crack growth data for a pre-strained 0.45% carbon steel

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