THE CALCULATION OF THE NECESSARY RADIUS OF CRACK ARRESTING BOREHOLES

M. Wingenbach*, H.A. Richard*

A method to calculate the necessary radius of crack arresting boreholes will be presented. It is based on the local strain concepts which are used to predict the crack initiation life. The local stress or the stress concentration factor is calculated by a stress field equation valid for a slit ending in a hole, where the stress intensity factor of the comparable crack is used for narrow notches ($\rho << a_0$). It will be shown that this is also valid for slightly curved cracks. Experiments and FE–calculations were done to prove the applicability of the method.

INTRODUCTION

A usual method to arrest a fatigue crack is to drill a hole at the cracktip. Using this method the question is how the radius of the hole has to be chosen in order to prevent another crack initiation. If such a hole is drilled one has no longer a crack but a notch. Therefore the calculation of the necessary radius can be reduced to the prediction of the crack initiation life at notches.

PREDICTION OF THE CRACK INITIATION LIFE

The local strain concepts are based on the assumption that the material at notches behaves like the material of unnotched specimens under the same stress–strain history. This is equal to the validity of a damage parameter–life–curve for different geometries of components. This concept requires the calculation of the stress–strain–history in the notch root. Once the local stress– strain path has been determined the closed hysteresis loops may be evaluated by use of a damage parameter. If the damage parameter of SMITH, WATSON and TOPPER (2) is used, the life curve is also valid for any R–ratio. This damage parameter is defined as:

$$P_{SWT} = \sqrt{(\sigma_a+\sigma_m) \cdot \epsilon_a \cdot E} = \sqrt{\sigma_{max} \cdot \epsilon_a \cdot E} \qquad (1)$$

For a large number of metallic materials life curves for this damage parameter have been collected in a special material data bank (3).

* Institute of Applied Mechanics, University of Paderborn, FRG

CALCULATION OF THE LOCAL STRESS–STRAIN PATH

For elastic–plastic material behaviour, the notch strain can be estimated with Neuber's rule (4)

$$\sigma \ \epsilon = \sigma_{max} \ \frac{\sigma_{max}}{E} = (\alpha \ \sigma_N)^2 \ \frac{1}{E} = \text{const.} \tag{2}$$

Essential for this calculation is the cyclic stress–strain curve of the material and the maximum notch stress or the stress concentration factor α. For the calculation of stress–strain–hysteresis loops caused by fatigue loading the Masing–hypothesis has to be assumed and memory effects have to be considered. Restricting to the region of the endurance limit the notch strains are mainly elastic and can be determined at once:

$$\sigma_{max} = \alpha \ \sigma_N \ ; \ \ \epsilon = \frac{\sigma_{max}}{E} \tag{3}$$

In both cases the knowledge of the maximum notch stress or the stress concentration factor is required.

A hole at the tip of a long crack is identical to circular notch with a slit to the edge. Especially for this geometry Kullmer (5) has developed stress field equations valid in the vicinity of the notch. These equations are similar to the Creager equations (6) which are valid for parabolic notches. There are little improvements for pure Mode I–loading with the new equations of Kullmer compared to the Creager– equations, whereas for Mixed Mode– or Mode II– loading ·vious differences exist. This was proved by FE–calculations. In regard to further ·estigations dealing with Mixed Mode–loading in this work the Kullmer equations are used. For Mode I–loading Kullmer indicates the following equations:

$$\sigma_r = \frac{k_I}{4\sqrt{2\pi r}}\left\{\left[5-2\frac{\varrho}{r}-3\left[\frac{\varrho}{r}\right]^2\right]\cos\left[\frac{\varphi}{2}\right]+\left[-1+\frac{7}{2}\frac{\varrho}{r}-\frac{5}{2}\left[\frac{\varrho}{r}\right]^3\right]\cos\left[\frac{3}{2}\varphi\right]\right\},$$

$$\sigma_\varphi = \frac{k_I}{4\sqrt{2\pi r}}\left\{\left[3+2\frac{\varrho}{r}+3\left[\frac{\varrho}{r}\right]^2\right]\cos\left[\frac{\varphi}{2}\right]+\left[1+\frac{1}{2}\frac{\varrho}{r}+\frac{5}{2}\left[\frac{\varrho}{r}\right]^3\right]\cos\left[\frac{3}{2}\varphi\right]\right\}, \tag{4}$$

$$\tau_{r\varphi} = \frac{k_I}{4\sqrt{2\pi r}}\left\{\left[1+2\frac{\varrho}{r}-3\left[\frac{\varrho}{r}\right]^2\right]\sin\left[\frac{\varphi}{2}\right]+\left[1+\frac{3}{2}\frac{\varrho}{r}-\frac{5}{2}\left[\frac{\varrho}{r}\right]^3\right]\sin\left[\frac{3}{2}\varphi\right]\right\}.$$

The factor k_I is defined as

$$k_I = \sigma\sqrt{\pi a_0} \ y_I \tag{5}$$

and describes the intensity of the stress field. It is in general dependent on the notch radius. To analyse this dependence FE–calculations with various notches in the CTSN– specimen (Fig. 1) were done with constant notch depth and variable radius. From the calculated nodal point stresses the factor y_I was determined. The results are shown in Fig. 3. In the same way as k_I, the stress intensity factor

K_I for a crack can be defined:

$$K_I = \sigma \sqrt{\pi a}\ Y_I(a) \qquad (6)$$

Additionally the factor $Y_I(a=a_0)$ of the comparable crack with the same orientation and depth is plotted. Fig. 3 reveals that for small notch radii ρ rather than for small ρ/a_0 there are only little differences between the y_I values and the crack factor, but for greater radii they can not be neglected. So for small ρ/a_0 ratios y_I can be compared with Y_I; this means k_I is equal to K_I. In this case it is possible to describe the elastic stress in the vicinity of the notch with the equations (4) and the stress–intensity factor of the comparable crack.

In practice cracks are not always straight. Often the crack path is slightly curved. To show that the equation (4) describes the stresses at crack arresting boreholes on curved cracks investigations were done on the CTR32– specimen (7), Fig. 2. The asymmetrical geometry of the specimen results in a Mode II factor so that a curved crack occurs. The path taken by the crack is one for which the local stress field at the tip is of a Mode I type. Therefore it can be assumed that if a crack arresting hole is bored the stress field in the vicinity of the hole is also one of a Mode I type. To examine the validity of this assumption further FE–calculations were made. The crack path and the geometry functions $Y_I(a)$ and $Y_{II}(a)$ were determined by a crack simulation with the program CRACKSIM (7). Afterwards a hole with the radius 1.6 mm was generated in the mesh at a crack depth of 40mm. Fig. 4 shows the results of this calculation. For the evaluation of the stresses the coordinate system has to be rotated in the direction in which the crack will propagate, Fig. 2. Additionally the stresses predicted by equation (4) are plotted in Fig. 4. For the prediction the stress intensity factor of the comparable crack was used. Fig. 4 illustrates the good agreement between the FE–results and the predicted stresses. It can be concluded that equation (4) is able to describe the elastic stresses in the vicinity of crack arresting boreholes and in particular the stress in the notch root can be determined. The maximum notch stress can be derived from equation (4)

$$\sigma_{max} = \sigma_\varphi(r=\rho;\ \varphi=0) = \frac{3K_I}{\sqrt{2\pi\rho}} = \frac{3\cdot\sigma_N\cdot\sqrt{\pi a_0}\cdot Y_I(a_0)}{\sqrt{2\pi\rho}} \qquad (7)$$

and the stress concentration factor can be written as

$$\alpha = \frac{\sigma_{max}}{\sigma_N} = \frac{3}{\sqrt{2}}\cdot Y_I(a_0)\cdot\sqrt{\frac{a_0}{\rho}} \qquad (8)$$

Now the local stress strain paths are calculable. Within the endurance limit the material behaviour can be idealized as linear elastic. For constant amplitude loading it is then possible to calculate the characteristic parameter of the hysteresis loop which is reduced to a straight line:

$$\sigma_{max} = \frac{3\Delta K_I}{\sqrt{2\pi\rho}}\,\frac{1}{(1-R)},\ \sigma_a = \frac{3\Delta K_I}{2\sqrt{2\pi\rho}},\ \sigma_m = \sigma_{max} - \sigma_a,\ \epsilon_a = \frac{3\Delta K_I}{2\sqrt{2\pi\rho}\ E} \qquad (9)$$

For the damage parameter of Smith, Watson and Topper follows

$$P_{SWT} = \frac{3\Delta K_I}{\sqrt{2\pi\rho}}\sqrt{\frac{1}{2(1-R)}} \tag{10}$$

Stress controlled constant amplitude tests on the CTSN–specimen have been carried out for various notch radius and R–ratio to prove the possibility of the crack initiation prediction with the local strain concepts. Complementary tests have been done on the CTR32–specimen with a crack arresting hole (ρ=1.6mm; a_0=40mm). The specimen material has been the aluminium alloy 7075 T651. For this material the damage parameter life curve and experimental results from strain controlled tests are given in (3). These data and the experimental crack initiation lifes are plotted in Fig. 5. It can be seen that all experiments lie in a small scatterband and are in good agreement with the data according to (3). Therefore it is possible to predict the crack initiation life with one damage parameter life curve calculating the damage parameter with equation (10).

CALCULATION OF THE NECESSARY RADIUS OF CRACK ARRESTING BOREHOLES

Crack arresting boreholes have to prevent the component from failure. So a new initiation has to be impossible. Using the threshold damage parameter P_{SWTth} of the endurance limit or a limiting value of cycles, the necessary radius can be calculated with equation (10)

$$\rho_{min} = \frac{9\Delta K^2}{P_{SWTth}^2}\,\frac{1}{4(1-R)\;\pi} = \frac{9\Delta\sigma^2 \cdot a_0 \cdot Y_I^2}{P_{SWTth}^2\; 4(1-R)} \tag{11}$$

The stress range and the R–ratio may be determined by the extreme values of the loading. So the load sequence is replaced by a single stage loading. The calculated radius has to be multiplied by a safety factor. If the loading is unknown the most critical case can be assumed to estimate the radius. The most critical loading is a stress intensity factor range of ΔK=2 K_{IC} and a R–ratio of R=–1. In that case it follows

$$\rho_{min} = \frac{9\,K_{IC}^2}{P_{SWTth}^2 \cdot 2\pi} \tag{12}$$

DISCUSSION

The result of equation (12) for the examined material is a radius of ρ = 15mm. Looking at Fig. (3) it is obvious that this radius couldn't be described with the geometry factor of the crack. For greater components(especially longer cracks)it can be assumed that it is possible. After calculating the radius you have to prove that the restriction $\rho << a_0$ is fulfilled. If not,the calculation has to be made in another way. It is thinkable to determine the local stresses for different radii with the FE–Method whereby the damage parameter can be calculated.

If the condition $\rho << a_0$ is satisfied, the equation (12) gives the largest necessary radius because the assumed load is the worst case. If the loading and the geometry factor of the crack is known equation (11) can be used and the calculated radius will be smaller. So it is possible to calculate the necessary radius of crack aresting boreholes if the life curve for the damage parameter from Smith, Watson and Topper and the stress intensity factor or the geometry factor of the crack and the loading are known. If the geometry factor and the loading are unknown the maximum necessary radius can be estimated with equation (12) using the fracture toughness of the material.

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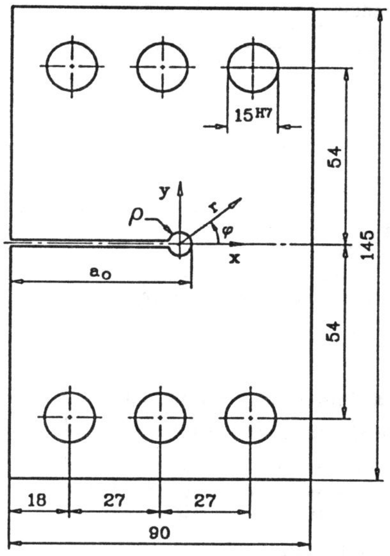

Fig. 1: CTSN–specimen

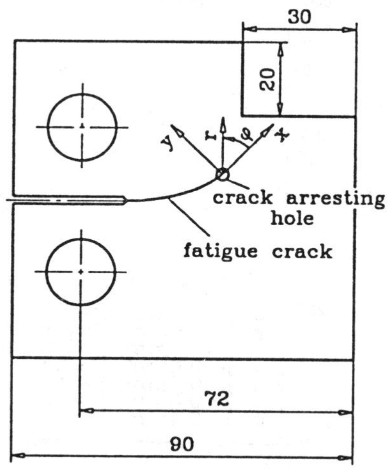

Fig. 2: CTR32–specimen

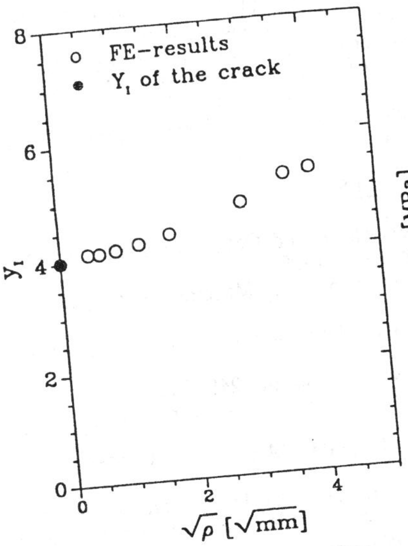

Fig. 3: FE–results for the geometry factor y_I for the CTSN–specimen

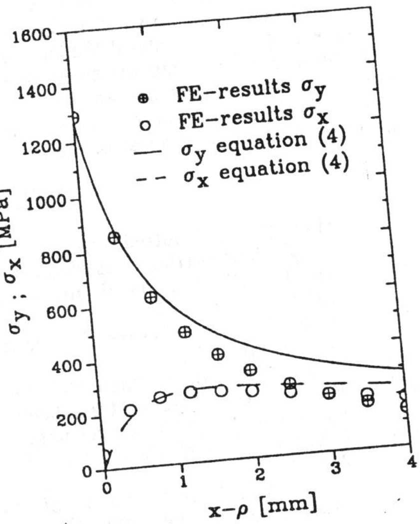

Fig. 4: Stresses at a crack arresting borehole in the CTR32–specimen

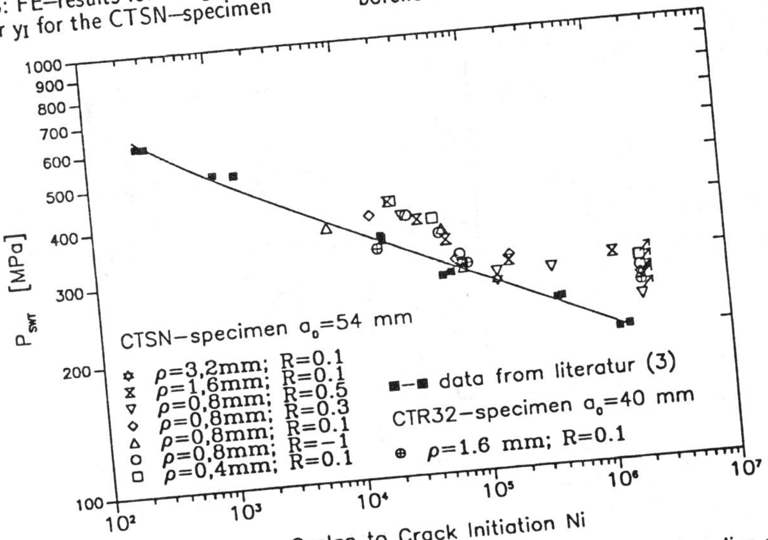

Fig 5: Experimental results of the crack initiation life for various notch radius and R–ratio