TEARING STABILITY OF SURFACE CRACKS IN PLATES OR SHELLS

H.J. Schindler *

Experiments have shown that the ultimate stress (or the leak stress in the case of containers or pressure vessels) of pre-cracked or notched plates or shells is only very weakly dependent on the sharpness of the notch and that there is a significant difference between the maximum load which can be achieved by subcritical crack growth and by tearing crack growth. This indicates that the ultimate stress is rather determined by the tearing behaviour than by the crack-initiation. By means of a simple analysis calculation based on a strip yield crack model and a two-parameter characterization of ductile crack-growth it could be shown that the parameter CTOA, which describes the tearing crack-growth of the material, affects the fracture behaviour and the ultimate stress very significantly.

INTRODUCTION

Burst experiments performed by Kieselbach and Cucci [1] on pressurized gas cylinders made of aluminum which were locally weakened by axial notches exhibited some interesting results: The ultimate pressure, i.e. the pressure which caused leaking or bursting, respectively, turned out to be only weakly dependent on the sharpness (notch radius and vertex angle) of the notch which was machined axially in the outer surface of the cylinder. In contradiction to what is expected from the basics of fracture mechanics, holding the depth of a notch constant and increasing the sharpness of a notch resulted rather in an increase in ultimate pressure than in the expected decrease. At a given depth of a notch, the highest pressure was achieved in the case of cyclic loading and the corresponding fatigue crack. Since the critical load of notches is definitively strongly dependent on the notch radius and the vertex angle, this behaviour indicates that crack initiation (onset of crack-growth at the notch root or the fatigue crack tip, respectively) is not the only strength-determining event: There must be further processes which also affect the ultimate stress.

Disregarding the possibility of cleavage fracture in the case of ferritic steel, crack initiation of ductile metals is in general followed by ductile tearing crack growth. Thus, one can guess that the tearing process may play an important role

* Swiss Federal Lab. for Materials Testing and Research, Dübendorf (CH)

in the fracture process. The purpose of the present investigation is to find the basic relations between the ultimate stress, the notch depth and the material parameters related to crack-growth. In order to obtain transparent, generalizable results which help to understand the basic physical behaviour, the problem was tried to be solved by using simple, analytical models .

A convincing simple model for a similar system was introduced by Paris et.al. [2]. They found the surprising result that the tearing stability is dependent on the tearing modulus T and the ratio of crack-width c and plate thickness t, but not on the initial crack-depth. This seems to be in contradiction with the experimental results mentioned above [1], where a significant dependence of the ultimate stress on notch depth was found (see also Schindler and Kieselbach [3] and Fig. 4 of the present paper). This indicates that for applying this model to the present problem some refinements are necessary.

CRACK MODEL

We consider a surface crack or a notch in a plate under uniaxial stress perpendicular to the crack (Fig.1, A). The aspect ratio (width c/depth a) is assumed to be high enough to enable crack-growth through the thickness of the plate prior to lateral growth. Furthermore, we assume that the remaining ligament ahead of the crack in thickness-direction is fully plastic before crack-initiation occurs, which is in general the case for relatively tough material and relatively small thickness t of the plate. The model which is used to analyze this situation is shown in Fig. 1: The real three-dimensional crack system A is split into two two-dimensional cracks B and C. The connection of the two cracks B and C is attained on one hand by assuming that the average plastic stress in the ligament of crack A and B, σ_a, acts as a cohesive stress $\sigma_a(1-a/t)$ on the crack-faces of the in-plane crack C, and on the other by the condition of equal displacements:

$$\delta = v = v_{el} + v_{pl} \tag{1}$$

where δ denotes the crack tip opening displacement (CTOD) of crack B and v the mouth opening displacement of crack C, v_{el} and v_{pl} being its elastic and plastic part, respectively. For simplicity, we approximate σ_a to be the flow stress σ_f. In fracture mechanics, the latter is usually assumed to be the mean value of the yield stress R_p and the ultimate strength R_m, thus

$$\sigma_a \cong \sigma_f = (R_p + R_m)/2 \ , \tag{2}$$

It is physically evident that this model requires an opening stress σ which is higher than the cohesive stress, thus

$$\sigma \geq \sigma_f \ (1 - a/t) \tag{3}$$

This model is basically the same as the one used by Paris et.al. [2], but extended by introducing Dugdale-type [4] plastic zones of length ℓ_p in the in-plane crack-model. Using the basic solutions given in [5], the corresponding condition of finite stresses at the fictive crack-tip at $z = c + \ell_p$ leads to

$$\ell_p = \{[\sin(\frac{t}{a}\frac{\pi}{2}(1 - \sigma/\sigma_f))]^{-1} - 1\}\ c \qquad (4)$$

Eq. (4) shows that for c>>a the length of the plastic zone remains small compared with the crack-width c for a large range of stresses and crack depths. This means that the condition for the applicability of linear-elastic fracture mechanics (LEFM) are in general fulfilled for the crack C. Thus, in order to calculate the displacement v, the corresponding expressions of LEFM can be applied. One obtains for the elastic part of v:

$$v_{el} = 4\ c\ [\sigma - \sigma_f(1 - a/t)]/\ E\ . \qquad (5)$$

The plastic part of v can be approximated by the fundamental relation between K_I and CTOD of a crack under LEFM conditions, thus

$$v_{pl} = \frac{(\sigma - \sigma_f + \sigma_f\ a/t)^2}{E\ (\sigma_f - \sigma)}\ \pi\ c\ . \qquad (6)$$

CRACK GROWTH MODEL

In the following, we assume that the crack-growth behaviour of a ductile metal can be characterized by two material-dependent parameters: The CTOD δ_i, which is necessary for onset of crack-growth, and the crack opening angle $CTOA_{mat}$, which is defined by

$$CTOA_{mat} = \frac{d\delta_{mat}}{d\Delta a} \qquad (7)$$

where δ_{mat} is CTOD at the original crack-tip measured during crack-growth and Δa is the amount of crack-prolongation. Roughly, $CTOA_{mat}$ can be assumed to be approximately constant during tearing crack-growth. According to this assumption, δ_{mat} is given by

$$\delta_{mat}\ (\Delta a) = \delta_i + CTOA_{mat}\ \Delta a \qquad (8)$$

By the assumption of constant $CTOA_{mat}$ the applicability of this model is restricted to situations where ductile tearing prevails in crack growth (i.e no transition to cleavage). Both δ_i and $CTOA_{mat}$ are considered to be material properties: δ_i can be obtained from the J-Integral by $\delta_i \cong J_c/\sigma_f$, $CTOA_{mat}$ can be estimated from a bending test by methods described by Demafonti [6] and Schindler [7].

According to this model for the tearing behaviour of the material, the following relation is fulfilled at any instant of crack-propagation:

$$\delta_{appl} = \delta_{mat} \qquad (9)$$

Hereby, δ_{appl} denote the "applied" CTOD, which results from the applied load or prescribed displacements.

The tearing process is stable, if (9) is fulfilled and

$$\frac{d\delta_{appl}}{d\Delta a} =: CTOA_{appl} \leq CTOA_{mat} \qquad (10)$$

APPLICATION TO SURFACE CRACKS IN A PLATE

In the present crack system (Fig.1), the displacement v of crack C represents the applied δ_{appl} for crack B, thus

$$\delta_{appl} = v \,, \tag{11}$$

and correspondingly

$$CTOA_{appl} = \frac{d\delta_{appl}}{d\Delta a} = \frac{dv}{da} \tag{12}$$

Equations (1), (5), (8) and (9) inserted in (11) lead to the following relation between applied stress σ and amount of crack-growth Δa:

$$\sigma(\Delta a) = \sigma_f[1 - (a_0+\Delta a)/t] + \frac{E}{4c}(\delta_i + CTOA\ \Delta a - \delta_{fp}) \tag{13}$$

Note that the plastic component of v can be neglected in this relation because of $\ell_p << c$. a_0 denotes the crack-depth prior to onset of growth. The quantity δ_{fp} is introduced to account for the fact that - for continuously increasing stress σ - there already is a certain amount δ_{fp} of CTOD present when the ligament becomes fully plastic, i.e. when the stress is high enough to fulfill criterion (3). Since the exact determination of CTOD at the instant of crack-initiation would make a detailed numerical 3D-analysis necessary and since - according to the experimental results - crack-initiation seems to be only of minor importance in the considered fracture process, we simply assume, that it occurs just when the applied stress is high enough to fulfill the criterion (3), i.e

$$\delta_{fp} \cong \delta_i \tag{14}$$

By inserting (12), (7), (6) and (5) in (10), one obtains the criterion for stability of the crack-growth described by (13) to be

$$\sigma \leq \sigma_f \{1 - \frac{a}{t}[\frac{2\ \pi}{2\pi - 4 + CTOA_{mat}\ E\ t/(c\ \sigma_f)}]\} =: \sigma_{ct} \tag{15}$$

DISCUSSION OF THE FRACTURE BEHAVIOUR

In order to obtain the ultimate load of a pre-cracked plate, we consider the growth of a crack of arbitrary initial depth a_0 under a continuously rising stress σ (Fig. 2). According to the assumption (14), the crack starts to grow when the stress reaches at $a=a_0$ line a, which represents (13) (and, with (14), the boundary line of criterion (3) as well). Subsequently, the $\sigma(\Delta a)$-curve (13) follows a line of a slope s, which can be obtained from (13) by

$$s = t\frac{d\sigma}{d\Delta a} = \frac{E\ t}{4\ c}CTOA_{mat} - \sigma_f \tag{16}$$

If $s \geq 0$, the tearing crack-growth is stable until the intersection with the line b which represents the limit stress σ_{ct} of criterion (15). At this point, i.e. at a crack-depth a_u and a stress σ_u, the ductile tearing becomes unstable. This means, that σ_u represents the ultimate stress in presence of a crack of initial depth a_0. Plotting σ_u in function of the initial crack-depths a/t, we end up with curve c. This line exhibits a kink at $a = a_l$ and $\sigma = \sigma_l$. The latter can be easily obtained from (15) by

$$\sigma_l = \sigma(a{=}t) = \sigma_f \{1 - [\frac{2\pi}{2\pi - 4 + CTOA_{mat} E\ t/(c\ \sigma_f)}]\} \qquad (17)$$

Obviously, cracks of initial depth $a_0 \geq a_l$ lead - under continuously increasing stress - to stable crack growth until the rear surface (a = t), causing a leak before the general instability is reached. Thus, a "leak before break" behaviour is guaranteed in this case. For shorter initial cracks, the ligament from a_u to the rear surface fails rapidly, in an unstable manner. If the toughness of the material is not high enough to stabilize the in-plane crack C (Fig. 1) at this moment, a burst process may be initiated.

If there is subcritical crack growth (fatigue or stress corrosion) the fracture behaviour is somewhat different from the one under continuously increasing stress. Consider, e.g, the subcritical crack growth under constant load σ_l (or cyclic loading with constant amplitude, resp.). In this case, the σ vs. a curve follows a horizontal line (see dashed line at $\sigma = \sigma_l$ in Fig. 2). When this line intersects with line a, the crack-growth changes from sub- to supercritical, but remains stable (one can show that $s \geq 0$ for $\sigma_l \geq 0$) until the rear surface is reached. Thus for $\sigma < \sigma_l$, leak before break will occur). For stresses $\sigma \geq \sigma_l$, the corresponding horizontal line intersects with line b, which means that the subcritical crack-growth is followed by unstable, rapid tearing. If the material is too brittle or too rate sensitive, this may lead to a general rupture (burst).

As shown in Fig. 3, the experimental results from [1] and the theory outlined above fit - at least qualitatively - quite well, both for the fatigue loading and the machined notches. If the flow stress would have been chosen slightly higher than according to (2), the agreement would be even better. It seems that the theory is able to describe the main features of the fracture behaviour of this system correctly.

CONCLUSIONS

- Tearing crack-growth plays an important role in the fracture of a pre-cracked or notched plate or shell. CTOA, being a suitable parameter for characterizing the tearing behaviour, strongly affects the ultimate stress.
- A deep crack which had been formed by subcritical growth is able to withstand considerably higher stresses than a machined notch of the same depth - despite the lower sharpness of the machined notch.
- The tearing behaviour plays an important role in the "leak before break" -behaviour
- Simple analytical models can be appropriate in analyzing complex fracture processes.
- To model the tearing behaviour correctly, it is important to extend the model of [2] by introducing plastic zones in the in-plane crack system.

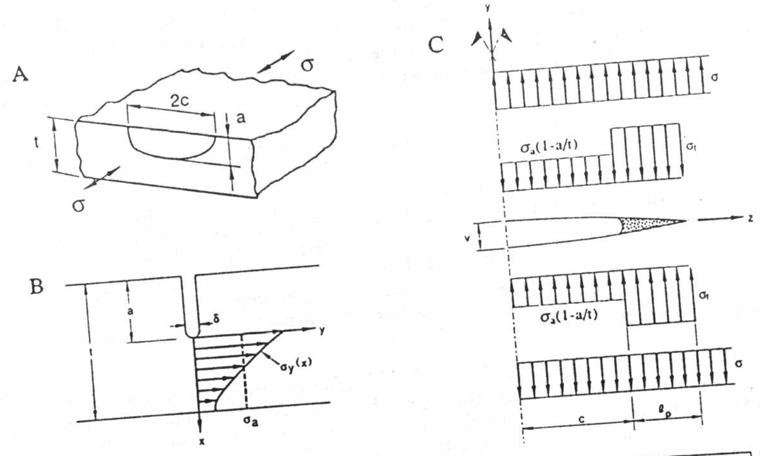

Fig. 1: Crack modeling

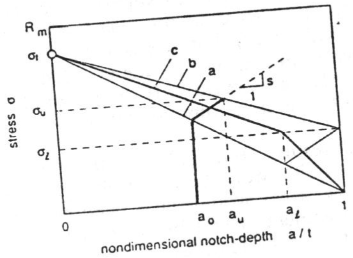

Fig.2: Limit of tearing instability (line b) and ultimate stress (line c)

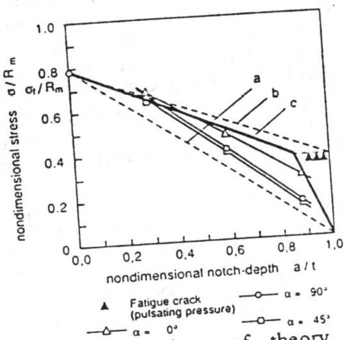

Fig.3: Comparison of theory and experiments (α: notch-angle)

REFERENCES

[1] Kieselbach, R., Cucci, E., EMPA-Bericht Nr. 102157, Jan. 1990

[2] Paris, P.C., et.al., in: "Elastic-plastic fracture", ASTM STP 668, pp. 5-36, 1979

[3] Schindler, H.J., Kieselbach, R., 23. Vortragsveranstaltung des DVM-Arbeitskreises Bruchvorgänge, Berlin 1991, pp. 73-82

[4] Dugdale, D.s., J. Mech. Phys. Solids, Vol 8, pp. 100-108, 1960

[5] Tada, H., et al., Stress Analysis Of Cracks Handbook, Del Research Corp., Hellertown, Pennsylvania, 1973

[6] Demafonti, G., Bufacchi, E., Proc. 6th Int. Conf. on Mechanical Behaviour of Materials, ICM6, Kyoto, 1991, pp. 123-128

[7] Schindler, H.J., E., Proc. 6th Int. Conf. on Mechanical Behaviour of Materials, ICM6, Kyoto, 1991, pp. 159-164