STRESS CONCENTRATION EVALUATION NEAR THIN ELASTIC INCLUSIONS

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The method of approximate evaluation of stress concentration near thin elastic inclusions in elastic bodies is proposed. Formulated by authors mathematical model of thin elastic inclusion deformation, way of problem reduction to the system of integro-differential equations in the jumps of stresses and inclusion surfaces displacements, the formulae connecting stress concentration factors near inclusions with remote field parameters, corresponding stress intensity factors form the basis of the elaborated method. It was approved on some specific problems.

Thin Inclusion Singular Model and Integro-Differential Equations of the Problem

Let us consider thin elastic inclusion placed symmetrically towards bounded by contour L medial region S. Following authors previous works (e.g. Stadnyk (1)) the equilibrium equations and Hooke's law are presented in displacements $[u_k^{(i)}]$ and stresses $[\sigma_{k3}^{(i)}]$ jumps on inclusion surfaces. Carrying out the discrete summation and appropriate integrating we shall obtain the following equations for inclusion surfaces

$$L_1\{(u_k^{(i)}),\ (\sigma_{k3}^{(i)}),\ [\sigma_{k3}^{(i)}]\} = A_1^{(i)},$$
$$L_2\{(u_k^{(i)}),\ [u_k^{(i)}],\ [\sigma_{k3}^{(i)}]\} = A_2^{(i)}, \qquad (1)$$
$$F\{(\sigma_{k3}^{(i)}),\ [u_k^{(i)}],\ h\} = A_3^{(i)}, \quad k=\overline{1,3}.$$

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Here L_1, L_2 - differential operators; F - appropriate functional; $(u_k^{(i)})$, $(\sigma_{k3}^{(i)})$ - sums of the displacements and stresses, respectively, in the opposite points of inclusion surfaces; $A_k^{(i)}$ - the known constants; h - inclusion thickness.

So, inclusion presence in solid leads to the changes of the remote stress field as well as in the case of slot with boundary conditions (1). In order to obtain equations of the problem, the same way was chosen as for integral equations construction of crack theory problems considered earlier by Panasyuk et al (2). In this case boundary conditions on slot surfaces are assumed in the form (1). As a result the problem is reduced to solving of the following integro-differential equations system

$$\iint_S M_j \{[\sigma_{k3}^{(i)}], [u_k^{(i)}]\} dS = P_j , \qquad (2)$$

$$k=\overline{1,3} ; \quad j=\overline{1,6} ,$$

where M_j - known differential operators, P_j - known functions of Descartes coordinates.

Unique solution of equations system (2) would be provided if the following equalities for displacements and stress jumps are valid on contour L of the medial surface S

$$[u_k^{(i)}]=0 , \quad k=\overline{1,3} ,$$

and stresses jumps satisfy the conditions

$$\iint_S [\sigma_{k3}^{(i)}] dS = 0, \quad k=\overline{1,3} .$$

Solution of the integro-differential equations system (2) can be obtained by means of successive approximations method, by presenting it in the product of polynomial with the unknown factors and basic solution of this sort equations or using numerical methods with the previous separation of function singularity.

Stress Concentration Evaluation in Matrix by Inclusion Contour L

The relations between stress concentration near elongated cavities and remote field parameters (corresponding stress intensity factors K_I, K_{II}, K_{III} near crack) were estimated using well-known approaches (Panasyuk et al (2), Cherepanov (3), Paris, Sih (4)). Taking into account inclusion longitudinal rigidity, new expressions for stress concentration in the points of contour L in the matrix were established

$$\left\{\sigma_{13}^{(M)};\ \sigma_{23}^{(M)};\ \sigma_{33}^{(M)}\right\}=\left\{(K_{II}\cos\theta-K_{III}\sin\theta)/\sqrt{\pi\rho}+\sigma_{13}^{(0)};\right. \tag{3}$$

$$\left.(K_{II}\sin\theta+K_{III}\cos\theta)/\sqrt{\pi\rho}+\sigma_{23}^{(0)};\ 2K_I/\sqrt{\pi\rho}+\sigma_{33}^{(0)}+\sigma_n\right\}.$$

Here $\sigma_n=\sigma_n^{(i)}-\sigma_n^{(0)}$, θ stands for coordinate angle of the polar system in the plane perpendicular to S with the beginning in the point O of its intersection with contour L; ρ - radius of curvature of inclusion section contour in the point O; $\sigma_{K3}^{(0)}$ - stress vector components in the points of contour L on the medial region S, calculated for the body without inclusion; $\sigma_n^{(i)}$ - normal to the contour L stresses inside the inclusion; $\sigma_n^{(0)}$ - normal to contour L stresses in the matrix in the case of the inclusion absence; n - coordinate on the normal to contour L.

It should be emphasized that introduced in equality (3) addends, σ_n in particular, sufficiently influence stress concentration. Depending on the inclusion rigidness the absolute value of σ_n can exceed $(2K_I/\sqrt{\pi\rho}+\sigma_{33}^{(0)})$ to a great extent.

Stress intensity factors K_I, K_{II}, K_{III} in plane of crack S are determined by the expressions

$$K_I=-\lim_{n\to-0}\sqrt{-2n}\left\{G\left[u_3^{(i)}\right]_n'/(2(1-\mu))+\right. \tag{4}$$

$$\left.+(1-2\mu)\left[\sigma_{n3}^{(i)}\right]/(4(1-\mu))\right\};$$

$$K_{II} = - \lim_{n \to -0} \sqrt{-2\pi n} \{ G[u_n^{(i)}]_n' /(2(1-\mu)) - $$
$$ - (1-2\mu)[\sigma_{33}^{(i)}]/(4(1-\mu)) \} ;$$

$$K_{III} = - \lim_{n \to -0} \sqrt{-2\pi n}\, G[u_t^{(i)}]_n' /2.$$

Here $[\ ]_n'$ stands for the derivative with respect to n ; G - shear modulus; μ - matrix Poison ratio; t - coordinate on the tangent to the contour L .

Thus, after the equations (2) being solved stress concentration by inclusion is found using formulae (3), (4).

Method Approximation

The elaborated method was approved in a number of problems treated by means of analytical methods thus confirming its effectiveness. In particular, the uniaxial extension of plate with elliptical inclusion by stresses p was considered. In this case the system of equations (2) has the exact solution presented in the form

$$[u_3^{(i)}] = D_1 \sqrt{-n(n+2a)}; \qquad (5)$$
$$[\sigma_{13}^{(i)}] = D_2 (n+a)/\sqrt{-n(n+2a)},$$

where D_1, D_2 - constants depending on matrix and inclusion elastic properties as well as on semiaxes of the inclusion.

Using the relations (3), (4) and basing on the solution (5) values of $\sigma_{k3}^{(m)}$ were calculated, which coincide with the exact solution of the problem presented by Hardiman (5) and Cherepanov (3) thus testifying effectiveness and high accuracy of the proposed method.

In the case $\rho \to 0$ the expression for stress concentration by linear thin elastic inclusion takes the form

$$\sigma_{33}^{M} = p\left[(1-\mu_1)^2 - (\mu_1 - \varepsilon\mu)^2\right] / \left[\varepsilon(1-\mu_1)(1-\mu)\right],$$

where $\varepsilon = G_1/G$, G_1, μ_1 - inclusion shear modulus and Poison ratio.

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