# STRESS ANALYSIS IN WELDED BUTT JOINTS

O.S. Yahşi* and E. Aydın*

In this study a symmetric contact problem of strips containing cracks is considered. Three strips are perfectly bonded over a finite length and are assumed to be isotropic. Uniaxial tension is applied to the middle strip away from the contact region. Integral transforms are used to reduce the problem to a system of singular integral equations of the second kind. The contact stresses and corresponding stress intensity factors are evaluated and presented for various strip geometries and material combinations.

## INTRODUCTION

Within the past two decades, problems of bonded dissimilar materials and load transfer problems in strips have attracted considerable attention. Since many engineering structures are comprised of more than one elastic material, the fracture at the interface between dissimilar material can be a critical phenomenon in many structures. For example, in [1] Keer analyzed the problem of a line bond between two layers. In this study, although the problem is formulated for dissimilar layers, numerical results are given only for the case of identical layers. In another study Yahşi and Göçmen [2] considered a finite contact problem for two perfectly bonded dissimilar infinite strips. In this study both of the strips are assumed to be isotropic and uniaxial tension is applied to the lower strip away from the contact region. However, the contact stresses that they obtained had a square root singularity without oscillation at the edge of the bond region; their solution therefore is appropriate only for identical materials. The same problem was reconsidered by Keer and Guo [3] and a system of singular integral equations of the second kind are obtained for the contact stresses, thereby yielding an oscillatory stress field near the crack tip. In a recent paper by Yahşi and Yazıcı [4] the contact problem for two bonded dissimilar homogeneous elastic strips is considered. In this study, the lower

* Department of Mechanical Engineering, Middle East Technical University, Ankara, Turkey.

strip was assumed to have two symmetric cracks perpendicular to the interface and subject to uniform tensile stresses far from the contact zone.

In this study, three symmetric isotropic strips are considered. Three strips are assumed to be perfectly bonded to each other through a finite length as shown in Fig. 1. The middle strip containing a pair of symmetric cracks is subjected to a uniform tensile stress far from the interface.

To solve the problem, the stress and displacement fields of an elastic infinite strip are obtained from the plane elasticity equations for an isotropic material by using Fourier transform techniques and a crack solution and a constant uniform stress solution is added to the middle strip solution. Finally, by using the appropriate boundary conditions the problem is reduced toa system of singular integral equations of the second kind.

## FORMULATION OF THE PROBLEM

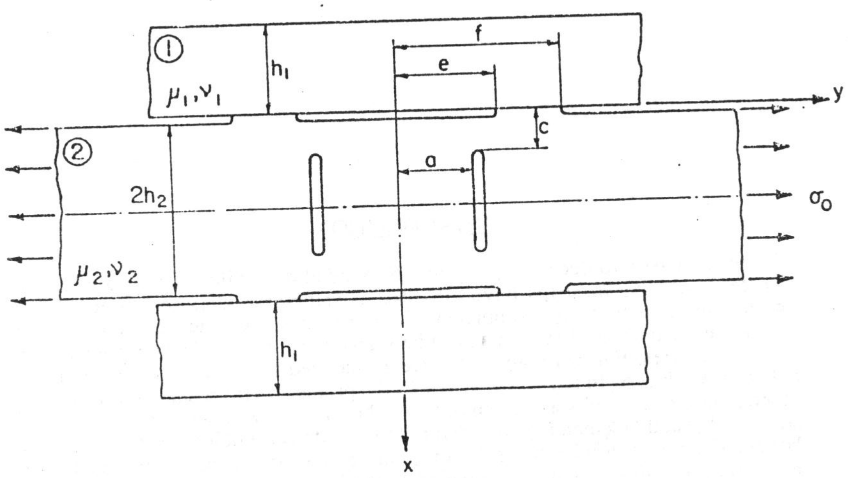


Figure 1 Geometry of perfectly bonded infinite strips.

Consider a strip of thickness $2h_2$ and elastic constants $\mu_2, \kappa_2$ bonded to two infinite strips of elastic constants $\mu_1, \kappa_1$ (Fig. 1), where $\kappa_j = 3\text{-}4\,\upsilon_j$ for plane strain and $\kappa_j = (3\text{-}\upsilon_j) / (1+\upsilon_j)$ for generalized plane stress, $\upsilon_j$ (j=1,2) being the Poisson's ratio. Thus, $x = h_2$ is a plane of symmetry and the boundary and continuity conditions may be expressed as

$$\sigma_{xx}^{(1)}(-h_1,y) = \sigma_{xy}^{(1)}(-h_1,y) = 0\,, \quad -\infty < y < \infty \tag{1a,b}$$

$$\sigma_{xx}^{(1)}(0,y) = \sigma_{xx}^{(2)}(0,y) = G_1(y)\,, \quad -\infty < y < \infty \tag{2a,b}$$

$$\sigma_{xy}^{(1)}(0,y) = \sigma_{xy}^{(2)}(0,y) = G_2(y)\,, \quad -\infty < y < \infty \tag{3a,b}$$

$$\sigma_{xy}^{(2)}(h_2,y) = 0\,, \quad u^{(2)}(h_2,y) = 0\,, \quad -\infty < y < \infty \tag{4a,b}$$

$$\frac{\partial r_i^{(1)}(0,y)}{\partial y} = \frac{\partial r_i^{(2)}(0,y)}{\partial y}\,, \quad e \le |y| \le f\,, \ i=1,2,\ r_1 = u\,,\ r_2 = v \tag{5a,b}$$

$$\sigma_{xj}^{(2)}(x, \pm a) = 0\ ; \quad j = x, y\ ; \quad c < x < d \tag{6a,b}$$

$$\text{where } G_i(y) = 0\,, \quad (i = 1,2)\,, \quad |y| < e \ \text{ or } \ |y| > f \tag{7}$$

In (1-6) and for the remainder of this section the superscripts (1) and (2) refer to the upper and middle strips, respectively.

Using the expressions given in [5] for an infinite plane for a pair of point dislocations with densities $f_1$ and $f_2$ located at the point $(x_o, y_o)$ and defined by

$$\frac{\partial}{\partial x}[r_i^{(1)}(x_o,0) - r_i^{(2)}(x_o,0)] = f_i(x_o)\, \delta(x-x_o), i = 1,2, r_1 = v, r_2 = u \tag{8a,b}$$

and standard Fourier transforms, the displacement and stress fields for the upper and lower strips can easily be obtained as given in [5].

Instead of dislocations if the middle strip contains cracks along $c < x < d$, $y_o = \pm a$ = constant, as shown in Figure 1, under a given set of surface tractions as stated in the equations (6a,b), by integrating the solution found for dislocations one would obtain the solution for the cracks.

By using the stress and displacement equations for the middle and upper strips given in [5] and the boundary conditions (1-4) and equations (7) and (8a,b) all the unknowns of the problem can be expressed in terms of $f_i(x)$, $G_i(y)$, $(i=1,2)$. Thus the complete solution of the problem is obtained once the unknown functions $f_1$, $f_2$, $G_1$, and $G_2$, are determined. Finally, by substituting the stress and displacement expressions obtained in terms of $f_i(x)$, $G_i(y)$, $(i=1,2)$, into the continuity conditions (5a,b) and the crack surface boundary conditions (6a,b) one would obtain the system of four singular integral equations given in [5] to determine the functions $f_i(x)$ and $G_i(y)$, $(i=1,2)$.

By extending the cracks to the upper and lower boundaries of the middle strip (i.e. $c = 0$, $d = h_2$) the problem can be reduced to a butt joint under the effect of uniform stresses. For this case by introducing the following substitutions into singular integral equations given in [5],

$$x = 0.5\, h_2\, (1 + \tau)\,, \qquad -1 < \tau < +1\,, \tag{9a}$$

$$t_2 = 0.5\, h_2\, (1 + \rho)\,, \qquad -1 < \rho < 1\,, \tag{9b}$$

$$t_1 = 0.5\,(f + e) + 0.5\,(f - e)\,\xi\,, \qquad -1 < \xi < 1\,, \tag{9c}$$

$$y = 0.5\,(f + e) + 0.5\,(f - e)\,\eta\,, \qquad -1 < \eta < 1\,, \tag{9d}$$

$$\phi(\xi) = G_2(\xi) + iG_1(\xi)\,,\ \ \psi(\tau) = f_1(\tau) + if_2(\tau)\,, \qquad i = \sqrt{-1}\,, \tag{10a,b}$$

these singular integral equations are reduced into the following form:

$$\gamma\phi(\eta) + \frac{1}{\pi i}\int_{-1}^{+1}\frac{\phi(\xi)}{\xi - \eta}d\xi + \int_{-1}^{+1}\{\psi(\rho)\,K_{11}(\rho,\eta) + \overline{\psi(\rho)}\,K_{12}(\rho,\eta)\}\,d\rho$$
$$+\int_{-1}^{+1}\{\phi(\xi)\,K_{13}(\xi,\eta) + \overline{\phi(\xi)}\,K_{14}(\xi,\eta)\}\,d\xi = h_1(\eta)\,,\ \ -1 < \eta < 1\,, \tag{11a}$$

$$\int_{-1}^{+1}\psi(\rho)\,K_{2S}(\rho,\tau)\,d\rho + \int_{-1}^{+1}\{\psi(\rho)\,K_{21}(\rho,\tau) + \overline{\psi(\rho)}\,K_{22}(\rho,\tau)\}\,d\rho$$
$$+\int_{-1}^{+1}\{\phi(\xi)\,K_{23}(\xi,\tau) + \overline{\phi(\xi)}\,K_{24}(\xi,\tau)\}\,d\xi = h_2(\tau)\,,\ \ -1 < \tau < 1\,, \tag{11b}$$

where the expressions of kernels $K_{ij}$ ($i$=1,2, $j$=S, 1,2,3,4) and known functions $h_1(\eta)$ and $h_2(\tau)$ are given in [5].

## NUMERICAL RESULTS AND DISCUSSION

The singular integral equations (11a,b) can be solved with the conditions

$$\psi(\pm 1) = 0\,, \qquad \int_{-1}^{1}\phi(\xi)\,d\xi = 2\,\frac{P + Q}{f - e} \tag{12a,b,c}$$

by using the method given in [5], where the solution of equations (11a,b) are approximated by

$$\phi(\eta) = F(\eta)(1-\eta)^{\alpha_1}(1+\eta)^{\beta_1}, \quad \psi(\tau) = R(\tau)(1-\tau)^{\alpha_2}(1+\tau)^{\beta_2} \qquad (13a,b)$$

$$\text{where } \alpha_1 = \frac{1}{2} - i\omega, \ \beta_1 = -\frac{1}{2} + i\omega, \ \alpha_2 = \beta_2 = -\frac{1}{2}, \ \omega = \frac{1}{2\pi}\log\left(\frac{1+\gamma}{1-\gamma}\right) \qquad (14a,d)$$

and the unknown functions $F(\eta)$ and $R(\tau)$ are approximated by a series expansion of orthogonal polynomials with unknown coefficients.

By using the following definition of stress intensity factor

$$k_1(e) - ik_2(e) = \lim_{t \to e} (t-f)^{-\alpha_1}(e-t)^{-\beta_1}(G_1 - iG_2) \qquad (15)$$

and equation (13a) the normalized stress intensity factors can be obtained as follows:

$$k_1(e) - ik_2(e) = \frac{i}{2}\frac{F(-1)}{\sigma_o}, \quad k_1(f) - ik_2(f) = \frac{i}{2}\frac{F(+1)}{\sigma_o} \qquad (16a,b)$$

Figures 2-5 give the first and second mode stress intensity factors, $k_1$ and $k_2$, versus $e/h_1$, respectively, for different material combinations of aluminium and steel. In the numerical calculations for plane strain, the following material constants are used.

aluminium : $\mu_a = 26.32$ GPa, $\upsilon_a = 0.33$
steel : $\mu_s = 80.77$ GPa, $\upsilon_s = 0.3$

From these figures, it is seen that at y=e, first mode stress intensity factor increases with the increase of $e/h_1$ ratio. On the other hand, as expected, it is decreasing with the increase of $e/h_1$ ratio at y=f which is the closest end to the applied tensile stress. In general, first mode stress intensity factors decreases as the contact length increases.

It is also found that the material dependence of $k_2$ is much more stronger than of $k_1$, and second mode stress intensity factor increase with the increase in contact length.

## REFERENCES


(1) Keer, L.M., J. Appl. Mech. Trans. ASME, Vol. 41, 1974, pp. 679-683.

(2) Yahşi, O.S. and Göçmen, A.E., Int. J. of Fracture, Vol. 34, 1987, pp. 161-177.

(3) Keer, L.M. and Guo, Q., Int. J. of Fracture, Vol. 43, 1990, pp. 69-81.

(4) Yahşi, O.S. and Yazıcı, Ş., Proc. SMİRT 11, Vol. G, 1991, pp. 13-19.

(5) Aydın, E., "Stress Analysis in Welded Butt Joints", M.Sc. Thesis, Middle East Technical University, Ankara, Turkey 1989.

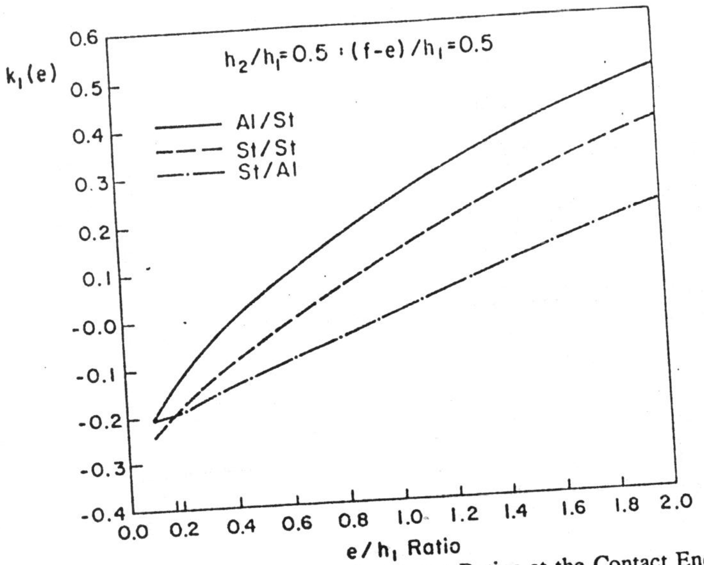


Figure 2. First Mode Stress Intensity Factor Ratios at the Contact End (y=e) of Perfectly Bonded Infinite Symmetric Strips ($h_2/h_1$=0.5, (f-e)/$h_1$=0.5, c=0).

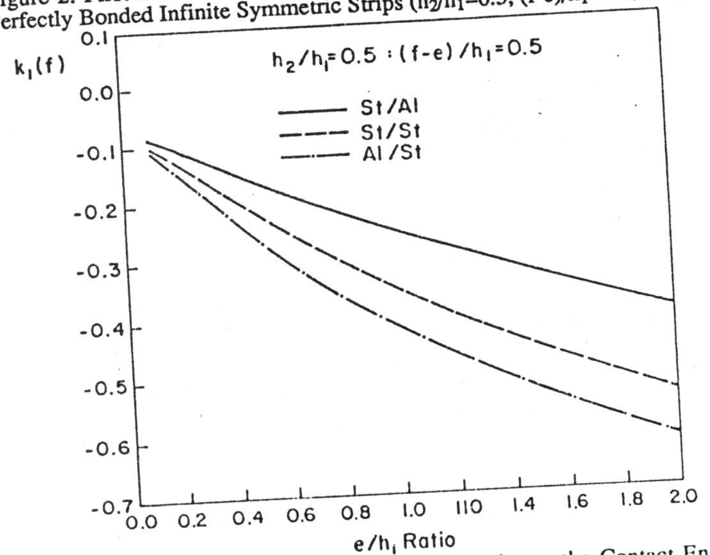


Figure 3. First Mode Stress Intensity Factor Ratios at the Contact End (y=f) of Perfectly Bonded Infinite Symmetric Strips ($h_2/h_1$=0.5, (f-e)/$h_1$=0.5, c=0).

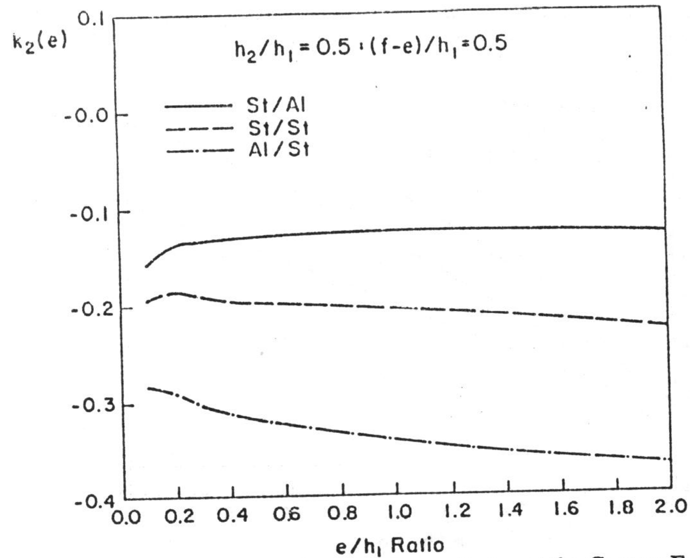


Figure 4. Second Mode Stress Intensity Factor Ratios at the Contact End (y=e) of Perfectly Bonded Infinite Symmetric Strips ($h_2/h_1$=0.5, (f-e)/$h_1$=0.5, c=0).

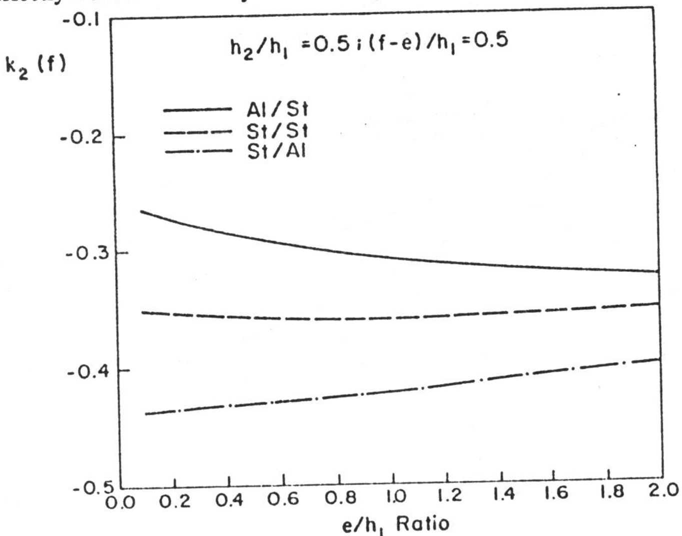


Figure 5. Second Mode Stress Intensity Factor Ratios at the Contact End (y=f) of Perfectly Bonded Infinite Symmetric Strips ($h_2/h_1$=0.5, (f-e)/$h_1$=0.5, c=0).