

# STOCHASTIC FATIGUE CRACK GROWTH PROBLEM OF HSLA STEEL WELDMENTS

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The stochastic fatigue crack growth problem of weldments was most conditioned by the weld joint properties and concentration of stress in crack area. This problem could be solved with the determination of the probability model of the fatigue propagation of crack by means of experimentally obtained results for assumed modes of loading.

## INTRODUCTION

The fatigue behaviour of weld joints and operation of the welded structure are influenced primarily by the weld joint properties. During fatigue crack propagation testing in the weld joint of several specimens the scattering of results can be noticed. It is mainly the result of the distribution of brittle microstructures and other hard and soft regions in the weld joint. Therefore it is necessary to determine a suitable model with which it is possible to establish reliably the probability that the fatigue crack will reach the length  $a_1$  in the number of cycles  $N_1$ . The task becomes more demanding when, during the fatigue, the given loading conditions change.

## DEFINITION OF THE MODEL

At the initial stage of the model defining it is necessary to determine the reliability range of the fatigue crack propagation.

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This can be done by dividing the experimentally determined fatigue curve into specified levels of the number of cycles  $N_i$  (Fig. 1). In this way the probability density function according to the lognormal distribution law is:

$$f(a) = \frac{1}{a \cdot \sqrt{2\pi} V_y} \cdot e^{-\frac{(\ln a - \mu_y)^2}{2 \cdot V_y}} \quad (1)$$

where the  $V_y$  variance and the  $\mu_y$  and expected value are determined according to the normal distribution law for  $y = \ln a$ .

Applying the mathematical theory of probability /1/, the reliability can be defined by:

$$R(a) = 1 - F(a) = \int_{a_{inf}}^{a_{sup}} f(a) da \quad (2)$$

With equation (2) it is possible to determine the limit values of the fatigue crack length ( $a_{inf}, a_{sup}$ ) in a selected reliability  $R(a)$  case. This means that after the cycles number  $N_i$  the crack length can assume any value within the interval ( $a_{inf}, a_{sup}$ ) for given loading conditions. The confidence intervals thus determined for the specified cycles number  $N_i$  form the reliability area of the crack propagation /2/.

The boundary (fatigue) curves of this area can be treated according to the Paris /3/ or Forman law, and the limit crack growth rates can be determined with the relevant values parameters  $C$  and  $n$ :

$$\frac{da}{dN} = C(\Delta K)^n \quad (3)$$

The values of the growth rate and fatigue crack length are located within the interval determined with the appropriate probability density function  $f(a)$ .

The next stage is the defining of the proper probability density function and the suitable probabilistic model. Therefore, it is necessary to determine a suitable criterion for establishing the reliability of operation, which would take into consideration the cracks and defects in the fatigue loaded structures components.  $\Delta K$  (range of the stress intensity factor) can be used as a suitable parameter because it can take into account the influences of the stress concentration, geometry, temperature, atmosphere etc.

During the fatigue test of the specimen, also  $\Delta K$  can be determined:

$$\Delta K = \frac{\Delta F \cdot S}{B \cdot W^{1.5}} Y\left(\frac{a}{W}\right) \quad (4)$$

$\Delta F = F_{\max} - F_{\min}$       W-specimen height  
 B-specimen width      S-distance between supports for 3PB  
 $Y\left(\frac{a}{W}\right)$  - parameter of crack length and type of specimen

The value  $\Delta K$  will be within the  $\Delta K(a_{\inf}) - \Delta K(a_{\sup})$  and its frequency will be determined with the density of the fatigue crack length in case of cycles number  $N_i$ . Thus it is possible to develop the function  $f(\Delta K)$  above the interval  $\Delta K(a_{\inf}) - \Delta K(a_{\sup})$ . In case of the assumed loading  $\Delta F_0$  a distribution of frequency of length  $a_0$  will occur. Therefore, it is possible to expect that the relevant value  $\Delta K_0$  will be located within an interval  $\Delta K_0(a_{\inf_0}) - \Delta K_0(a_{\sup_0})$ , as well.

The influence of microstructural properties on the fatigue curves with lower loading condition  $\Delta K_0$  will not be greater than in case of  $\Delta K$ . Therefore, the variance will be  $V(a_0) \leq V(a)$  and  $V(\Delta K_0) \leq V(\Delta K)$  and it is possible to determine the probability function  $f(\Delta K_0)$  above the appropriate interval.

The probability  $P(\Delta K_0)$  that the fatigue crack in the  $\Delta K_0$  case with the respective density  $f(\Delta K_0)$  will reach the crack length  $a$ , which is reached in the  $\Delta K$  case with  $f(\Delta K)$  after  $N_i$  cycles, can be determined according to the following expression:

$$P(\Delta K_0) = P(\Delta K_0 \leq \Delta K) = \int_0^{\infty} f(\Delta K_0) \int_{\Delta K_0}^{\infty} f(\Delta K) d(\Delta K_0) d(\Delta K) \quad (5)$$

### EXPERIMENTAL RESULTS

The fatigue crack propagation was determined for the weld joint of HSLA steel NIOMOL 490K, on 20 three-point bend specimens (3PB). The constant amplitude fatigue loading was applied in load control conditions at the  $F_{\min}/F_{\max}$  ratio  $R=0.1$  ( $F_{\max}=120\text{kN}$ ) and frequency 25Hz. The fatigue crack propagation was observed up to the cycles number  $N=145000$  cycles (Fig. 1).

### PROBABILISTIC ANALYSIS

Figure 2 shows the limit values of the fatigue crack lengths ( $a_{\inf}, a_{\sup}$ ) in case of reliability  $R(a)=99\%$  calculated according to the equation (2). By means of this limit values ( $a_{\inf}, a_{\sup}$ ), and

by expecting that the crack growth rates will not be greater than determined experimentally, it is possible, by the use of the equation (3), to determine the crack growth intervals  $\Delta a_0$  for assumed loading conditions  $\Delta F_0$ .

The applicability of this model is conditioned by the initial state determined by the fatigue conditions  $F_{max}=120\text{kN}$ ,  $R=0.1$ , cycles number  $N_1=30$  kilocycles and interval of crack length  $(a_{inf}, a_{sup})_1$  with appropriate probability density function  $f(a)_1$ . As we can claim with 99% reliability that in case of the cycles number  $N_1$ , the value of the parameter  $\Delta K_1$  will be in  $[\Delta K(a_{inf}), \Delta K(a_{sup})]_1$ , the fatigue crack length  $a_1$  will be in  $[a_{inf}, a_{sup}]_1$  with the same reliability.

Assuming that the specimen loaded from  $N_1=30$  kilocycles onwards with  $F_{max}=108\text{kN}$  ( $R=0.1$ ), by 10% lower loading range than fatigue testing, it is possible to determine the values  $\Delta K(a)$  and  $\Delta K_0(a_0)$  by applying equation (4). For this reason, it can be established that the probability that the fatigue crack  $a_0$  will reach the experimentally determined length  $a$  ( $a_0 \in [a_{inf}, a_{sup}]_1$ ) is the same as the probability  $P(\Delta K_0)$  that  $\Delta K_0$  will reach the value of the interval  $[\Delta K(a_{inf}), \Delta K(a_{sup})]$ .

### ANALYSIS OF RESULTS

The direction of the scatter band is determined by the parameters of Paris law C and n. In spite of lower crack growth rate under assumed fatigue conditions, the results obtained (in the form of too great crack growths) are conservative and are used as auxiliary results. An outstanding difference between  $f(a_0)$  and  $f(\Delta K_0)$  is that  $f(\Delta K_0)$  at the level  $N_1$  undergoes a drop to lower values due to the drop of the load from  $\Delta F$  to  $\Delta F_0$ , Fig. 3.

As crack growths are too great, it is necessary to improve the probabilistic model with the criterion  $f(\Delta K_0)$ . For the reliability  $R(a)=99\%$  in case of 10% lower fatigue load, the probability  $P(\Delta K_0)$  that the fatigue crack will reach the length reached during the experiment is very low, Fig. 4.

In this case the probability varies between the individual levels of cycles  $N_i$ , which is clearly shown in Fig. 4. The assumed fatigue load cannot be decreased without limits because in case of  $\Delta K_0 < \Delta K_{th}$  (lower than  $\Delta K$  threshold) the crack propagation does not occur.

### CONCLUSION

The described stochastic fatigue crack growth problem for changed modes of loading could be possibly solved by a probabilistic model. This model with the selected reliability of 99%, and at assumed fatigue conditions, makes it possible to determine the value of probability  $P(\Delta K_0)$  in which case the fatigue crack will reach the length which is reached under the experimental load range. As the obtained results are applied on welded structure parts, they ensure, in case of the operating load  $\Delta F_0=108\text{kN}$  ( $R=0.1$ ), a reliable operation of the structure at least up to  $N=145$  kilocycles, despite the high scatter band and without the danger that the fatigue crack could reach a length of  $a=22.29$  mm.

The disadvantage of the described probabilistic model is that the size of the assumed loading condition is limited. Another disadvantage is that it is not possible to determine the length of the fatigue crack.

The advantage of this model is that it is possible to interpret and use the experimentally obtained results for the loading conditions, which are not directly tested and that with a conservative approach the reliability is maintained.

### REFERENCES

- (1) I. Pavlič "Statistika, teorija i primjena" Tehnička knjiga, Zagreb, 1988.
- (2) J. Legat, N. Gubeljak, Č. Primec, The Proceedings of the 10<sup>th</sup> Congress on Material Testing, Vol. 2, pp 507-512, Budapest '90.
- (3) P.C. Paris, F. Erdogan, Trans. ASME, J. Basic Eng., Vol. 85, No. 4, pp. 528
- (4) N. Vujanović, in "Teorija pouzdanosti tehničkih sistema" Vojno izdavački i novinski centar, Beograd, 1987.
- (5) J. Tang, B.F. Spencer, Jr., Eng. Frac. Mech., Vol. 34, No 2 (1989), pp. 419-433
- (6) H. Ghonem, S. Dore, Eng. Frac. Mech., Vol. 27, No 1 (1987), pp. 1-25

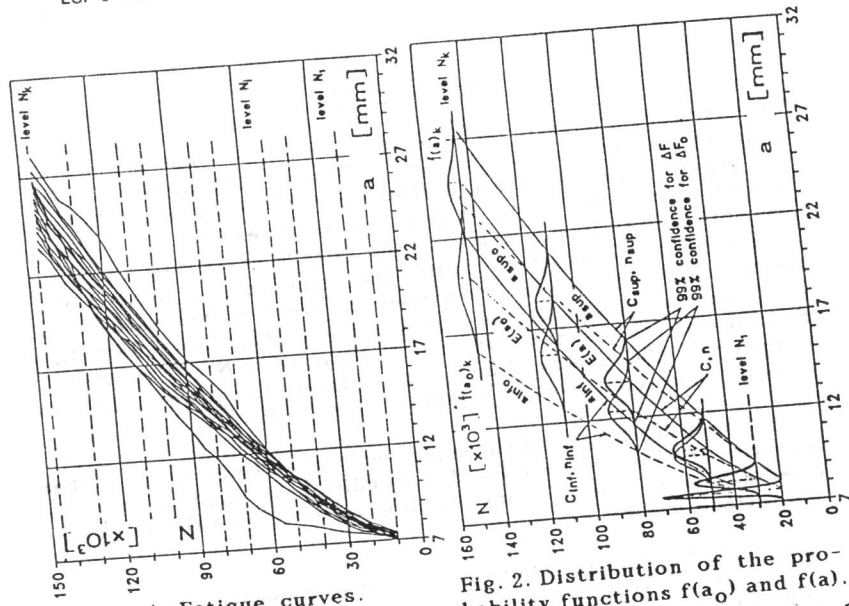


Fig. 1. Fatigue curves.

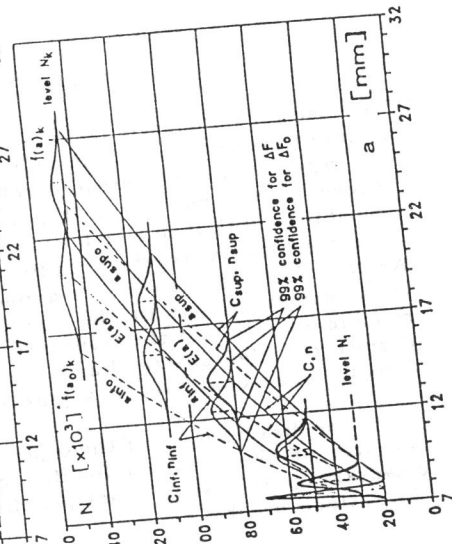


Fig. 2. Distribution of the probability functions  $f(a_0)$  and  $f(a)$ .

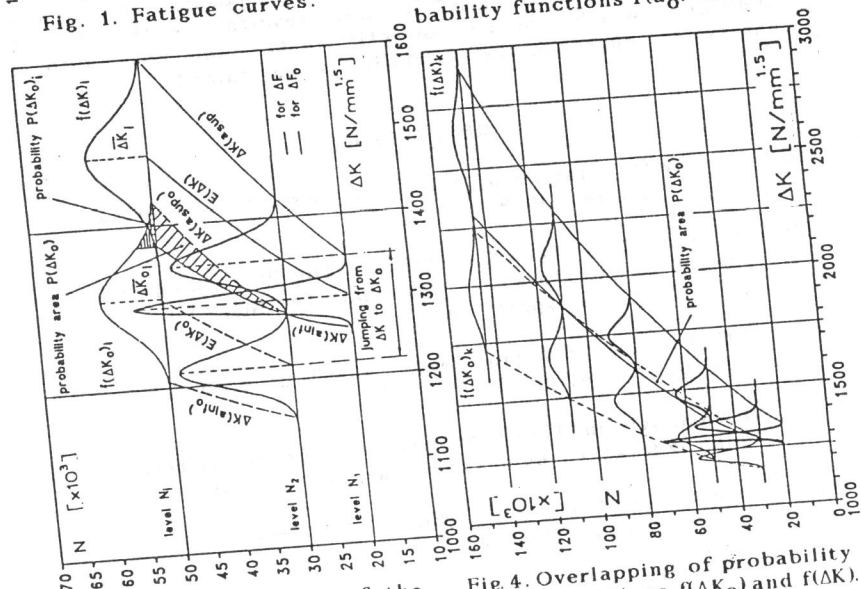


Fig. 3. Initial drop of the value  $\Delta K_0$ .

Fig. 4. Overlapping of probability density functions  $f(\Delta K_0)$  and  $f(\Delta K)$ .