Sensitivity of the load carrying capacity of plastic products to the material model parameters

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In order to predict the cleavage fracture of brittle plastic (PMMA) products with stress concentrations the stress and strain field must be calculated. The sensitivity of the stress and strain field to the material model parameters have been investigated by parametric studies. Especially the magnitude of the peak stress as a function of the material parameters has been determined. The possibility of predicting the probability of cleavage fracture using the Weibull statistics based on the results of FEM calculations, is presented.

INTRODUCTION

Plastic products often fail in a brittle manner due to stress concentrations. The effect of stress concentrations on the stress and strain field in the vicinity of stress concentrations can be predicted by carrying out Finite Element Method (FEM) calculations. PMMA was selected in our research because of its brittle behaviour at room temperature and the brittle-to-ductiletransition at $40\,^{\circ}C$ which effect on the mechanical behaviour will be investigated in the next future. The constitutive relations of plastics are, unlike the majority of metals, rather complex and not completely determined. Moreover an adequate three dimensional constitutive relationship in the non-linear visco-elastic range for PMMA was not available. The calculations have been carried out applying an isotropic elasto-plastic Prandtl-Reuss model in order to approximate the multi-dimensional non-linear visco-elastic constitutive behaviour. The effects of modelling the stress-strain curve on the peak stress have been investigated and are presented in the first part of this contribution. The fracture of plastic products due to stress concentrations have also been investigated. Because

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of the brittle behaviour of PMMA the application of the Weibull statistics, has also been investigated. The results of applying the Weibull distribution to characterize the rupture stresses of test bars with a stress concentration are presented.

THE ELASTO-PLASTIC CALCULATIONS

Sensitivity analysis

The parameters of the isotropic plasticity model for PMMA have been determined by measuring the stress-strain curve on a standard test bar. This curve has been linearized by a number of increments with variable elasticity as depicted in figure 2, to enable Finite Element Method (FEM) calculations. The stress-strain curve of plastics and the magnitude of the yield stress varies heavily upon the temperature and rate of testing. In order to determine this effect of the course of the stress-strain curve on the stress and strain field in a test bar with a central hole, a parametric study was carried out. In this study the effect of three parameters was investigated:

- the course of the stress-strain curve up to the yield stress,
- the magnitude of the yield stress and
- the number of increments to linearize the curve stepwise,

Taking the experimentally determined stress-strain curve as a starting point each parameter was varied, keeping the other two constant. For each variation of a parameter the resulting stress and strain field has been calculated. The maximum load applied in the FEM calculations was obtained from the tensile tests on specimens with a central hole.
The FEM analyses have been carried out with the MARC K4 program using plane strain (element nr. 27) and plane stress (element nr. 26) elements. The Von Mises stress has been used as the yield criterion. Only the physical non linearity was to be accounted.

Results and discussions

The course of the stress-strain curve up to the yield stress. From the results of the FEM analyses it was learned that proportional stretching of the stress-strain curve with respect to the strain axis did not alter the stress field. The strain field alters proportional with the stretching factor. Using the model of elastic-perfectly plastic material behaviour means that the stress field will not change in case of varying the modulus of elasticity.
Effect of the magnitude of the yield stress The magnitude of the occurring peak stress depends on the magnitude of the yield stress. The FEM analyses revealed that a change of 20 % of the yield stress resulted in a change of 20 % of the peak stress in case of plane stress and only 10 % in case of plane strain.

Number of increments to linearize the stress-strain curve. If the elastic-perfectly plastic material model, with only 2 increments is applied, the highest peak stress is calculated (figure 2 and 1). If the number of increments is increased up to 3 increments the peak stress decreases, but above 3 increments the peak stress increases slightly. Applying more than 6 increments does not change the stress field noticeably. It seems that the discontinuities in the stress-strain curve coincide with the discontinuities in the stress field. The representation of the stress-strain curve by an elastic-perfectly plastic material results in a conservative value of the peak stress.

APPLICATION OF WEIBULL STATISTICS TO CHARACTERIZE THE PROBABILITY OF RUPTURE

The Weibull distribution is often applied to characterize the experimentally determined rupture stress as well as the model for the weakest link analysis of the material behaviour. The general expression of the probability of failure using the Weibull distribution is (2):

$$P_F = 1 - e^{-\left(\frac{\sigma}{\sigma_u}\right)^m} \quad \text{for } \sigma \geq \sigma_{ys} \qquad P_F = 0 \quad \text{for } \sigma < \sigma_{ys} \tag{1}$$

and its parameters can be estimated using Weibull probability paper.

Experimental set up

The statistical distribution of the rupture stress of PMMA (ICI CMG 302) test bars (ISO r 527) was determined experimentally. A central hole was drilled in the bars , which were submerged in water to minimize thermal degradation. The test bars were annealed after the central holes were drilled. The tensile test were carried out at a room temperature of $23\,^\circ C$ and the cross head speed was 5 mm/min and 500 mm/min. For each separation speed a series of 25 bars was tested.

Predicting of the cleavage fracture probability

In the Weibull statistics the material is modelled as a chain of elementary cells. For the failure mode rupture the "weakest link analysis" is applied and the cells are thought to be arranged in a series configuration.
The so called Weibull stress is a parameter that determines the probability of fracture of a structure and can be calculated according to the method proposed in (1)(3). The expression for the Weibull stress is:

$$\sigma_w^m = \sum_{plastic\ zone} \sigma_1^m \frac{\Delta V}{V_0} \tag{2}$$

This stress has been determined from the stress field around a stress concentration according to the procedure as described in more detail in (3) and using the results from experiments with a stress concentration. Each rupture test generates a rupture stress at a distance far enough from the stress concentration. The non homogeneous stress field has been calculated with the non linear FEM calculations in a FEM model that includes the central hole. For each rupture stress the highly stressed area in which the stress exceeds the yield stress is determined to find the plastic zone. The maximum principal stress σ_1 is then computed in all the elements in the plastic zone to calculate the Weibull stress. The Weibull parameter m (shape parameter) have been found using the procedure described in [3]. The final results are presented in table 1. Once the Weibull stress σ_w has been determined the probability of cleavage fracture can be calculated using the three parameter Weibull model:

$$P_F = 1 - e^{-\left(\frac{\sigma_w - \sigma_{th}}{\sigma_u - \sigma_{th}}\right)^m} \qquad (3)$$

In the case of a two parameter Weibull model $\sigma_{th} = 0$.

Weibull distribution:				
Strain rate:		2-parameter:		3-parameter:
5 mm/min	m=71.0	$\sigma_u = 87.5\ N/mm^2$	m=3.5	$\sigma_{th} = 84.2\ N/mm^2$ $\sigma_u = 2.3\ N/mm^2$
500 mm/min	m=76.0	$\sigma_u = 107.7\ N/mm^2$	m=5.0	$\sigma_{th} = 104.3\ N/mm^2$ $\sigma_u = 4.0\ N/mm^2$

Table 1: Parameters for the 2 and 3 parameter Weibull models

Discussion of the results.

- The variation in the rupture stresses is low and the coefficient of variation was 2.81 % (5 mm/min) and 2.57 % (500 mm/min). If a two parameter Weibull model is applied the Weibull shape parameter m is very high. If a three parameter Weibull model is applied the scatter in the experimental results can be determined more accurately.
- In the case of 5 mm/min the variation in the experimental results is larger than those of 500 mm/min. The higher the strain rates the higher the probability of cleavage fracture and also the higher the scatter in the results should be. This could not be concluded from the results.
- The three parameter Weibull model is very sensitive to changes in the threshold value. Small variations of this value result in a large change in the shape parameter m.
- Because the values of the shape parameter of the three parameter Weibull model are close to 3, the application of the normal distribution has also been considered. The rupture stresses could be very well characterized by the normal distribution and this was tested with the Kolmogorov-Smirnov test. The hypothesis of the validity of using normal distribution could not be rejected.

Conclusions and future research

- The two parameter Weibull model is very sensitive for changes in the parameters. A small variation has a great influence on the predictability of the model. The three parameter Weibull model is very sensitive for the estimated threshold value. A small variation in the threshold value influences the shape parameter considerably.
- Changes in strain rate by a factor of 100 have little influence on the variation of the experimental results and hence on the shape factor.
- Further investigations on the applicability of the Weibull and the normal distribution will be carried out in order to determine the appropriate distribution.
- 3D FEM analysis will be carried out also to determine the accurateness of the plane stress and plane strain approximations and the results will be presented on the poster.
- The applicability of the cleavage fracture at higher temperature will also be investigated especially in the transition range (ductile to fracture behaviour).
- The dependence of the model for other geometrical changes like notches will be studied in order to determine the generality of the model.

SYMBOL LIST

P_F = Failure probability
σ = Normal stress
$\sigma(x)$ = Non uniform normal stress
V_{ys} = Volume plastic zone
V_0 = Characteristic volume
σ_{ys} = Yield stress
σ_{th} = Threshold stress
m = Shape factor
V = Volume
σ_w = Weibullstress
σ_u = Scale parameter

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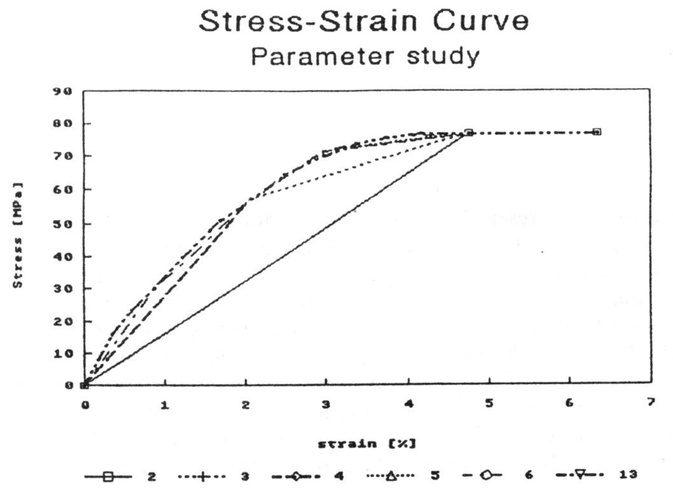

Figure 1: Increments in stress-strain curve

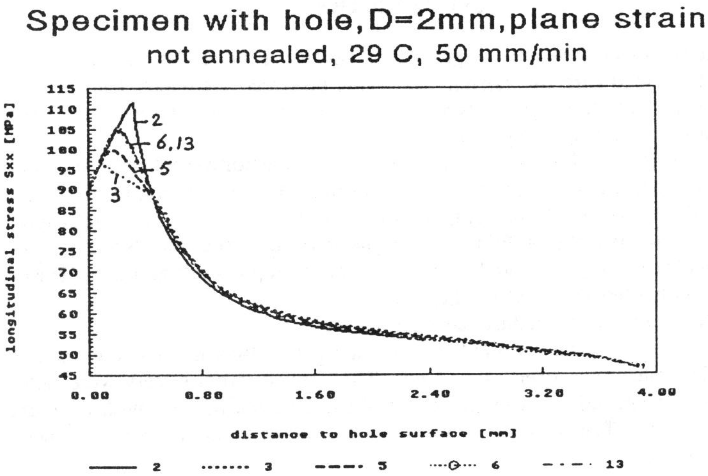

Figure 2: $\sigma_{longitudinal}$ versus distance to hole