RESIDUAL STRESSES IN BONDED DISSIMILAR MATERIALS

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The stresses near the free edge of the interface between dissimilar materials under thermal loading can be described by one or two singularity terms and a term which is independent of the distance from the free edge. Some examples are given for the effect of the material properties on the parameters describing the stress distribution.

INTRODUCTION

If two materials with dissimilar elastic properties and thermal expansion coefficients are bonded together high stresses occur at the free edge of the interface under mechanical loading or after a change in the temperature. In ceramic-metal joints these stresses may cause failure in the ceramic material. The stresses depend on the mentioned physical properties, but also on the size and geometry of the joint and the plastic deformation behaviour of the different components, especially of the solder in a brazed metal-ceramic joint.

In this paper only a joint of two materials is considered and elastic deformation is assumed.

BASIC RELATIONS

In Fig. 1 the general configuration at the free edge is shown, which is characterized by the two angles θ_1 and θ_2. The stresses can be calculated by applying Airy stress function and the boundary and continuity conditions at the free edge and the interface. The stresses near the free edge of the interface can be expressed by the relation

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$$\sigma_{ij}(r,\theta)=\sum_{k=1}^{N}\frac{K_k}{(r/L)^{\omega_k}}\cdot f_{ijk}(\theta)+\sigma_o\, f_{ijo}(\theta) \qquad (1)$$

where L is a characteristic length of the component, ω_k are the stress exponents, K_k stress intensity factors and f_{ijk} are angular functions. Dependent on the geometry and the elastic constants, one or two ω_k are positive leading to stress singularities.

For mechanical loading there is $\sigma_o = 0$ (excluding the case of a body with a crack). For thermal loading, however, σ_o is a major contribution to the stress field also near the singularity point. For thermal loading σ_o and K_k are proportional to $\Delta\alpha \cdot \Delta T$ and functions of the Dundurs parameters and the angles θ_1, θ_2. ΔT is the difference between the actual temperature T_2 and the temperature T_1, where the joint is stress free. $\Delta\alpha$ is the difference between the effective thermal expansion coefficients:

$$\Delta\alpha = \begin{cases} \alpha_1 - \alpha_2 & \text{for plane stress} \\ \alpha_1(1+\nu_1) - \alpha_2(1+\nu_2) & \text{for plane strain} \end{cases} \qquad (2)$$

The Dundurs parameters are dependent on the elastic constants of the two materials (1).

The ω_k and the angular functions depend only on the elastic constants and the geometry. σ_o, ω_k and the f_{ijk} can be calculated analytically, whereas the K_k have to be determined with numerical methods, for instance the finite element method.

THE JOINT WITH $\theta_1 = -\theta_2 = 90°$

For this joint the σ_o-term is given in cartesian coordinates by

$$\sigma_x = \tau_{xy} = 0 \qquad \sigma_y = \sigma_o \qquad (3)$$

with

$$\sigma_o = \Delta T \cdot \Delta\alpha \, \frac{1}{E_1^{*-1} - E_2^{*-1}} \qquad (4)$$

$$E_i^* = \frac{E_i}{\nu_i} \text{ for plane stress} \qquad \frac{E_i}{\nu_i(1+\nu_i)} \text{ for plane strain}$$

In Figures 2-4, σ_o, ω and K (determined from FEM) are plotted versus $(E_1^*-E_2^*)/(E_1^*+E_2^*)$. It can be seen that for $E_1^* \to E_2^*$ there is $\omega \to 0$, $\sigma_o \to \infty$ and $K \to \infty$. The ratio K/σ_o, however, is finite and approaches −1 for $\omega \to 0$. It was found that there is a unique function between the ratio K/σ_o and the exponent ω (Fig. 5) which can be expressed for H/L>1 by (2)

$$-K/\sigma_o = 1-2.89\,\omega + 11.4\omega^2 - 51.9\omega^3 + 135.7\omega^4 - 135.8\omega^5 \qquad (5)$$

Thus it is possible to calculate the stress distribution analytically from eq. (1), without any finite element caculation.

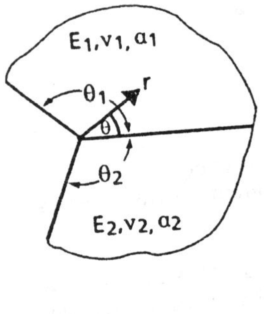

Fig.1 General configuration at the edge of a joint between dissimilar materials

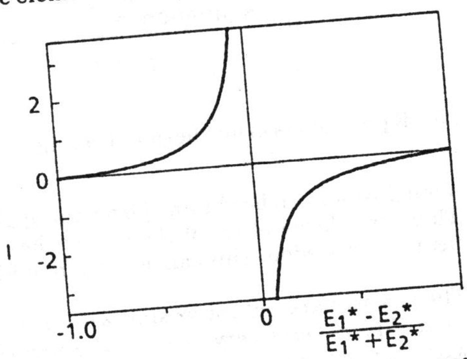

Fig.2 σ_o for a joint with $\theta_1 = -\theta_2 = 90°$

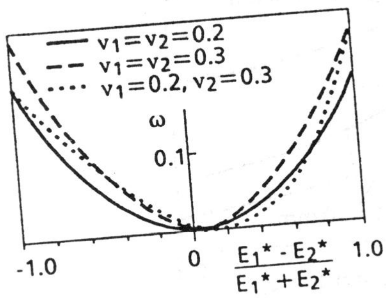

Fig. 3. ω versus differences in effective moduli ($\theta_1 = -\theta_2 = 90°$)

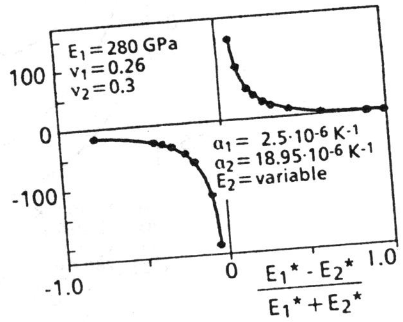

Fig. 4. Effect of difference in effective moduli on $K_I / \Delta T$ ($\theta_1 = -\theta_2 = 90°$)

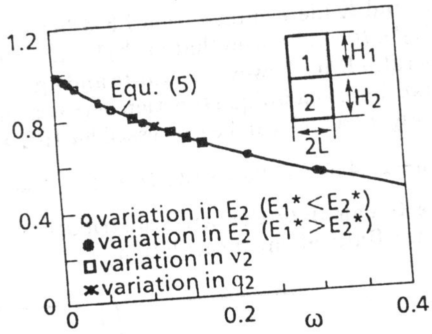

Fig 5. K_1/σ_o versus ω for thermal loading

Two conclusions can be obtained from the application of eq. (1).

1. There is a large effect of the size of the component on the stresses near the free edge. This can be seen from Fig. 6.

2. The stresses are not increasing with increasing ω. This can be seen from Fig. 7. Only very close to the free edge the stresses increase with increasing ω.

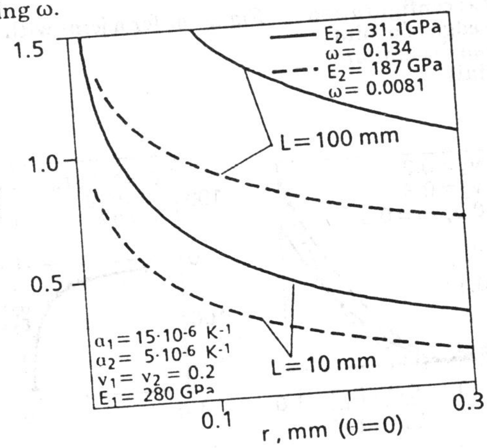

Fig 6. Effect of component size on $\sigma\theta/\Delta T$ for thermal loading.

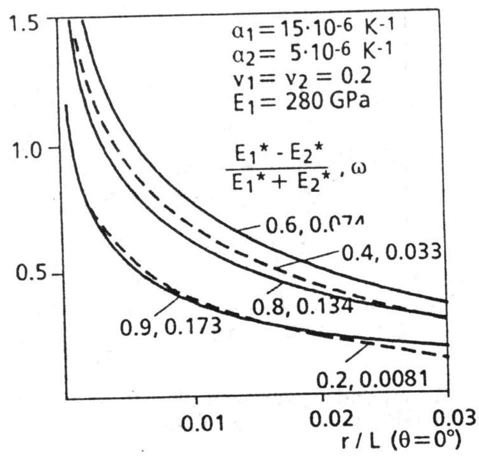

Fig. 7. $\sigma_\theta/\Delta T$ for $\theta = 0°$ versus r/L for different $E_1^* - E_2^*$

JOINTS WITH ARBITRARY ANGLES θ_1 AND θ_2

As an example a joint with $\theta_1 = 165°$ and $\theta_2 = -55°$ is considered. In Fig.8. σ_o, K_1, K_2, ω_1, ω_2 are plotted versus E_2/E_1 for $E_1 = 280$ GPa, $\nu_1 = 0.26$ and $\nu_2 = 0.30$. It can be seen that

- ω_1 and ω_2 can be <0 (no singularity) or >0. For $E_2/E_1 > 30.2$ the ω_k are complex and specific considerations are necessary.
- σ_o approaches infinity for $\omega_1 \to 0$ or $\omega_2 \to 0$.
- K_1 approaches infinity for $\omega_1 \to 0$ and K_2 approaches infintiy for $\omega_2 \to 0$.

In Fig. 9 K_1 / σ_o and K_2 / σ_o are plotted versus ω_1 and ω_2 respectiveley. It can be seen that $K_k / \sigma_o = -1$ for $\omega_k = 0$. Thus the stresses remain finite.

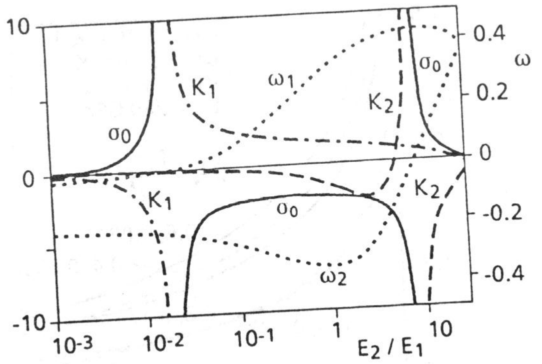

Fig. 8. σ_o, K_1, K_2, ω_1 and ω_2 versus E_2/E_1 ($\theta_1 = 165°$, $\theta_2 = -55°$, $E_1 = 280$ GPa, $\nu_1 = 0{,}26$, $\nu_2 = 0{,}30$)

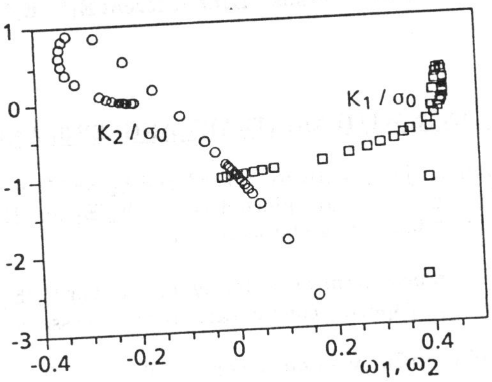

Fig. 9. K/σ_o versus ω for combination with $\theta_1 = 165°$, $\theta_2 = -55°$

REFERENCES

(1) Dundurs, J., J. of Applied Mech., Vol. 36, 1969, p. 650.

(2) Munz, D. and Yang, Y.Y., J. of Applied Mech., to appear 1992.