PROBABILITY APPROACH TO THE RELIABILITY CONCEPT OF VEHICLE STRUCTURES

M. Matolcsy, CS. Molnár*

The fatigue life of a structural element is determined by two processes: one is the service strength and the other one is the extreme load history. The service strength is the actual, momentary load bearing capacity of the element, which is decreasing after the fatigue crack initiation and during its propagation. The crack initiation and propagation is a stochastic process which can be described with the method of theory of probability. On the other side, the extreme loads are also probabilistic variables so the fatigue life distribution can be resulted from these two stochastic processes. On the basis of this probability approach a new concept can be defined for the reliability, in which the safety factor, the fracture sensitivity are time dependent and the function of statistical and load parameters.

THE RELATION BETWEEN THE STRESS HISTORY AND THE FRACTURE PROCESS

The initial strength (load bearing capacity) of the individual structural parts is usually reduced by the fatigue damage process caused by the stress alternation, that means the service strength of the components is a monotonously decreasing function of the time and also a stochastic process. The final fracture of the individual elements will occur when the momentary extreme stress exceeds the damage, i.e. reduced service strength. This fracture model - see Fig. 1 - shows that the service stress history has a double role in the developing of the final fracture

- the repeating alternation of the stress causes fatigue damage in the dangerous cross section of the element
- one of the extreme values of the stress history (the rare, very high peaks of the stress history) causes the final fracture of the element.

Studying the problem of the service strength of the components, and their damage process, the following three stages have to be analyzed and described mathematically:

- The initiation of the fatigue crack. This problem is connected to the stress concentration. The incubation time - which is needed to the crack initiation - depends on certain parameters of the material, the constructional features of the component and the stress history. In the first approximation the effect of the extreme stresses can be neglected.
- The stable crack propagation under stochastic stress history,

* IKARUS BUS FACTORY, Budapest, Hungary

where problem belongs, to the dangerous cross section. This process depends on the material behaviour, the geometric configuration of the component and the stress history, but it can be described by different parameters then those used in crack initiation. The extreme stresses play an essential role in this process, delaying or increasing the crack propagation rate.

- The unstable crack propagation under the effect of one extreme stress. This very rapid process in the final fracture, which could be characterized also by three parameter-families, being different from the previous ones. Only the extreme stresses are of interest in this period, the other parts of the stress history can be ignored from this point of view.

From this it is clear that the extreme stresses have a particular importance in the problem of structural strength and service life. The other main effect of stress history is the fatigue process, caused by its continuous alternation. From this point of view, the stress history has two essential parameters. These are the mean value and the root mean square (RMS) value, which is closely related to the standard deviation:

$$\overline{x(t)} = \frac{1}{\tau}\int_0^{\tau} x(t)dt; \qquad x_{RMS} = \left[\frac{1}{\tau}\int_0^{\tau}\left(x(t)-\overline{x(t)}\right)^2 dt\right]^{1/2} \tag{1}$$

If the stress history x(t) is stationary, ergodic process with a wide frequency band, these parameters can describe the fatigue process. Figure 2 shows the basic concept of the equivalent stress history derived from the original one.

THE THEORY OF EXTREME STRESSES (1)(2)

The complex stress history x(t) can be derived into three essential components:

- stresses from manufacturing
- stresses from the pay load
- stresses from the operation.

Figure 3a shows the x(t) stress history. Let us assume that an "i" number of realizations $x^{(1)}(t), x^{(2)}(t), \ldots, x^{(i)}(t)$ are available for a representative length. There are "n" local maxima in an interval Δt. Furthermore, we assume, that the stress history is a stationary and ergodic process, thus it is enough to study only one realization in the time instead of "i" realizations in a given Δt time-interval. Therefore the X_i extreme stresses may be defined as the absolute maximum stresses in the intervals $\Delta t^{(1)}, \Delta t^{(2)}, \ldots, \Delta t^{(i)}$, being equal to each other (Fig.3b). Not going into details of the derivations we obtain the extreme value distribution for three essential stress component

$$\phi_1(X_1) \equiv F_1(x_1); \qquad \phi_2(X_2) = \exp\left[-\left(\frac{\omega - x_2}{\omega - m_2}\right)^{\alpha_2}\right];$$
$$\phi_3(X_3) = \exp\left[-\exp\left(-\alpha_3(x_3-m_3)\right)\right] \tag{2}$$

where ω is the upper limit of the pay load, m_2, m_3 and α_2, α_3 are strongly related to the extreme stress distribution (1),(2).

Introducing the concept of the return period $T(X)$, which can be expressed by the distribution function:

$$T(X) = \left[1-\phi(X)\right]^{-1} \tag{3}$$

$T(X)$ indicates the average number of Δt intervals needed to exceed the extreme stress level X.

Putting Eq(2) into Eq(3) we can get three equations for the return period of the three essential stress components

$$T_1(X_1) = \left[1-\phi_1(X_1)\right]^{-1};\ T_2(X_2) = \left[1-\exp\left\{-\left(\frac{\omega - x_2}{\omega - m_2}\right)^{\alpha_2}\right\}\right]^{-1}$$

$$T_3(X_3) = \left[1-\exp\left\{-\exp\left(-\alpha_3(x_3-m_3)\right)\right\}\right] \tag{4}$$

Not going into the detailed mathematics (using some assumptions, rearrangements and derivations) we can express from Eq(4) the extreme peak stresses of the three stress components:

$$X_1 = \text{const.};\ X_2 = \omega - (\omega-m_2)T_2^{-1/\alpha_2};\quad X_3 = \left(m_3 + \frac{2.31}{\alpha_3}\right) + \frac{2.31}{\alpha_3}\lg T_3$$

These equations show how long the time T_i is needed (measured in the numbers of Δt intervals) to exceed a certain extreme peak stress X_i, or in other words, what is the return period of a certain extreme peak stress? It is easy to understand that the function of the stresses from the manufacturing process is constant, independent from time, and the third function (stresses from the operation) is a simple semi-logarithmic rule, which represents a straight line in a semi-logarithmic coordinate system.

FATIGUE PROCESS

The fatigue of the structural elements is a stochastic, time dependent process. There are no sharp borders, theoretical limits in this process, but the engineering practice generally divides it into three phases:
- the crack initiation period
- the stable crack propagation
- the final fracture, or in other words the unstable crack propagation.

Some major questions of these periods will be mentioned in the following.

The Crack Initiation. Discussing the life problems and the fatigue process of a structural element the first step is to understand and describe the crack initiation period and to know, how to calculate the incubation period, that is the service time needed for initiation of the crack. Altogether to solve the problem of crack initiation at least the following main problems have to be cleared (4)
- the definition of a crack
- the physical and mechanical model of crack initiation
- the effect of the extreme stresses on the crack initiation
- the calculation of incubation period.

Only one question, the definition of a crack will be discussed here. It is an essential question: what size of a material discontinuity could be already considered as a crack?

One possible definition of the crack - which is based on the most important effect of the crack - is given in the following: in the case of the structural part, that material discontinuity, developed on the effect of stresses arising from the manufacturing process and/or service, could be called crack, which already causes measurable reduction in the load bearing capacity (service strength) of the structural element. This lowest limit crack in the following will be called a critical crack.

Considering the main characteristics of this new crack definition, the following can be fixed:

a) this definition is based on the strength behaviour of the structural element, on the most important structural effect of the crack, namely, it decreases the service strength, the load bearing capacity;
b) the critical crack does not have a definitive, absolute size, its size depends on the size and geometry of the structural element itself, as well as on the loading conditions;
c) a lot of three dimensional material discontinuity, like: inclusions, shrinkage cavities, casting discontinuities, gas cavities, etc. can not be called and treated as cracks.

<u>The Crack Propagation due to Fatigue</u>. The crack propagation equations found in literature are based on the Paris equation, considered as a classical benchmark

$$\frac{dl}{dn} = A(\Delta K)^n = B\Delta\sigma_\infty^n l^{n/2} \tag{6}$$

which has recently been further developed by Forman:

$$\frac{dl}{dn} = \frac{A(\Delta K)^n}{(1-r)K_c - \Delta K} \tag{7}$$

where dl/dn is the crack propagation per cycle, ΔK is the stress intensity variation per cycle at the crack tip, $\Delta\sigma_\infty$ the stress variation at a sufficient distance from the crack, r indicates the asymmetry coefficient of the stress cycle, A,B and n are material constants, while K_c is the critical stress intensity factor. These relationships are based on many assumptions which are not satisfied in reality, but the most difficult problem is the assumption of an elastic material law.

In the case of a stationary random stress history the traditional stress cycle does not have meaning (5). Therefore it seems to be better to define the crack propagation as the function of time. In the fracture process of machine parts the current dimension (size) of the crack plays more important role than the momentary rate of propagation. If ψ denotes the characteristic crack parameter, then the general crack propagation function could be expediently approximated

$$\psi(t) = \psi_k f\left(\frac{t}{t_k}\right) \qquad \text{if } t > t_k \tag{8}$$

where ψ_k is the parameter of the critical crack, or in other words: it is the critical crack, while t_k is incubation period, needed to the formation of the critical crack size $f(\ldots)$ is an appropriate function. Eq(8) is valid only in the case of stationary stress history. The crack propagation is a stochastic process itself, therefore Eq(9) gives only the trend, the mean value of the crack size, which can only be described by a probability distribution at all times. The standard deviation (scatter) of the crack size may be given on the basis of the probabilistic theory of "scatter expansion". Assuming that the parameters in Eq(8) are independent by pairs, and taking into consideration only the first derivations, generally can be written:

$$S_\psi^2 = \left(\frac{\partial\psi}{\partial\psi_k}\right)^2 S_{\psi_k}^2 + \left(\frac{\partial\psi}{\partial t_k}\right)^2 S_{tk}^2 + \sum_i \left(\frac{\partial\psi}{\partial c_i}\right)^2 S_{ci}^2 \tag{9}$$

where C_i denotes further parameters in the function $f(\ldots)$ and $S_{\psi_k}^2$, S_{tk}^2, S_{ci}^2 are the standard deviation of the parameters being marked in the index.

<u>Residual Strength of Cracked Structural Elements, Unstable Crack Propagation</u>. As the result of the complex service stress history the fatigue crack initiates and propagates in the critical cross section, decreasing the original, initial strength (load bearing capacity) of the structural element. The final fracture occurs when the reduced strength cannot resist the momentary extreme stress, which occurs accidentally during the service. We have to know the residual strength of a cracked element, or in other words, the stress which is enough to propagate the crack rapidly, to cause the final unstable propagation. One of the well known equations is the Irwin's formula, which is regarded as the basic equation of the fracture mechanics. This theory has a lot of disadvantages which are not realized in technical practice, e.g. the theory is based on sheet specimens, basically elastic material properties, energy criteria of fracture and finally the Irwin's equation does not contain the original (not cracked) load bearing capacity of the element, as a parameter.

The essence of the final fracture can be given as the answer to the question: what are the conditions - geometrical, material and load conditions - of the rapid, unstable crack propagation, when this process becomes a self supporting one, it works without increasing the outer load, or σ_∞?

In the case of an arbitrary, cracked structural element, having an arbitrary load system, the critical stress $\sigma_{\infty cr}$, which causes the unstable, final crack propagation - and which means the residual strength of the cracked element - can be approached by the following type of function:

$$\sigma_{\infty cr} \equiv R(\psi) = R_c h\left(\frac{\psi}{\varphi_o}\right) \tag{10}$$

where φ_o is the geometrical parameter of the original, uncracked cross section being related to the crack (e.g. the width B of the cracked plate), R_c is the critical load bearing capacity of the element without any crack, and $h(\ldots)$ is an appropriate function, which has to meet the following requirements:

- if the crack is smaller or equal to the critical crack size, then it does not cause strength reduction. That means:

$$R(\psi_k) = R_c \tag{11}$$

From Eqs.(10) and (11) one can express the mathematical formula of the critical crack size:

$$\psi_k = \varphi_o H(1) \tag{12}$$

where H(1) is the inverse function of h at the value of 1.

- the residual strength has to be zero if the cross section is completely cracked:

$$R(\varphi_o) = 0 \tag{13}$$

- increasing of the crack with $d\varphi$ causes bigger reduction in the residual strength if the crack is smaller:

$$dR(\psi) = g^{-1}(\psi)d\psi \tag{14}$$

where $g(\psi)$ is a monotonously increasing function of ψ. Differentiating Eq(10) and comparing it to Eq(15) one can write

$$g(\psi) = \frac{\varphi_o}{R_c}\left[h'\left(\frac{\psi}{\varphi_o}\right)\right]^{-1} \tag{15}$$

The residual strength of a cracked element is also a random variable, and it can be only described by statistical methods, that means only by distribution function.

<u>The Service Strength of Structural Elements</u>

The service strength of an element means its actual, momentary load bearing capacity during the service. The aim of the engineers and manufacturers is to assure the calculated (initial) load bearing capacity during the total service life, but according to the undesired fatigue crack initiation and propagation this strength will be reduced. Knowing the general form of the residual strength - Eq(10) - one gets for the service strength:

$$R(t) = R_c h\left[\frac{\psi_k}{\varphi_o} f\left(\frac{t}{t_k}\right)\right] \cong R_c[1-K(t)] \tag{16}$$

where K(t) is the function of the mechanical damage. The service strength depending on lot of random effects is also a stochastic process.

Let us examine - assuming the initiation of the crack - how the service strength decreases during a small interval dt? If the service strength is R(t) at the time t, then after the interval dt:

$$R(t+dt) = R(t) - dR \tag{17}$$

or the relative reduction:

$$\frac{R(t+dt)}{R(t)} = 1 - \frac{dR}{R(t)} \tag{18}$$

Taking into consideration Eq(16) and by differentiation, on gets:

$$\frac{R(t+dt)}{R(t)} = 1 - \frac{\left[\frac{\psi_k}{\varphi_o t_k}\right]\frac{dh}{df} f\left(\frac{t}{t_k}\right)dt}{h\left[\frac{\psi_k}{\varphi_o} f\left(\frac{t}{t_k}\right)\right]} \tag{19}$$

The detailed study of this equation shows that the second part of the right side is smaller with more orders comparing to the unit, and this is the theoretical base of the assumption which was used for defining the extreme stresses: if a certain extreme stress does not cause fracture in a given time t, then the other stresses, being smaller than the extreme one have no probability to cause fracture in the next interval Δt.

FRACTURE, SERVICE LIFE DISTRIBUTION

The up-to-date life calculation is based on the theory of stochastic process. Two processes discussed before have to be analyzed:

- the stochastic process of extreme stresses, or in other words the time dependent function of the extreme peak stresses.
- the fatigue process or in other words the time dependent function of the service strength.

One of the most important things in life calculation is the definition of the fracture.

Definition of Final Fracture. The final fracture of a vehicle structural element occurs, when a momentary extreme stress exceeds the momentary service strength:

$$R(t) \leq X(t) \tag{20}$$

Figure 4 shows that the new explanation of the fracture differs from the classical interpretation in two aspects:

- the final fracture is not a static critical condition but the end of a time process
- the final fracture is not a deterministic critical condition, but a stochastic process, the combination of probabilistic fields.

It is interesting to note that the fracture generally is caused by a relative overload (overstress) and not an absolute one. The structural element had withstood many times higher extreme stresses during its service life then the final one which caused the fracture, but its service strength was higher (the mechanical damage was lower) in the earlier stage.

Service Life Distribution Function. On the basis of Fig.4, the life distribution of a structural element F(T) can be derived from its service strength distribution F[R(t)] and the distribution of the extreme peak stresses in its dangerous cross section $F_2[X(t)]$. In the general cases we can get very complicated expressions for life distribution, but in the case of a tensiled plate element we get the following expression for the mean value of the service life (6):

$$T = T_k^{\varepsilon}\, \omega \tag{21}$$

where ω - reliability factor, ε - fatigue coefficient.

The logarithm of the standard deviation can be determined from the triangle 1-2-3 in Fig.5:

$$S_{\ln T} = (S_R + S_X)(\nu R_c)^{-1} \tag{22}$$

It can be established from this equation that the scatter of service life depends on the original strength of the structural element, on the scatter of its service strength and that of the extreme stresses which were assumed to be constant in time, and finally on the fracture sensitivity of the element.

Interpretation of the Safety Factor. In engineering practice the safety factor used to be the ratio of the strength and stress. Maintaining the assumptions given before, the expected value of the safety factor may be written as follows:

$$h(t) = R(t)X^{-1}(t) = n_o N(t) \tag{23}$$

where $N(t)$ is an appropriate time function. Taking into consideration Eq(20) and Eq(4) with the conditions belonging to them, one can get from Eq(18) the following formula:

$$n(t) = n_o\left[\frac{1 - K(t)}{1 + v(t)}\right] \tag{24}$$

This interpretation of safety factor differs from the traditional concept in two respects:

- the safety factor is also time dependent being a monotonously decreasing function of the time. Therefore it is not sufficient to prove that the safety factor is greater than a required value at the whole service period.
- the safety factor - being the ratio of two random variables - is a stochastic parameter, which could be characterized by its distribution function.

The Fracture Probability and Sensitivity. The final fracture occurs when the condition $n(t)<1$ is fulfilled, namely the probability that the safety factor is less than one, is the probability of fracture P in the same time. The fracture probability P may be defined by the distribution function of the safety factor $Q(n)$:

$$P = \dot{Q}(n{=}1) = \int_o^1 g(n)dn \tag{25}$$

where $g(n)$ is the density function of the safety factor. Because the safety factor is time dependent, the fracture probability is a monotonously increasing function of time. The density function of the safety factor $g(n)$ may be derived from the functions and parameters used before, so we can generally write:

$$P = P(R_c, t_k, S_R, S_X) \tag{26}$$

Figure 6 shows the probability of fracture in the function of initial safety factor n_o, while the parameters are S_R and S_X. The reliability of a structural element is characterized by the reliability factor ω, showing the the time ratio between the crack initiation and the final crack. Figure 7 shows the reliability factor as a function of the initial safety factor n_o. The several curves have different value of fracture sensitivity ν. It is interesting to recognize the well known engineering experience: when the structure has a high fracture sensitivity, it is useless to increase the value of the initial safety factor, or in other words it is no solution to increase the initial strength using a better material, or increasing the thickness, the fracture could not be avoided, the life does not increase. On the other hand, if ν has a small value, a very small increasing in the thickness results in an order of magnitude increase of the reliability. On the basis of the fracture sensitivity, the structures could be derived into three groups:

- If $\nu > 1$, the structure is unreliable, the fracture sensitivity is high. Essential modifications are needed to get a structure suitable for the service.
- If $1 > \nu > 0.1$, the structure has a medium reliability, generally not enough for the service, some local modifications could solve the problem.
- If $\nu < 0.1$, the structure is reliable, unsensitive to the fracture, the required service life could be guaranteed by the choice of the appropriate cross section area.

REFERENCES

(1) Matolcsy,M., Shet,N.J. Determination of extreme structural loads. SAE Paper (1987), No.780107, p.394-404

(2) Matolcsy,M., Stress concentration in the joints of bus frames. XIX Congress of FISITA, Wien (1986),Paper No.845121,p.4101-4109

(3) Matolcsy,M., Analysis of stresses caused by useful load in vehicle framework structures. Acta Technica Academiee Scient Hungaricae (1977) Tom 84(3-4) p.379-399

(4) Matolcsy,M., General fatigue problems of stochastically loaded (vehicle) structures. Materialprüfung, 18 (1976) No.4 p.115-122

(5) Matolcsy,M., Berke,P. The problems of fatigue crack propagation and residual strength in the case of shaft or axle type components. Acta Techn.Acad.Scient.Hung.(1977) Tom.84(3-4) p.251-267

(6) Matolcsy,M., Molnar,Cs. Fatigue life distribution of vehicle frame structures. Proc. of Vehicle Structures Conf. Craanfield (1984) IMechE No.C169/84 p.121-130

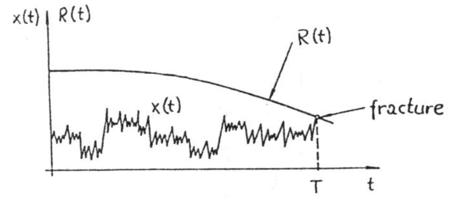

Fig. 1

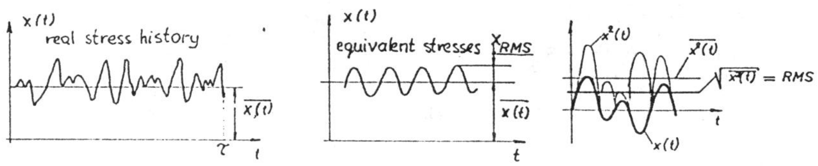

Fig. 2

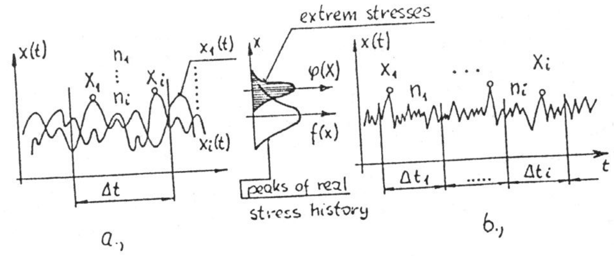

Fig. 3

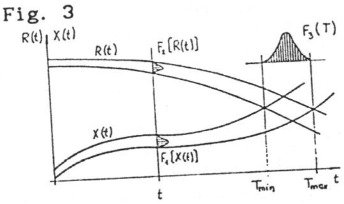

Fig. 4

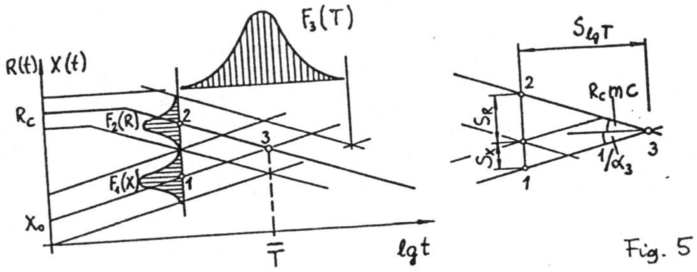

Fig. 5

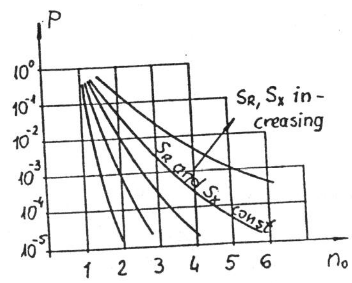

Fig. 6

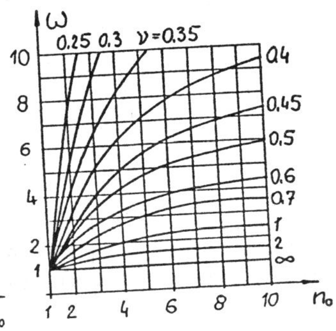

Fig. 7