PROBABILISTIC MODELS OF FATIGUE DESIGN

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As long as the normal stress range of the cross section is investigated it is sufficient in fatigue crack to suppose small deflections. If the cracking of web loaded in shear (or loaded to the combination of the tension and shear) is researched it is necessary to consider large web deflection, e.g., to solve the Föppl-Kármán-Marguerre non--linear partial differential equations. The larger depth-to-thickness ratio of web, the smaller the number of cycles of the final fatigue life.

INTRODUCTION

We shall consider the randomization of the Paris-Erdogan equation and with the properties of stochastic process solutions. Fatigue crack growth under constant amplitude cyclic loading can be related to the stress intensity factor, ΔK, through the equation

$$\frac{da}{dN} = C(\Delta K)^n \tag{1}$$

where $a = a(N)$ is the crack length at time N, C is an experimentally determined function and n is an experimentally determined constant. Clearly, that n and C are not negative. Any randomization of the P.E equation is to be based upon a randomization of three parameters.

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THE RANDOMIZED PARIS-ERDOGAN EQUATION

We shall consider in detail the randomization of the function C.

Let us have

$$D = 1/(C(n/2 - 1)(f\,\Delta\sigma\sqrt{\pi})^n \,.\, a_o^{n/2-1}) \qquad (2)$$

where $\Delta\sigma$ is either a normal or modified shear stress range, f is a calibrating function and a_o an initial crack length.

The main as well as the variance of the cycle number N (a) can be obtained by

$$E\,\{N\,(a)\} = E\,(D\,(1 - (\frac{a_o}{a})^{n/2-1}) \qquad (3)$$

$$Var\,N\,(a) = Var\,D\,(1 - 0{,}9\,(\frac{a_o}{a})^{n/2-1})^2 \qquad (4)$$

The normal distribution of probability is used.

Let us have zinc alloy (5.5% Zn, $\sigma_{0.1}$ = 495 MPa), n = 3.7 (Hellan (1), pp. 196). If the crack length a = 50 mm and initial crack length a_o = 9 mm the mean and the variance of cycle number

$$E\,\{\,N\,(a)\} = 1143\,(1 - (\frac{a_o}{a})^{n/2-1}) . \frac{10^9}{(f.\Delta\sigma\sqrt{\pi})^{3.7}} \qquad (5)$$

$$Var\,(a) = 53361\,(1 - 0.9\,(\frac{a_o}{a})^{n/2-1})^2 . \frac{10^9}{(f.\Delta\sigma\sqrt{\pi})^{3.7}} \qquad (6)$$

The dependence of the cycle number and of crack length for the mean and 0.1% fractil of random number of cycles for zinc alloy are on Fig. 1.

Fractil of random number of cycles for probability P = = 0.001 and standard fractil $t^{(P)}$ = 3 will be as follow

$$N^{(0.001)} = (877 - \sqrt{33343.15}) . \frac{10^9}{(f\Delta\sigma\sqrt{\pi})^{3.7}} \qquad (7)$$

$$= 329.19\,\frac{10^9}{(f\Delta\sigma\sqrt{\pi})^{3.7}}$$

The solution for zinc alloy (5.5% Zn) agrees with empirical distribution functions of Kozin and Bogdanoff (2), pp. 61.

CRACKING IN LARGE WEB DEFLECTION

Combination of the tension and shear

Let us have, e.g., cross beam or a stringer of a railway bridge, loaded in shear.

Studying the combination of the Type I and II (the tension and shear of the crack propagation), let us have

$$K_I/K_{IC} + \frac{3}{2}\,(K_{II}/K_{IC})^2 = 1 \tag{8}$$

where K_{IC} the fracture toughness.

The stresses taking part in the crack oppening are as follows

$$\sigma_y = \sigma\,(\eta \sin^2\beta + \cos^2\beta) \tag{9}$$

$$\tau_{xy} = \sigma\,(1 - \eta)\,\sin\beta\,\cos\beta \tag{10}$$

For steel webs is sufficient to suppose $\eta = \sigma_2/\sigma_1 = 0$. Then

$$K_I = \sigma\sqrt{\pi a}\,.\,\cos^2\beta\,.\,f(a) \tag{11}$$

$$K_{II} = \sigma\sqrt{\pi a}\,.\,\sin\beta\,\cos\beta\,.\,f(a)$$

The critical stress for $\eta = 0$ can be found in the form

$$\sigma_c = \frac{K_{IC}}{\sqrt{\pi a}}\;\frac{-1 + \sqrt{1+6\mathrm{tg}^2\beta}}{3\sin^2\beta\,.\,f(a_c)} \tag{12}$$

As it follows from Föppl-Kármán-Marguerre theory the stresses σ_y and τ_{xy} are not to be constant in the web area. For a given material the stresses are the functions of the width-to-thickness ratio of web and depend on the critical web stress. E.g., for the quadratic web simply supported the critical stress in shear $\tau_{cr} = 9.32\,\sigma_E$, where $\sigma_E = \pi^2Et^2/12(1 - \nu)b^2$ and b, t is width and thickness of the web sheet.

For the solution of large web deflection the Papkovich method is used.

For shear the coefficients w_{mn} are in sum even/m+n is even/, e.g. w_{11}, w_{13}, w_{31}, ... or odd. e.g. w_{12}, w_{32}, w_{23}, ...

Some numerical results have been published (3).

Example :

The transverse crack has been discovered in the web of the cross beam of railway bridge. The girder operated in maximum shear stress $\bar{\tau}$ = 40 MPa. The number of cycles of fatigue life and full length of the crack is to be determined for cyclical fatigue loading.

The fracture toughness is K_{IC} = 55 $MPa.m^{1/2}$. The crack growth exponent for structural steels, n = 3, and crack growth function for probability P = 0.001 was found to be C = 68.9 . $10^{-13} MPa^{-3}.m^{-1/2}$.

We apply the large web deflection. The stresses σ_y and τ_{xy} express the membrane plus bending stresses of web.

Let us have $\lambda = \bar{\tau}/\sigma_E$ = 18.64. The eigenvalue λ is for quadratic simply supported web with restained edges.

After some iterations we determine the critical length of crack a_c = 0.1921 and the callibrating function $f(a_c)$ = 0.85. Final number of cycles N = 5663 . 10^6.

If we repeat the solution for the eigenvalue $\lambda = \bar{\tau}/\sigma_E$ = 9.32 or for the depth-to-thickness ratio of web b/t = 180, we get final number of cycles N = 8.183 .10^6.

For depth-to-thickness ratio of web b/t = 300 the final number of cycles N = 5563 . 10^6. The example is given for loading in shear $\bar{\tau}$ = 40 MPa.

Fig. 2 shows membrane plus bending stresses of quadratic web loaded in shear. Simply supported web with restrained edges is supposed for $\tau/\sigma \geq 18$, and elastic constraint of web for $\tau/\sigma_E < 18$ is compared. The degree of elastic constraint is increasing if the depth-to--thickness ratio is sinking.

REFERENCES

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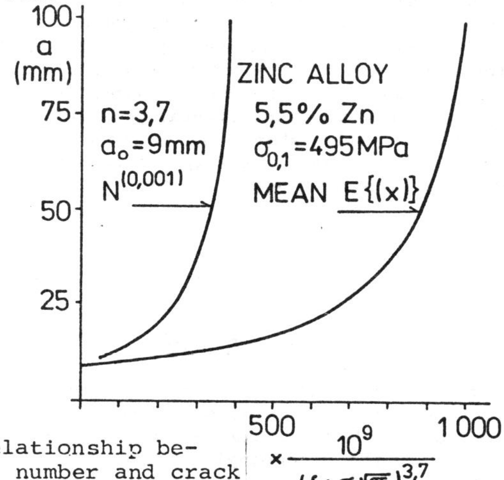

Figure 1 Relationship between cycles number and crack length

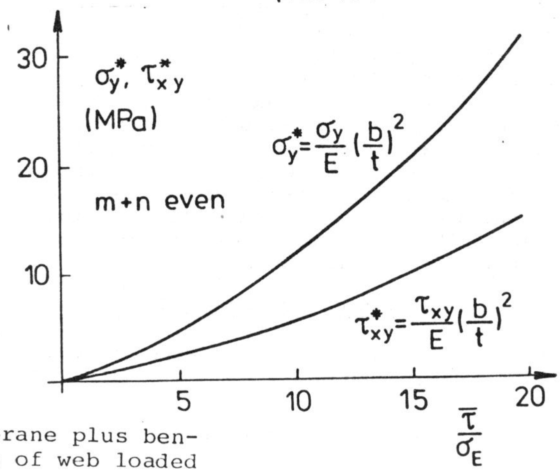

Figure 2 Membrane plus bending stresses of web loaded in shear