PLASTIC DAMAGE IN ROUND NOTCHED TENSILE BAR USING CONTINUUM DAMAGE MECHANICS

Bonora N.[1], Marchetti M.[1] and Milella P.P.[2]

Damage due to plastic deformation in metal that exhibit ductile behaviours was investigated by many authors using different approaches. The continuum damage approach proposed first by Lamaitre and Chaboche was here used to develop a post process routine for damage evaluation. A simple but accurate resolution scheme suggested by Owen was modified and used to predict damage accumulation process in a notched tensile bars specimen. Results were compared with traditional cavity growth measurement performed by the authors.

INTRODUCTION

In the last thirty years ductile fracture mechanisms have been the subject of investigation of many researchers. The former basic investigation on this subject was developed by McClintock [1]. He found that ductile fracture is intrinsically related to the void growth and the void growth rate is a function of the plastic strain rate, of the triaxiality ratio and of the hardening coefficient of the material. Later, using variational approach to the void growth into a material that exhibit ductile behaviour, Rice and Tracy [2] found a result really close to that experienced by McClintock. They found that the cavity growth rate is dependent only on the triaxiality ratio and on the plastic strain rate. Rice and Tracy first postulated that ductile fracture occurs as soon as a critical value of the cavity growth is reached. That value is characteristic of the material under investigation and is really difficult to be determined theoretically. Cavity growth criterion is really easy to be applied but unlucky does not give accurate results for high stress gradients or when void interactions can't be neglected [3-5]. To take into account of all the complex phenomena that occurs at microscopic level different models, that go under the common name of Continuum Damage Mechanics, were developed. Pineau, Gurson, Rousselier et al [6-8] re-defined the constitutive law of a porous material that exhibit ductile behaviours introducing the effect of the volume fraction of voids on the yielding surface. Lamaitre and Chaboche [9-11], in the framework of the damage mechanics proposed a different kind of approaches followed also by other authors [12-13]. Here, damage mechanisms are localized into a micro scale that does not interact with the macro scale up to all the micro volume is damaged and a macro crack appears. That kind of hypotheses permit to formulate the interaction between damage and plasticity using standard continuum mechanics approach. Different kind of coupling levels could be developed. In all these models, some material constants appear and have to be determined by experimental

[1] Aerospace Department, Univ. Of Rome "La Sapienza", via Eudossiana 18-00184 Rome, Italy.

[2] ENEA-DISP, via Vitaliano Brancati 48, Rome, Italy

measurements. One of the most common specimen geometry used for these purposes is the round notched tensile bar (RNTB). In fact this geometry permits to obtain different triaxiality ratios and it is easy to be study using finite element in axial-symmetric formulation. The characterization of these kind of specimen using the Lamaitre model was made. A simple post processor procedure to solve the coupled damage-plasticity equations is proposed. The results are also compared with the classic Rice and Tracy model results and with some experimental data performed by the authors in previous works [14-15].

MODEL

Damage is always defined as a progressive deterioration of the material, physically it is related to micro-cracking, micro defects, void growth and dislocation pile-ups. As consequence of these observation a main hypothesis can be made: the damage volume is at least of an order of magnitude smaller of the usual representative volume element used in the continuum mechanics. As a consequence of this hypothesis it is possible to affirm that:

A) The influence of the micro scale phenomena could be neglected at the macro scale up to all the micro volume is damaged and a macro crack nucleates.

B) Strain Equivalence. The total strain field is the same at micro and macro scale. The stress in the micro scale is replaced by the effective stress.

C) Knowing the strain history the coupling between damage and strain at micro-level can be solved.

Damage is defined as the ratio between the initial section, S_0, of a given volume element and the effective resistant section, S_d that take into account of the net area, the interaction between the micro defects, etc.:

$$D = 1 - \frac{S_d}{S_0}$$

Then the effective stress into the micro element is defined as:

$$\tilde{\sigma}_{ij} = \frac{\sigma_{ij}}{1 - D}$$

The strain equivalence hypothesis allows to use the same constitutive equations used in the continuum mechanics where the stress is replaced with the effective stress and an additional equation for the damage is also added. The yielding function for a Von Mises material is given by:

$$f = \frac{\sigma_{ij}}{1 - D} - R - k$$

where R is the hardening stress and k is the elastic limit under which no damage occurs (that is usually given by the limit fatigue stress.). The complete set of constitutive equations is given by:

$$\dot{\varepsilon}^T_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^p_{ij} \quad \dot{\varepsilon}^{el}_{ij} = \frac{(1+\nu)\dot{\sigma}_{ij}}{E(1-D)} - \frac{\nu}{E}\frac{\dot{\sigma}_{kk}}{(1-D)}\delta_{ij} \quad \dot{\varepsilon}^p_{ij} = \dot{\lambda}\frac{\partial f}{\partial \tilde{\sigma}_{ij}} = \frac{3}{2}\frac{\dot{\lambda}}{(1-D)}\frac{s_{ij}}{\sigma_{eq}}$$

where λ is the plastic multiplier. The hardening stress is given in the Ramberg-Osgood form:

$$R = k_y \cdot \varepsilon^{\frac{1}{M_y}}_{eq} \qquad \dot{R} = \frac{dR}{d\varepsilon_{eq}} \cdot \frac{d\varepsilon_{eq}}{dt} = R'(p)\dot{p} \qquad \dot{p} = \sqrt{\frac{2}{3} \cdot \dot{\varepsilon}^p_{ij} \cdot \dot{\varepsilon}^p_{ij}}$$

K_y and M_y are the Ramberg Osgood material constant. The damage constitutive law is given by:

$$\dot{D} = -\dot{\lambda}\frac{\partial \dot{\varphi}}{\partial Y} = -\frac{\partial \dot{\varphi}}{\partial Y}\dot{p}(1-D) = -\frac{D_{cr}}{(\varepsilon_{th} - \varepsilon_r)(1-D)}\left[\frac{2}{3}(1+\nu) - 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right]$$

where D_{cr}, ϵ_{th}, ϵ_r are material constants Y is the strain energy release rate and

$$\sigma_H = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) \qquad s_{ij} = \sigma_{ij} - \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})\delta_{ij} \qquad \sigma_{eq} = \sqrt{\frac{3}{2}s_{ij}s_{ij}}$$

Finally the expression of the plastic multiplier is given by:

$$\lambda = \frac{3}{2\sigma_{eq}}\frac{s_{ij}\dot{\sigma}_{ij}}{(1-D)R'(p) + [k + R(p)]\partial\dot{\varphi}/\partial Y}$$

In that formulation the damage and plasticity are fully coupled even if the dissipation effects in the full potential are disengaged.

RESOLUTION SCHEME

During the solution of the equations showed in the previous section, the principal constraint that has to be verified is that at the end of each increment the material point has to belong to the yielding surface. That require a proper correction on the stress tensor components that means a precise direction of the plastic flow. For this purpose the correction scheme proposed by Owen [16] for the elasto-plastic problem in finite element method, was here modified to take into account of the coupling between damage and plasticity. Underscored letters are used for vector and double underscored letters are used for a tensor quantities. First of all the plastic multiplier is re-formulated in terms of total strain rate that is also the given input in the problem under investigation. The normality rule gives:

$$df = \frac{\partial f}{\partial \sigma_{ij}}d\sigma_{ij} + \frac{\partial f}{\partial K}dK + \frac{\partial f}{\partial D}dD = 0$$

that can be rewritten as:

$$a^T d\underline{\sigma} - B d\lambda = 0$$

where $\mathbf{a}^T$ and B are

$$a^T = \left\{ \frac{\partial f}{\partial \underline{\sigma}} \right\} \qquad b = -\frac{1}{d\lambda} \left[\frac{\partial f}{\partial K} dK + \frac{\partial f}{\partial D} dD \right]$$

taking into account of the expressions given in the previous section we obtain

$$B = \left[H' + \Xi \cdot (\sigma_y + H' \varepsilon^p_{eq}) \frac{\partial \dot{\phi}}{\partial Y} \right]$$

where H' is the local slope of the hardening curve and Ξ is equal to zero if the equivalent plastic strain is lower then a threshold value under which damage is zero, otherwise is equal to one. The total strain equation can be rewrite as

$$d\underline{\varepsilon}^{tot} = d\underline{\varepsilon}^{el} + d\underline{\varepsilon}^{pl} = \underline{\underline{D}}^{-1} d\underline{\sigma} + d\lambda a$$

where D is the elastic tensor modified to take into account damage.
Pre-multiplying by $\mathbf{a}^T\underline{\underline{D}}$ and eliminating the term $\mathbf{a}^T d\sigma$ we have for the plastic multiplier:

$$d\lambda = \frac{1}{[B + a^T \underline{\underline{D}} a]} \underline{\underline{D}} a \, d\underline{\varepsilon}^{tot}$$

The principal steps into the resolution scheme can be given in this way:
- At the beginning of each step the rate of stress is computed assuming elastic behaviours.
- For each Gauss point the total stress are accumulated according to

$$\underline{\sigma} = \underline{\sigma}^{(r-1)} + d\underline{\sigma}^{(r)}_{el}$$

where r is the iterate number.
- Check if the Gauss point had yielded during the (r-1)th iterate.
  *If the Gauss point has not yielded then check if $\sigma^{(r)}_{eqel} > \sigma_y$
    ** if NO: the Gauss point is still elastic updated and go to the next step.
    ** if YES: the Gauss point has yielded during the current increment. The stress portion greater then the yield stress has to reduced on the yield surface. The reduction factor $\Re$ is given from Figure 1 as AB/AC$=(\sigma^{el(r)}_{eq}-\sigma_y)/(\sigma^{el(r)}_{eq}-\sigma^{(r-1)})$
  * If the Gauss point has already yielded then check if $\sigma^{(r)}_{eqel} > \sigma^{(r-1)}_{eq}$
    ** if NO: the point is under elastic unloading.
    ** if YES: all the excess of stress has to be reduced on the yielding surface and, as in Figure 2, $\Re=1$.
- For the yielded Gauss point the portion $\Re d\underline{\sigma}'$ can be reduced using:

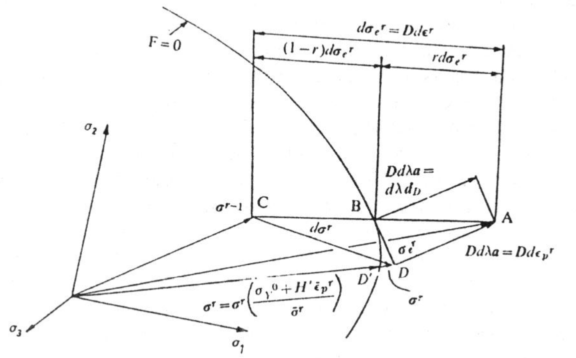

Figure 1 Incremental stress changes at a point in an elasto-plastic continuum at initial yield.

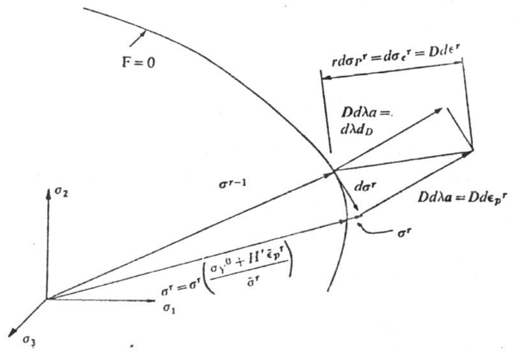

Figure 2 Incremental stress changes in an already yielded point in an elasto-plastic continuum.

That produces an increasing of the plastic flow and of the damage. Relaxing the excess of stress in several stages the accuracy of the solution is improved. More details on this kind of process or on different kind of scheme to the same problem are discussed in [17, 18].

- At the end of each step the stresses, strain and damage are updated.

NUMERICAL RESULTS

The damage accumulation process in a round notched specimen was evaluated and results were compared with the cavity growth calculation performed by the authors [14, 15] in a previous works. The geometry of the specimen under investigation is depicted in Figure 3. Only a quarter of the specimen, using a notch radius of 2 mm, was simulated in axial-symmetric formulation. Loading was applied imposing a displacement on the remote section. Damage calculation shows that fracture starts at the center of the specimen were even if the plastic strain is not too much, if compared with the element at the tip that reaches the 45% of plastic deformation in correspondence of the fracture load, the3 1, 4re 4 shows the trend of the triaxiality factor along the minimum section at different load steps. In that figure there is clear evidence that the triaxiality factor is almost constant during the loading phase but it has the highest values at the center of the specimen. Damage accumulation history is shown in Figure 5 where the damage values along the minimum section are plotted for different load steps. The highest curve is the damage values in correspondence of the fracture load. The damage trend is anyway very similar to the cavity growth trend and this kind of comparison is shown in Figure 6 where the value of the cavity growth, in correspondence of the fracture load, is normalized by a factor of 9.5 so to have the same scale. Damage and cavity growth shows the same trend for the elements close to the symmetry axis. Some differences appears in the element close to the notch where the cavity growth criterion predicts a more severe damage state. Anyway continuum damage mechanics permits to evaluate also the loss of capability of carrying load of the damaged elements. In Figure 7 the macroscopic axial stress is plotted versus the total strain in correspondence of the Gauss point closest to the axial axis for the central element.

CONCLUSIONS

A very simple resolution scheme for the damage equations directly based on the total strain formulation proposed by Owen was used. This kind of resolution scheme allows to obtain with a very short time calculation damage measurement from any strain history previously obtained by finite element analysis. Damage measurements using continuum damage mechanics were compared with traditional damage model as cavity growth criterion proposed by Rice and Tracy. Both model are in agreement with the experimental data but continuum damage mechanics gives much more information on the phenomena during accumulation process. Using the same scheme an approach to damage, fully coupled with finite element technique, could be developed.

REFERENCES

[1] McClintock,F.A., A criterion for ductile fracture by the growth of holes, J. App. Mech., 1968 vo1.35, pp. 363-371.
[2] Rice J.R. And Tracy D.M., On ductile enlargement of voids in triaxial stress fields.,J. Mech. Phys. Solids 1969, vol. 17.pp 2O1-217.
[3] J.Koplick and A. Needleman, Void growth in porous plastic solids, Int. J. Solid Structures, 1988 vol 24, p.835-853.
[4] Needleman A., Void growth in elastic-plastic medium J. Appl. Mech., 1972 vol 39, pp. 964.
[5] Tvergaard V., Material failure by void coalescence in localized shear bands. Int J. Sol. Struc. 1982, vol. 18.

[6] Pineau A. And Amar E., Application of a local approach to ductile-brittle transition in a low alloyed steel,1987 Nucl. Eng. and Design vol 105 pp. 89-96
[7) Rousselier G. Ductile fracture models and their potential in local approach of fracture, 1987 Nucl. Eng. and Design vol. 105 pp 97-111.
[8] Gurson A. L. Continuum theory of ductile rupture by void nucleation and growth: Part I., 1977 J. Eng. Mater. Technol. 99,2.
[9] Lemaitre J., Micro mechanics of crack initiation, 1990 Int J. of Fract. vol 42 pp 97-99
[10] Chaboche J. L. And P. M. Lesne, A non continuous fatigue damage model, 1988 Fat. Fract Eng. Mater. Struct. vol. 11 pp. 1-17.
[11] Lamaitre J. How to use damage mechanics, 1984 Nucl. Eng. and Design vol. 80 pp.233-245.
[12] J. W. Ju, On energy based coupled elastoplastic damage theories: constitutive modeling and computational aspects 1989 Int. Jour. Solid. Struct. vol.25 pp 803-833.
[13] J. C. Simo and J.W. Ju, Strain and stress based continuum damage models. Int J. Solid Str. 1987 vol 23 pp 821-840.
[14] Bonora N., Marchetti M., Milella P.P, Maricchiolo C. and Pini A., Local approach criteria: an application of cavity growth theory to ductile fracture in low alloy steel, AMD-vol. 137, presented at the ASME summer meeting 1992 Phoenix, Arizona.
[15] Bonora N., Marchetti M., Milella P.P, Maricchiolo C. and Pini A., Ductile fracture criteria: an application of local approach based on cavity growth theory, presenter at the 92 PVP conference, New Orleans 1992.
[16] Owen D. R and Hinton E., Finite elements in plasticity: theory and application, Pineridge Press Limited Swansea. U.K.1980.
[17) Krieg R. D. and Krieg D. B., Accuracies of numerical solution methods for elasto perfectly plastic mode Trans. ASME J. of Pres. Vess. Tech. 99 pp 510-515 1977.
[18] Schereyer H.L., Kulak R. F., Kramer L. M. Accurate numerical solution for elastic plastic models, Trans. ASME J. Press Vess. Tech. 101, pp. 226-234 (1979)

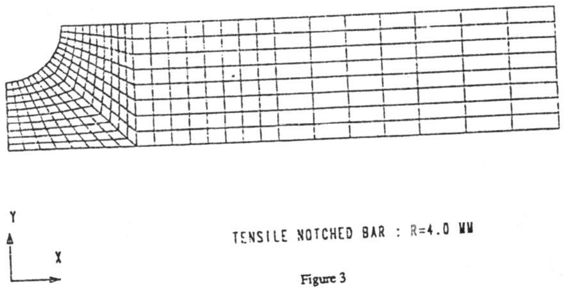

Figure 3

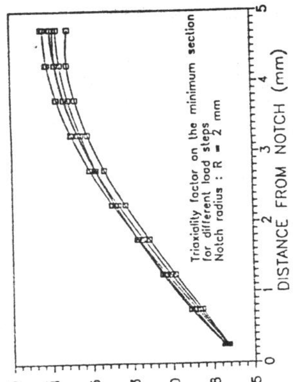

Figure 4

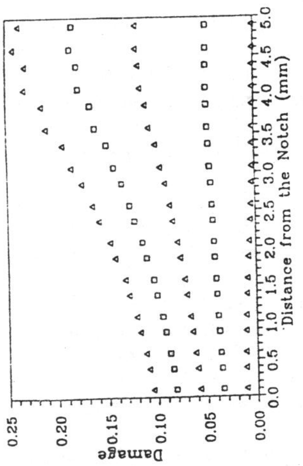

Figure 5.- Damage on the minimum section for several loads steps.

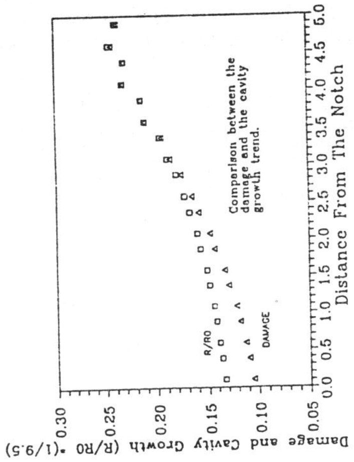

Figure 6.- Comparison between the Damage and R/R_0 along the minimum section in corrispondance o. the fracture load.

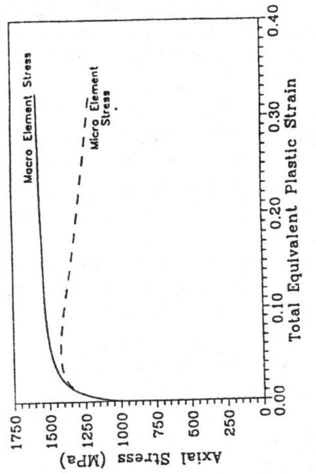

Figure 7. - Different load curves for the same Gauss point in the different volume scales.