OPTIMAL DESIGN OF PLATES AND SHELLS WHEN ACCOUNTING FOR MATERIAL DAMAGE

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The paper considers axisymmetrically loaded structure elements-plates and shells. Calculation of the elements carrying capacity is performed by accounting for the material damage, gained during loading at low and moderate rates of strain. The approach is based on a proposed micro damage model. The material characteristics are experimentally found by testing specimens of mild steel. An approach of how to determine the element safety, regarding critical void nucleation and growth, is outlined.

INTRODUCTION

As is well known, the record of material damage is an essential part of structure safety calculations. Most of the metals are characterized by a residual volume change, gained over a definite stress limit and by the accompanying phenomenon of micro fracture. Such a process, combined with void nucleation and growth and developing under different rates of strain, has been studied by a number of authors - see for example Rice and Tracey (1), Curran et al (2), Zukas et al (3) etc. The aim of this paper is to study the micro fracture mechanism of metals at low and moderate rates of strain. The material is assumed to deform elasto-viscoplastically. Damage, occurring during the loading process, is taken into account, while material induced anisotropy is disregarded. The deformation and damage processes are studied together, and it is assumed that a thermo-fluctuation process develops and Boltzmann's statistical law is valid on micro level. A mechano-mathematical model is used, following Baltov and Minchev (4), in order to describe the inelastic deformation of plates and shells.

OUTLINES OF THE MECHANO-MATHEMATICAL MODEL

As shown in (4), the mechano-mathematical model is based upon the following assumptions:

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1. The strain rate consists of an elastic and inelastic part

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{in}_{ij}, \qquad i,j = 1,2,3, \tag{1}$$

where the superimposed dot denotes time differentiation, $\dot{\varepsilon}^{e}_{ij}$ and $\dot{\varepsilon}^{in}_{ij}$ are the elastic and the inelastic strain rates respectively.

2. The elastic strain rate is related to the Cauchy stress tensor σ_{ij}, according to the Hooke's law

$$\dot{\varepsilon}^{e}_{ij} = H_{ijkl}\dot{\sigma}_{kl} \tag{2}$$

where H_{ijkl} is the elastic resistance tensor.

3. The visco-plastic deformation is given by the inelastic deviatoric part of the strain rate,

$$\dot{e}^{in}_{ij} = \Lambda \exp\left[D\zeta\right].S_{ij}/\tau_{o}; \quad \tau_{o} > \tau_{s}(\gamma^{a}).\mathrm{sign}(\tau_{s}) \tag{3}$$

where

$$\tau^{2}_{o} = \tau^{2}_{p}(\gamma^{a}, \beta), \quad \gamma^{2}_{a} = \left[\frac{1}{2} e^{in}_{ij} e^{in}_{ij}\right]$$

is a yield condition, expressing plastic hardening at the subsequent strain rate β and $\tau_{s} = \tau_{p}(\gamma_{a}, \beta_{s})$ is the yield limit at pure shear for a quasi static strain rate β_{s}. Λ and D are material functions, depending on the process factors, i.e on the inelastic shear strain and strain rate intensity. ζ is a dimensionless energetic parameter, expressing the exceed over a certain energetic level, where viscoplastic deformation takes place.

4. The inelastic volume increase is given by the spherical part of the inelastic strain tensor and it consists of two parts - ξ_{N}, expressing the void nucleation, and ξ_{G}- giving the void growth. However,

$$\xi = \xi_{N} + \xi_{G} \tag{4}$$

5. The rate of void nucleation $\dot{\xi}_{N}$ is determined by accounting for two micro mechanisms - a mechanism of thermo fluctuation and such of an internal bond destruction. Moreover, as noted by Krzeminski (5) and Perzyna (6), $\dot{\xi}_{N}$ decreases with the increase of the void volume ξ, i.e. a void saturation (to a certain extent) takes place. The rate of void growth $\dot{\xi}_{G}$ can be determined by considering growth and coalescence of the existing voids. This can result in a macro crack formation, which may be considered as a loss of carrying capacity of the structure element. All these phenomena can be described by introducing subsequent characteristics and parameters of the actual deformation/damage process, such as the energy limit of void initiation and growth, material mechanical characteristics etc. - see ref.(4) for details.

The critical void volume ξ_F, causing macro crack formation, can be taken as constant when high rates of strain are considered- see refs.(5) and (6). Here we take into account low and moderate strain rates and it is worth determining ξ_F as depending on $\dot{e}^{in}_{ij}$.

DETERMINATION OF THE MATERIAL CHARACTERISTICS

The mechanical characteristics of the material, considering different rates of strain, can be deter- mined by performing a number of experiments. The split Hopkinson bar for compression, tension, torsion etc. is most often used for material testing at high strain rates, and a lot of such results have been already published - Harding (7) and many others. As was noted, we consider low and moderate rates of strain. Uni- axial tension of mild steel specimens to fracture is performed on a universal testing machine. The steel specimens are previously annealed, in order to improve the material homogeneity. The relations $\sigma_{11}-\varepsilon_{11}$ are obtained for different strain rates, and the material characteristics are given in Table 1. The tested specimens are analyzed metalographically and samples are cut from them in the vicinity of the macro crack.

TABLE 1 - Material Characteristics - Mild Steel

Strain Rate, s^{-1}	$\tau_s \cdot 10^6$, N/m^2	$\sigma_s \cdot 10^6$	$E \cdot 10^{10}$ N/m^2	ξ_F %
0	-	-	-	10.01
0.0066	121.28	210.06	20	12.51
0.066	128.48	222.67	20	12.62
0.66	135.49	238.82	20	13.21
6.6	144.34	250.01	20	13.94

The sample critical void volume ζ_F is measured by using a porosimeter. ξ_F varies here with the strain rate, while for high rates of strain, $\dot{\varepsilon} \geq 10^3 sec^{-1}$, ξ_F can be taken as constant (5,6). The dependence $\xi_F - \dot{\varepsilon}$ is plotted in Fig.1 after an appropriate transformation.

DESIGN OF PLATES AND SHELLS

The outlined method is used for the design of circular plates and shells, made of steel. The plate dimensions are: δ = 0.004 m - thickness and R = 0.4 m - radius. They are clamped and loaded by a uniformly distributed load. The shells are uniaxially compressed and their dimensions are: δ = 0.004 m - thickness, R = 0.4 m-radius and H = 0.8 m - height.

The plate deflections and the shell radial and axial displacements are calculated by using the finite system method (FSM), given in ref.(4), where plate and shell mechanical behaviour is also modelled. A computing programme, using an implicit scheme with respect to time, is developed. The applied load has the form

$$P(t) = \begin{cases} P_M, & t \in [0, t_1] \\ P_M \exp[-(t - t_1)], & t > t_1 \end{cases}$$

The critical load, under which macro fracture of the elements occurs, is determined. Deflections of a plate with and without damage are given in Fig.2. The comparison between the two deflections shows that the account for damage is significant. The shell radial and axial displacements are given in Table 2.

It is seen for both the elements that micro damage, being an essential part of the whole deformation/damage process, affects significantly the element carrying capacity. Since the damage record is an important part of the safety calculations, this implies an optimization of the element by a material damage control. These problems will be discussed in another paper.

TABLE 2 - Shell Radial and Axial Displacements

Shell height, m	Radial displ., m	Axial displ., m
0.80	-0.0137	0.0323
0.72	-0.0124	0.0318
0.64	-0.0071	0.0314
0.56	0.0146	0.0292
0.48	0.0213	0.0274
0.40	0.0382	0.0262
0.32	0.0308	0.0227
0.24	0.0194	0.0182
0.16	0.0083	0.0092
0.00	0.0000	0.0000

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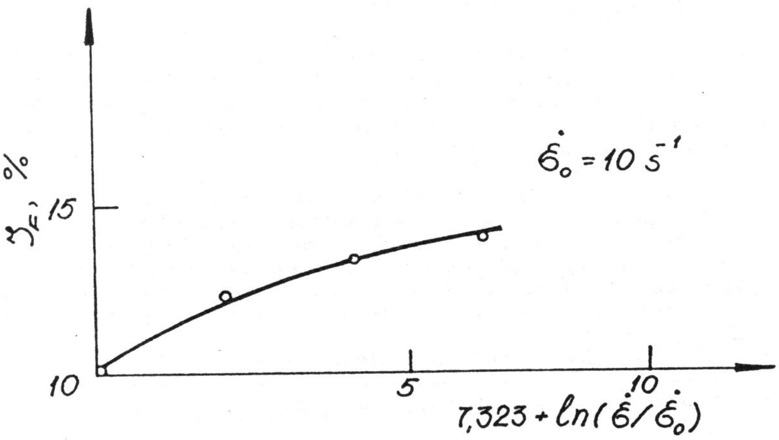

Figure 1 Void volume vs. strain rate

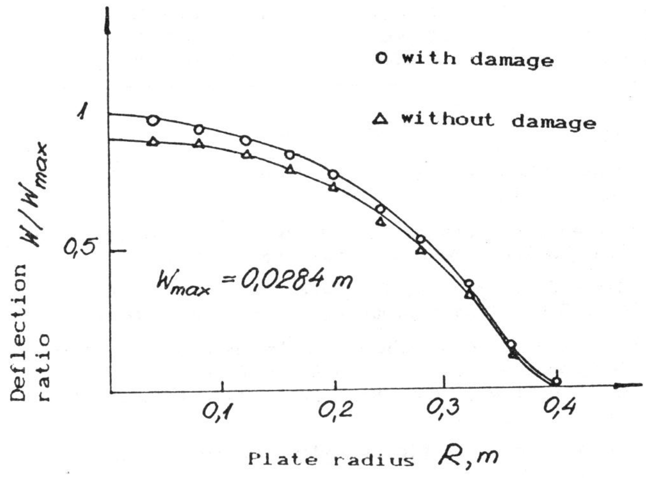

Figure 2 Deflections of a circular plate