ON THE SHAPE OF THE SOFTENING FUNCTION FOR CONCRETE

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When concrete cracking is modelled by using a cohesive crack, one needs to know the shape of the strain-softening diagram. At least four parameters are needed to describe reasonably this function: the tensile strength f_t, the specific fracture energy G_F and two more parameters characterizing the shape of the softening function.

In this contribution, an outline of a simple procedure to derive these parameters from stable three point bend tests on notched beams is presented. It is shown that a long-tailed softening is obtained which gives a better description of the observed behaviour than the more usual short-tailed softening curves.

INTRODUCTION

Localized crack growth in concrete has been successfully described by cohesive crack models following their introduction by Hillerborg and co-workers in the mid 70's [1]. A basic ingredient of the model is the *softening* curve, a material property. This function relates the stresses —the cohesive stresses— acting across the crack to the corresponding crack openings.

In principle, this curve could be experimentally obtained from direct tensile tests. Unfortunately this procedure has many drawbacks [2, 3]. In particular, to run a tensile test in which the opening of the crack is kept always uniform —as required to get sound results— is extremely difficult. This is why most of the procedures to infer the softening function rely on indirect methods based on the parametric fitting of the experimental results of bending beams or compact specimens [4, 5]. The available fitting procedures require some kind of iterative optimization technique where a complete "numeric test" has to be run (using finite elements or equivalent numeric procedures) in each iteration. Needless to say, these kind of analyses are reserved to very specialized research groups possessing the sophisticated numerical tools required.

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Recent work has lead to the development of a method of parametric fitting of the softening curve (with up to four parameters) requiring, by contrast, very simple analytical or graphical procedures to obtain reasonably good approximations. The purpose of this paper is to present the essentials of the method which is based on stable fracture tests on notched three point bend beams

THE SOFTENING FUNCTION

Even though the procedure to follow may be used to fit any softening curve with 4 parameters, for definitiveness, the softening function will be approximated by a bilinear function. This function is completely characterized when the following four parameters are known, as shown in Fig. 1: the tensile strength f_t, the specific fracture energy G_F, the abscissa of the centroid of the softening area $\bar{w}$, and the horizontal intercept w_1 of the initial tangent. For clarity, the softening function for a particular concrete will be derived from experimental measurements already performed by the authors [6].

Tensile Strength, f_t.

Owing to the difficulty in performing reliable simple tension tests, a Brazilian splitting test (ASTM C-495) may be used to approximate f_t. For our concrete, this approximate strength turned out to be $f_t = 2.8$ MPa.

Specific Fracture Energy, G_F.

The area enclosed under the softening function is the specific fracture energy G_F. This parameter was measured according to the RILEM procedure [7] with some improvements [8, 9, 10]. Basically, G_F was obtained by dividing the measured work of fracture W by the ligament area:

$$G_{FMeas} = \frac{W}{Bb} \tag{1}$$

where the ligament b is $b = D - a$, according to Fig. 1. These raw measured values, G_{FMeas}, display a clear size effect, increasing with specimen size as shown in row 1 of Table 1. They have to be corrected to take into account various sources of spurious energy dissipation —hysteresis of the testing equipment, bulk dissipation and energy dissipation at the supports—, and the effect of interrupting the test at some fixed rotation angle. With these corrections, a nearly size independent value of the fracture energy is obtained as shown in row 2 of Table 1. The corrected average value, for our concrete, was $G_F = 81$ N/m.

TABLE 1 – Specific Fracture Energy Determination.

	Size I	Size II	Size III	Size IV
G_{FMeas}	57 ± 2	75 ± 13	82 ± 2	94 ± 5
G_F	71 [13]	85 [23]	79 [7]	89 [11]

Values in [ ] are estimated variation intervals.

Abscissa of the Centroid of the Softening Curve, $\overline{w}$. In [10] it was shown that the abscissa $\overline{w}$ of the centroid of the area under the softening curve can be evaluated by fitting a theoretical expression to the far end of the P - δ curve recorded in the tests. It was shown that, asymptotically, the bending moment per unit thickness at the central section, M, varies proportionally to the inverse square of the rotation angle, θ, and that the proportionality coefficient is related to $\overline{w}$. More specifically,

$$M = \overline{w}\, G_F \frac{1}{\theta^2} \qquad (2)$$

Following the fitting procedure detailed in [10], the value $\overline{w} \approx 61$ µm was found.

Initial Tangent Intercept, w_1.

A method to get a reasonable approximation of the initial softening tangent is based on the observation that for small enough specimen sizes the peak load is reached, for the bilinear softening, before any point in the cohesive zone reaches the kink point on the softening curve. This means that for these specimen sizes the peak load for the bilinear softening exactly coincides with that found for an imaginary linear softening having the same horizontal intercept w_1, as represented by the dashed line in Fig. 1.

Further analysis —too lengthy to be reproduced here— provides an easy semi-graphic construct to determine w_1. When a log-log plot is used to represent the peak load versus the size of geometrically similar specimens (using the dimensionless variables shown in Fig. 2) the unique "master" curve shown as a solid curve in Fig. 2 may be proved to exist, representing the size effect curve for a linear softening. This curve is obtained once and for all for a given specimen geometry using a suitable numerical computation such as the influence method [11]. The curve shown in Fig. 2 corresponds to the geometry in Fig. 1 (notch-to-depth ratio equal to 0.3). Similar "master" curves may be obtained for other geometries by cross-plotting size effect results found in the literature (for example those of Petersson [12]). If the actual concrete did soften linearly, the experimental points would lie on this "master". However, if the concrete softening is bilinear, the experimental plots are horizontally displaced towards the left. For small sizes, the horizontal displacement is constant so that the experimental points should lie on a line obtained by translating the "master" curve a horizontal distance Δx towards the left, as shown by the dashed line in Fig. 2. This horizontal displacement in the log-log plot is related to the initial tangent intercept w_1 by

$$\Delta x = \log \frac{2G_F}{w_1 f_t}, \quad \text{or} \quad w_1 = \frac{2G_F}{f_t}\, 10^{-\Delta x} \qquad (3)$$

In principle, a single size experimental result is enough to determine Δx and w_1, but a range of sizes is required to check that the specimen size is small enough to guarantee that the experimental results do lie on a parallel to the master curve. For our concrete, the experimental results in Fig. 3 comply reasonably well with this condition, and the value $w_1 = 37$ µm is found.

General Bilinear Fitting (GBF) for the Softening Curve.

From f_t, G_F, $\overline{w}$ and w_1, the characteristic points of the bilinear softening may be computed using simple geometrical relations. For our concrete, the critical crack opening w_c is found to be equal to 367 µm, and the coordinates of the kink point (35 µm, 0.176 MPa).

DISCUSSION AND CONCLUSIONS

The bilinear softening obtained using the procedure just outlined is a long tailed one. Indeed, the dimensionless critical crack opening, $w^*_c = w_c f_t / G_F$, turns out to be equal to 12.7. This is to be compared with the dimensionless critical crack opening of the well known Petersson's bilinear softening [12], which amounts to 3.6. This means that the GBF softening is longer than Petersson's softening by a factor of nearly 4.

To explore the difference between the GBF and Petersson's softening, the experimental load-CMOD curve for a specimen of 100 mm depth (size II in Fig. 1) was compared with the numerical simulations found for these models using the influence method [11]. The resulting curves are shown in Fig. 3 in dimensionless form. Up to the maximum load, both models, Petersson's and ours (GBF), give the same values and agree quite well with the experimental curve, which is the average of two tests. However, the post-peak behaviour is better fit by GBF than by Petersson's approximation, which predicts a higher stress level in this region.

The reason for this behaviour is that Petersson's model was primarily set to fit the around-peak region rather than the far end of the P-δ curve, while ours has been set to fit both regions. Indeed, the first segment of the GBF and Petersson's softening happen to be very close (w_1=35 µm for Petersson's and 37 µm for ours). Thereafter, both models give a close description of the fracture behaviour for not too large sizes *and* not too large deflections. For large deflections the GBF softening appears to give better predictions.

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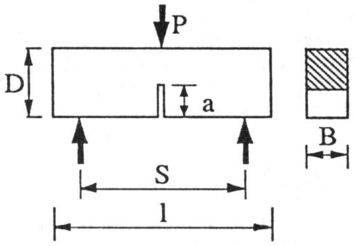

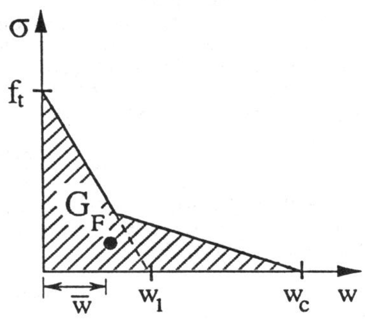

Size	l	S	D	a	B
IV	800	750	300	100	100
III	550	500	200	67	100
II	550	250	100	33	100
I	300	125	50	17	100

in mm

Figure 1. Sample geometry and softening function. The heavy dot represents the centroid of the dashed area.

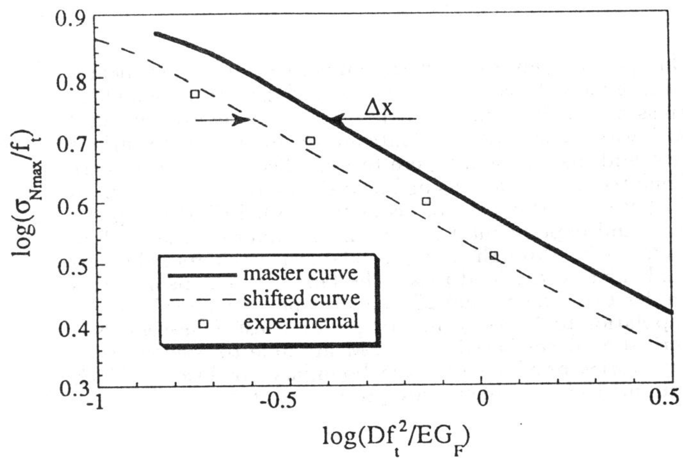

Fig. 2. Size effect plot. The nominal maximum stress is $\sigma_{Nmax} = \frac{3P_{max} S}{2B\ D^2}$.

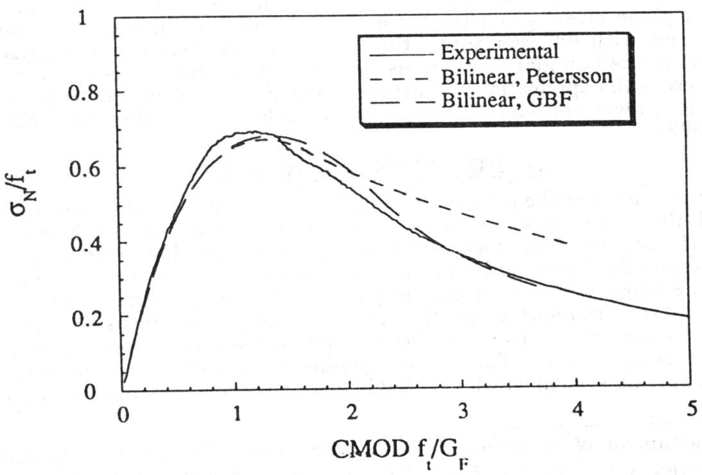

Figure 3. Load - CMOD curves. The nominal stress is $\sigma_N = \frac{3PS}{2BD^2}$.