ON THE INELASTIC BEHAVIOUR AND THE FAILURE MECHANISMS IN FIBRE-REINFORCED COMPOSITES

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The influence of the inelastic behaviour of a class of fibrous composites on the local failure mechanisms is considered from the viewpoint of an approach recently developed by the authors. The approach is shown to imply useful qualitative conclusions and quantitative results concerning the types and the critical instants of occurence as well as the locations of the typical of the fibrous composites considered failure modes.

INTRODUCTION

The heterogeneous composite materials structure creates various local stress concentration fields and gives rise to a variety of intensive local deformation processes. Their highly nonuniform and complex nature forms a variety of critical circumstances favouring the probable occurence of deformation instabilities. The physical appearances of the latters are the local failure modes observed in the real composite structures.

The types, the instants of occurence, and the locations of these modes are governed by specific failure criteria. These relate, on the one hand, the characteristics of the deformation processes and of their interactions, and, on the other hand, the mechanical properties of the constituents and the geometries of the phases. The failure mechanisms appear thus to be specific branches of these processes.

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For this reason the analysis of the local failure modes should be necessarily coupled with that of the deformation processes. In what follows the potentials of an authors' approach to such a coupling are briefly discussed.

BASIC APPROACH

The approach concerns the response of a class of composites consisting of ductile matrices reinforced by continuous parallelly aligned elastic fibres. It is based upon the matrix plastification model proposed by Herrmann and Mihovsky (1,2). The latter governs axisymmetric stress fields in a unit composite cell,i.e. a circular cylindrical fibre with a perfectly bonded coaxial matrix coating. The model implicitly assumes that the load transfer is due to local shear stress distributions near the ends of the cell and involves thus the plane cross-sections hypothesis.

As is shown by Herrmann and Mihovsky (3) the approach is unified in the sense that it reduces a class of problems of thermal and/or mechanical loadings of such composites to particular cases of a certain general problem of the plasticity theory. The latter proves to be qualitatively in much similar to the known plane-stress perfect plasticity problem, cf.Kachanov (4). The similarity concerns most of all the types (hyperbolic, parabolic,and elliptic) of the set governing the general problem. But types and changes in types mean mathematically specific features of specific solutions. Mechanically these appear as specific nature and features of the plastic deformation processes and, accordingly, of the local failure mechanisms.

The analysis of the thermal (matrix cooling) and longitudinal extension model problems carried out by Herrmann and Mihovsky (5,6) proves the potentials of the unified approach to reliable predictions of the overall composites response. The extension response predicted in (6) by the simplest version of the approach is identical with the known lower bound estimation due to Hill (7). A more precise version predicts in (5) a thermal response which is shown in (3) to be in perfect agreement with the experimental data reported by Larsson (8).

GENERAL PLASTICITY PROBLEM

Let the cell be referred to cylindrical coordinates (r,θ,z) (z -axis being the fibre axis) and r_f and r_m be fibre and cell radius,respectively. Let E_i , ν_i , α_i

denote the Young's moduli, Poisson's ratios, and thermal expansion coefficients of fibre ($i=f$) and matrix ($i=m$) and σ_Y be the tensile yield stress of the matrix. The stresses σ_i, $i=r,\theta,z$ in both of the phases are principle ones.

The approach reduces the mentioned class of problems to a plane problem with an yield (von Mises) ellipse defined as

$$(\sigma_r-\sigma_\theta)^2+[\sigma_r+\sigma_\theta-(2E_m\varepsilon^*\cot an\Phi)/\sqrt{3}]^2\tan^2\Phi-4\sigma_Y^2/3=0 \qquad (1)$$

where

$$\left.\begin{matrix}\sigma_r\\ \sigma_\theta\end{matrix}\right\}=\frac{E_m\varepsilon^*}{1-2\nu_m}+\frac{\sigma_Y}{\sqrt{3}\sin\Phi}\cos(\omega\pm\Phi),\ \sin\omega=\frac{\sqrt{3}(\sigma_\theta-\sigma_r)}{2\sigma_Y},\ \tan\Phi=\frac{1-2\nu_m}{\sqrt{3}} \qquad (2)$$

The constant strain ε^* is specific for a given "composite-loading"-combination, cf. (5,6). The governing set is specified by equations (1),(2) and the Hooke's law and the equilibrium equations

$$\sigma_z=E_m\varepsilon^*+\nu_m(\sigma_r+\sigma_\theta),\quad \partial\sigma_r/\partial r+(\sigma_r-\sigma_\theta)/r=0. \qquad (3)$$

The cases $|\omega|<\Phi$ (or $|\omega-\pi|<\Phi$), $\Phi<\omega<\pi-\Phi$ (or $\Phi<\omega-\pi<\pi-\Phi$), and $|\omega|=\Phi$ (or $|\omega-\pi|=\Phi$) correspond to elliptic, hyperbolic, and parabolic types of the set. Equations (2),(3) imply the implicit $\omega(r)$-dependence in the form ($r_f\le r\le R$, $\omega_R\le\omega\le\omega_f$), cf. (1)

$$R^2\sin\omega_R=r^2\sin\omega\,\exp[(\omega-\omega_R)\cot an\Phi] \qquad (4)$$

where $R\le r_m$ is the current plastic zone radius, $\omega_R\equiv\omega(R)$, $\omega_f\equiv\omega(r_f)$.

Equations (2) prove the existense of a natural limitation of the shrinkage. Its maximum is achieved at $\omega_f=\pi-\Phi$. The $R(\pi-\Phi)\equiv R^*$-value follows from equation (4) with $r=r_f$, $\omega=\pi-\Phi$ and appears to be the maximum possible plastic zone radius. For given combinations of mechanical properties and r_f/r_m -ratios R^* may be smaller or larger than r_m . In the $R^*<r_m$-cases entire matrix plastification is simply impossible.

So far the approach has been restricted to such cases only. It was quite recently shown by Mihovsky (9) that it governs the cases $R^*\ge r_m$ as well. As is shown in (6) entire matrix plastification is always achievable in the longitudinal extension case and the phenomenon is a sudden one. In the thermal case (5) the plastic zone expands monotonously. The type of the set in these cases is elliptic and hyperbolic, respectively. The considerations below will be restricted to the latter case.

PARTIALLY PLASTIFIED MATRIX ($R^* < r_m$)

The essential feature of this case is that at the instant $\omega_f = \pi - \phi$ the set governing the matrix plastification at the interface changes its type from a hyperbolic to a parabolic one. In the terms used by Freudenthal and Geiringer (10) in the plane-stress analysis of a ring under internal pressure and due to the similarity mentioned above the plastic annulus $r_f \le r \le R^*$ at this instant undergoes a transition from "plastic equilibrium" to "free plastic flow". The latter sets in a thin layer just surrounding the fibre where the radial and axial strain rates tend to infinity. The cell behaviour at this state will obviously depend on the interaction between the tendency to plastic instability and the fibre which tends to prevent the occurence of the singular strain rates field. This interaction will obviously create over the interface shear stresses equal to the shear yield stress τ_Y of the matrix.

Now, if τ_S is the shear strength of the interface and $\tau_S \le \tau_Y$, then interfacial debonding will take place in the middle portion of the cell. But this is hardly to be expected since if $\tau_S \le \tau_Y$ such a debonding will occur along the end portions due to the same shear stresses transfering the load between the phases.

In the opposite $\tau_S > \tau_Y$ -case radial and/or transverse matrix cracking should be expected to occur within the layer in the case of matrix cooling. In the case of heating opening mode interfacial cracks are to be expected as well (with probable subsequent fibre breaking and pull-out effects).

ENTIRELY PLASTIFIED MATRIX ($R^* \ge r_m$)

In this case the condition $R = r_m$ defines through equation (4) (with $r = r_f$, $\omega = \omega_f$) the instant of entire matrix plastification and the corresponding ω_f^* -value. According to the analysis in (9) further loading, i.e. matrix cooling, does not change the governing set (hyperbolic) but leads to a stable weakening of the shrinkage (decrease of ω_f from its ω_f^* -value) and unloading of the fibre in both the radial and axial directions. The experimentally studied in (8) thermal response allows now to assume that the changes in the length of the cell with entirely plastified matrix are negligible. Along with the assumption that the cross-sections remain plane in the middle portion of the cell this leads to the curious (but at first sight only) result that the axial strain within this portion becomes tensile. The intervals ($\tilde{\omega}$, ω_f^*) and ($\tilde{\omega}$, ω_R) where $\tilde{\omega} = \arctan(-3 \operatorname{tag} \Phi)$ correspond to thinning and thickening of the fibre, respectively. The first of them exists, of course, provided $\omega_f^* > \tilde{\omega}$.

This deformation regime can not activate the failure modes discussed in the $R^* < r_m$ -case. But as in the latter case one should not overlook the undesired possibility of creation of critical circumstances favouring the occurence of fibre buckling. From the point of view of its buckling the fibre is simply a rod under both axial and radial pressure. The corresponding axial and radial compressive stresses either simultaneously increase ($R^* < r_m$) or decrease ($R^* \geq r_m$), but for obvious reasons they depend on each other in a strongly nonlinear way. The thermal analysis in (5) allows for a reliable determination of this nonlinearity and thus for the prediction of the instant of possible buckling of the middle portion of the fibre, provided, of course, an appropriate buckling criterion is available.

At the same time the considered $R^* \geq r_m$ -regime of matrix cooling is capable of creating another critical situation. The point is that due to the implicitly assumed load transfer mechanism the change in sign of the strain in the middle portion of the cell creates additional interfacial shear stresses over certain portions of increasing length L_2 next to these of specific length L_1 created by the $r_f \leq R \leq r_m$ -regime. The two shear stress distributions are of opposite signs and result into further progressive axial compression of the end portions of the fibre of length $L_1 + L_2$. The situation is of obvious interest from the point of view of occurence of fibre buckling (or breaking in the case of matrix heating) close to the free ends of the cell. The approach leads to reliable approximate estimations of the specific L_1 -length as well as of the dependence of the increasing current L_2 -length on the loading parametr.

PREEXISTING DEFECTS

The approach is applicable to the investigation of the sensitivity of the considered composites to certain preexisting microdefects in the phases as well. It allows, in particular, to justify the applicability of the known Dugdale crack model to the class of radial matrix cracks and to predict the equilibrium an the growth of the latters by means of a correspondingly modified Dugdale type analysis,cf.(11).

CONCLUSIONS

It is hoped that these brief considerations illustrate at least qualitatively the ways in which the unified approach couples the specific features of the deformation processes with the occurence of local failure modes and implies quantitative predictions concerning their locations and instants of occurence. The approach forms thus a sound basis for the investigation of such modes in the considered composites. But of course this approach covers only one of the two basic aspects of this investigation. The other one is first of all dependent on the availability of appropriate fracture criteria.

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