NUMERICAL INVESTIGATION OF THE CREEP DAMAGE RUPTURE IN METALS

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This paper presents a continuum constitutive model for the creep damage rupture of metals. The damage variables are associated with anisotropic damage processes. A description, based on the conception that the damage may be specified by a combination of vectorial and scalar damage parameters, is used. The constitutive equations of creep and creep damage are formulated by employing the damage vector and the scalar damage parameter as internal state variables. The computational procedure in the finite element analysis solving the coupled problem was proposed.

INTRODUCTION

The development of new materials and the improvment of computational techniques in engineering mechanics leads us to consider more and more complex types of rheological behaviour. In this context, the main purpose of the continuum damage mechanics is to describe, in terms of continuum mechanics, internal structure change that occurs in certain materials under stress. The starting point of continuum damage mechanics is proposed by Kachanov L.M. in 1958 scalar damage measure (1). The *CDM* approach takes into account microscale level physical processes such as: nucleation and growth of grain boundary defects, voids, cavities, micro-cracks and other microscopic defects. The main differences between the models concern the derived damage measures (scalar, vectorial or tensorial of second, fourth or eigth rank).

The theory of anisotropic damage mechanics was developed by Sideroff and Gordebois (2). Prior to this latest development, Murakami S. (3), Betten J. (5), Krajcinovic D. (4) investigated brittle and creep fracture using appropriate anisotropic models.

One of the main features of *CDM* is to take into account the coupling effects between damaging processes and stress-strain behavior. Using different approaches the models for coupling between damage and creep have been proposed (6,7). A coupled elastic damage model is proposed in (8). The model is numerically implemented using the finite element method with an Updated Lagrangian description. The problem of crack initiation in a thin plate with a center crack that is subjected to uniaxial tention is analyzed using that model. Chow C.L. propose a finite element formulation of an anisotropic theory of continuum damage mechanics for ductile fracture. The formulation is based on

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a generalized model of anisotropic continuum damage mechanics of elasticity and plasticity (9,10).

This paper presents a coupled creep damage model and a method of finite element analysis. The damage constitutive equations used in the model are proposed by Datcheva M. (11). It is assumed that the material damage in creep can be represented by a combination of vectorial and scalar damage parameters. Such a representation is shown to give a possibility to avoid the restrictions of the theories based on the pure vectorial approach – a need of infinite set of vectors to describe the damage in point of body (4) and an impossibility to define the damage in case of uniform loading. The proposed damage parameters are not connected with microscale level mechanisms of damaging as in model of Hayhurst (14).

THE COUPLED CREEP DAMAGE MODEL

Creep is slow time dependent deformation which occurs in the many materials. Plastic deformation may also be present, but for simplicity we shall not consider plastic effects.

A basic assumption in the formulation of the model is that the total strain tensor can be expressed as the sum of elastic and creep strain tensor:

$$\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^c_{ij} \quad , \tag{1}$$

so the total strain rate tensor can be expressed as

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^c_{ij} \quad , \tag{2}$$

where the superposed ($\dot{}$) implies differentiation with respect to time t.

The total stress rate depends on the elastic strain rate according to

$$\dot{\sigma}_{ij} = D_{ijkl}\dot{\varepsilon}^e_{ij} \quad , \tag{3}$$

where D_{ijkl} is the elasticity tensor. The creep strain rate tensor is related to the stress tensor and the internal state variable $\omega_{\bar{\alpha}}$ by

$$\dot{\varepsilon}^c_{ij} = f(\sigma_{ij}, \omega_{\bar{\alpha}}) \quad . \tag{4}$$

The constitutive creep law was assumed in the next form

$$\dot{\varepsilon}^c_{ij} = \gamma(\omega_{\bar{\alpha}}) < \Phi(F) > \frac{\partial F}{\partial \sigma_{ij}} \quad , \tag{5}$$

in which F is a potential function governing the creep deformation of the material. This potential function for an isotropic material in which the principal axes of stress and strain rate are coincident, can be given by yield functions used in plasticity. For example for creep of metalls a Mises potential function may be used

$$F^2 = \frac{3}{2} s_{ij} s_{ij} \quad , \tag{6}$$

where $s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$ is the deviatoric stress tensor. The term $< \Phi(F) >$ is a positive monotonic increasing function for $x > 0$ and the notation $<>$ implies

$$< \Phi(F) >= \begin{cases} \Phi(F), & \text{for} \quad x > 0 \\ 0, & \text{for} \quad x \leq 0 \end{cases}$$

Different choices have been recommended for the function Φ. For example the exponential function may be used

$$\Phi(F) = \left(\frac{F - F_0}{F_0}\right)^N \quad , \tag{7}$$

where F_0 is the threshold uniaxial yield value and the constant N is a material parameter. The influence of damage was expressed by the function

$$\gamma(\omega_{\vec{\sigma}}) = \gamma_f \left(\frac{1}{1 - \omega_{\vec{\sigma}}{}^l}\right)^N \quad , \tag{8}$$

where γ_f and l are material parameters.

A lot of metallurgical investigations indicate that the creep damage of polycristaline metals occurs by the nucleation and growth of grain boundary defects. The equation (8) describes the softening of the material due to micro cracking.

The damage state at a body point P is characterized by two internal variables: a vector $\vec{\omega}$ and scalar Ω. In the definition of the damage variiables specific damaging mechanisms or other microcharacteristic are not taken into account, but the metallographics observations are guidelines for the present model — especially the fact that the damage process in the maximum principal stress cross–section is more significant than the other cross–sections. The damage of cross–section through the point P with normal $\vec{n}$ is defined by

$$\omega_{\vec{n}} = \vec{\omega}\vec{n} + \Omega \quad , \tag{9}$$

where $\vec{\omega} = (\omega_1, \omega_2)$.

In two dimensional stress state in an initial fixed coordinate system Ox_1x_2 the differential equations describing evolution of ω_1, ω_2 and Ω are

$$\begin{aligned} \dot{\omega}_1 &= V \cos\alpha \quad , \\ \dot{\omega}_2 &= V \sin\alpha \quad , \\ \dot{\Omega} &= \frac{V|\omega_1 \sin\alpha - \omega_2 \cos\alpha|}{\sqrt{(\omega_1^2 + \omega_2^2)}} + \dot{\Omega}_1 \end{aligned} \tag{10}$$

where α is the angle between the maximum principle stress σ_1 and the Ox_1 axis, Ω is a scalar parameter.

The function V and $\dot{\Omega}_1$ must be represent in the following form:

$$\begin{aligned} V &= \frac{b(\sigma_1 - \sigma_2)^\beta (\sigma_1 - \sigma_3)^{\nu-\beta}}{(1 - \omega_{\vec{\sigma}}^\rho)^\nu} \quad , \\ \dot{\Omega}_1 &= \frac{c\sigma_2^\mu}{(1 - \omega_{\vec{\sigma}}^\rho)^\mu} \quad . \end{aligned} \tag{11}$$

where the following designations are assumed: $\sigma_1 > 0$ is the maximum principle value of the stress tensor, the second nonzero principle stress is equal to σ_2 if it is tension and to σ_3 if it is compression ; b, c, β, μ and ρ are material parameters.

As rupture criteria the Kachanov-Rabotnov type criteria is assumed: "The rupture of structure occurs when the damage measure of the maximum damaged section acheaves 1". The rupture time t_r is then given by:

$$t_r = \min_t \{ \mid \vec{\omega} \mid + \Omega = 1 \} \quad . \tag{12}$$

FINITE ELEMENT ANALYSIS

The computational procedure presentede in this paper is based on the finite element method approach. In the field of numerical analysis of time dependent problems the finite element technique is widely used (12). The coupled creep damage model presented in the previous section may be considered as two coupled processes: creep deformation is the first one and damage is the second one.

Consider the equilibrium state at time $t_{n+1} = t_n + \Delta t_n$ assuming that all state variables are known at time t_n. The finite element discretized equations of equilibrium that must be satisfied at any instant of time t_{n+1} are

$$\int_V [B]^T \{\sigma_{n+1}\} \, dv + \{f_{n+1}\} = 0 \quad , \tag{13}$$

where $\{f_{n+1}\}$ is the sum of the vector of equivalent nodal loads due to applied surface tractions, body forces and forces equivalent to the microcracking stage, $[B]$ is standard approximation matrix defined by means of the derivatives of the chosen shape functions (13).

During the time increment the equilibrium equations must be satisfied in the incremental form of eq.(13)

$$\int_V [B]^T \{\Delta\sigma_n\} \, dv + \{\Delta f_n\} = 0 \quad , \tag{14}$$

where $\{\Delta f_n\}$ represents the change in loads and damage state during the time interval $\{\Delta t_n\}$. Using the basic assumption from section 2 the stress increment are related by

$$\{\Delta\sigma_n\} = [\bar{D}^n] \left[[B^n]\{\Delta u_n\} - \{\dot{\varepsilon}^c\}\Delta t_n - \{E^n\}\Delta\gamma_n \right] \quad , \tag{15}$$

and the creep strain increment is

$$\{\Delta\varepsilon_n^c\} = \{\dot{\varepsilon}_n^c\} \Delta t_n + \{C^n\} \{\Delta\sigma_n\} + \{E^n\} \Delta\gamma_n \quad , \tag{16}$$

where $[\bar{D}^n] = [[I] + [D]\{C^n\}]^{-1}[D]$, $[D]$ – elasticity matrix, $[I]$ – unit matrix,

$$\{E^n\} = \theta\Delta t_n \left\{ \frac{\partial\dot{\varepsilon}^c}{\partial\gamma} \right\}^n , \qquad \{C^n\} = \theta\Delta t_n \left\{ \frac{\partial\dot{\varepsilon}^c}{\partial\sigma_{ij}} \right\}^n ,$$

the parameter $\theta \in [0,1]$; $\Delta\gamma_n$ from eq.(15) is expressed as

$$\Delta\gamma_n = \gamma_f \frac{N}{\left(1-(\omega^l_{\bar{\alpha}})_n\right)^{N+1}} l(\omega_{\bar{\alpha}})^{l-1}_n (\Delta\omega_{\bar{\alpha}})_n \quad ,$$

where

$$(\Delta\omega_{\bar{\alpha}})_n = (\dot{\omega}_{\bar{\alpha}})_n \Delta t_n \quad , \tag{17}$$

$$(\dot{\omega}_{\bar{\alpha}})_n = (\dot{\omega}_1)_n \cos\alpha_n + (\dot{\omega}_2)_n \sin\alpha_n + (\dot{\Omega})_n \quad , \tag{18}$$

where α_n is the change of the angle α during the time increment Δt_n.

Using eq.(14) and eq.(15) and applying standard finite element technique (12,13), the displacement increment $\{\Delta u_n\}$ occuring during the time step Δt_n can be calculated as:

$$\{\Delta u_n\} = [K^n]^{-1}\{\Delta V_n\} \quad , \tag{19}$$

where $[K^n]$ is the global stiffness matrix and

$$\{V_n\} = \int_V [B]^T[\bar{D}^n]\{E^n\}\Delta\gamma_n \, dv + \{\Delta f_n\} \quad . \tag{20}$$

The first term in the right hand side in eq.(19) represent the influence of the damage processes on the deformation processes during the time step and $\{\Delta f_n\}$ – is the incremental pseudo load.

Substituting eq.(15) into eq.(14) the following incremental stiffness equation for obtaining the displacement increments can be derived. The well known Newton – Raphson iteration procedure is applied (13). In the above mentioned solution procedure material softening effect due to microcracking is included in the last term of eq.(15).

The damage constitutive equations eq.(10) are solved at each time step (see fig.1) with zero initial boundary conditions. The Runge-Kutta method is applied.

NUMERICAL RESULTS

The numerical example consists in the coupled creep damage analysis of a plate with a circular notch. Fig.1 shows the quarter of the plate and its division into 8-nodes isoparametric finite elements.

During the numerical calculations the strees tensor and the damage measure $\omega_{\bar{\alpha}}$ are calculated in each Gauss integration point (marked with symbol $\times$). In that way the rupture criteria was checked in these points.

The Fig.2 shows the distribution of $\omega_{\bar{\alpha}}$ after 500 hours, $\omega_{\bar{\alpha}} \in [0, 0.38]$. The values of the material parameters are (for cooper) (11): $\nu = 2.91, \beta = 1.5, l = 1, \rho = 5, c = 4.18E-7, b = 7.53E-8, \mu = 1, \gamma_f = 1.E-14$.

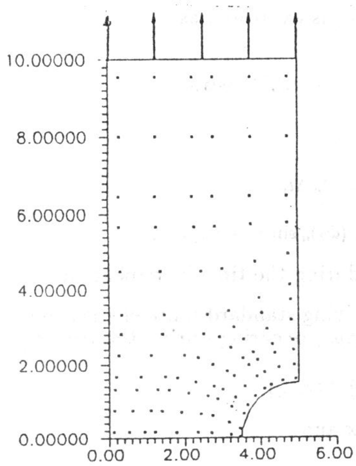

Fig.1

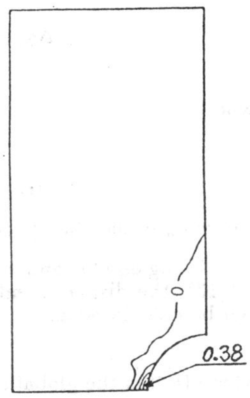

Fig.2

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