

NONLINEAR MODEL OF INTERLAMINAR DEFECTS
IN COMPOSITE SHELLS

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The process of separation into layers with subsequent fracture of the composite shells experiencing predominant compression is studied. The model is suggested which allows for geometric non-linearity, variability of the contact zone at the separation boundary and possibility of secondary branching.

The fracture process is analysed.

The experimental and theoretical studies (1,2,3) demonstrate that the process of fracture of composite shells involving layers separation can consist of two stages. During first stage, one of the layers in defect zone loses stability and during second stage the failure commences as a result of growth of the defect initiated by local bulging of the layer. The processes of shell deformation and growth of the defect are interconnected and define different forms of exhaustion of the carrying capacity of structures.

The investigations of nonuniform thin-walled structures (4) demonstrate that the phenomenon of local loss of stability is connected with secondary branching of non-linear solutions or existence of the separate equilibrium branches. Therefore the elaborated model takes into account the geometric non-linearity of beha-

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viour of the layers and package, possibility of normal contact of the layers at the crack zone, natural kinematic and static conditions at the boundary of layer separation zone, which is mobile and can be defined on the basis of generalised energy fracture criterion.

The formulated problem can be solved by using variational approach. To this end, on the basis of the Lagrangian functional, a complete functional of the investigated linear boundary-value problem with variable boundaries (contact zone boundaries and macrocracks boundaries) shall be created. This functional has the following form:

$$\mathcal{E}^* = \mathcal{E}_0 + \sum_{i=1}^N (\mathcal{E}_i^+ + \mathcal{E}_i^-) + \sum_{i=1}^N \iint_{S_{ic}} q_i (\omega^+ - \omega^-) dS + \sum_{i=1}^N \mathcal{E}g_i \quad (1)$$

where: \mathcal{E}_0 - is potential energy of monolithic region

$\mathcal{E}_i^+, \mathcal{E}_i^-$ - are potential energies of layers in N regions of layer separations

q_i - are Lagrangian factors for conjunctive contact conditions in the contact regions of layer separation zones

$\mathcal{E}g_i$ - is the energy of formation of a new layer separation surface

The functional is a complete one in a sense that out of it all resolving relationships inside the region can be derived as well as all boundary static and kinematic conditions, static and kinematic conditions of mating, conditions of contact, and defects growth equations. The condition of the defect growth is the equality of invariant J -integral and limiting value of J_R . For varying the functional, the formula for varying the varying boundaries functional is used.

In a case when the characteristics of the defect and stress and strain state of the shell are of the axially symmetric nature, the non-linear boundary-value problem is solved by reducing the boundary-value problem to the Cauchy problem applying Newton's method. In this case, the coordinates of the contact zones and the defects boundaries are included into argument-vector and the corresponding conditions of transversality and fracture are included into the implicitly defined Newton's discrepancy functions. In case the deformation is axially non-symmetric one, the problem is reduced to a single-dimensional one by applying Vlasov-Kantorovich method with utilization of the finite-ele-

ments approximation. By continuing to solve the problem for loading parameter, it becomes possible to perform non-linear analysis of the structure behaviour within the complete range of load variations and obtain the equilibrium branches (including separate and secondary ones) which correspond to general and local forms of the loss of stability and fracture of the structure.

The analysis performed with the aid of the suggested model demonstrates that the phenomenon of local loss of stability at the layer separation zone can be related to existence of the separate equilibrium branches of non-linear solutions.

Depending on the nature of interaction of the stress and strain states within the zone of defect and outside it, type of the structure and its geometric shape, there can exist general, local and mixed forms of bulging corresponding to different levels of the critical loads, loss of stability and fracture. Here the possibility exists either of the local loss of stability with consequent growth of the crack, or of the general bulging of the shell with separate deformation of the layers within the defect zone, or total loss of stability at contact of the layers (shapes II, III and I in Fig.1). As the characteristics of a defect change, the possibility of the local loss of stability of the layer disappears. The degeneration of separate branches of solution with decrease of the relative distance of layers separation is demonstrated in Fig.2.

The region of existence of the local loss of stability critical loads is determined only by the size of defect and slightly depends on the geometric shape of the structure. As the zone of damage increases and the bulging layer becomes thinner the critical loads decrease.

The realization of this or that form of the loss of stability and the possibility of defect spreading are determined by the type and intensity of initial disturbances in a system. For a number of systems (spherical segment, endless cylindrical shell, etc) there exists a minimal (critical) value of amplitude of the initial camber in the defect zone at which the layers separate and growth of the defect continues (Ref. curve 3 in Fig.1). At the amplitudes of initial camber below the critical value the defect does not develop and the general form of loss of stability of the damaged shell is realized (Ref. curve 2 in Fig.1).

The process of spreading of the defect zone on account of normal separation of the bulged layer is determined by the non-linear character of the system deformation. The critical loads of local loss of stability and initiation of the defect growth not always determine the exhaustion of the carrying capacity.

Depending on the type of the structure and its strength characteristics the growth of the defect can continue until the complete failure of the structure, or the spreading of layer separation ceases as the load increases. In Fig.3 the equilibrium curves are presented which describe the shell behaviour prior to the initiation of the defect (curves 1 to 5) and the process of fracture (curves 6 and 7). From Fig.3 it follows that after bulging of a layer and commencement of fracture the appearance of ascending branches of solution becomes possible.

In the process of performing analysis, the possibility of the exhaustion of carrying capacity of the shells on account of local loss of stability in the layer separation zone has been revealed (e.g. formation of the 'Chinese lantern' shape at axial compression of the varimodulus cylindrical shell); in this case the equilibrium branch of the solution describing the local bulging is obtained as a result of bifurcation of the initial solution rather than formation of a separate branch.

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