MODIFIED C* INTEGRAL FOR MULTI-MATERIAL BODIES

M.Pavišić*, A.Sedmak*, N.Savović*

Modified C* integral for multi-material bodies has been introduced in order to recover path independency of C* integral, lost because of material heterogeneity. Procedure is analogous to the Rice's J integral and its modification for multi-material body. Modified C* integral is applied to the weldment treated as three-material body with different properties in base and weld metal and heat-affected-zone.

INTRODUCTION

Welded high-temperature components, like turbine pipelines and rotors, often fail because of excessive creep damage, including stable creep crack growth. If the conditions of steady state creep are fulfilled the appropriate fracture mechanics parameter is C* integral, being path independent under the same conditions as Rice's J integral, including material homogeneity, at least in crack direction. Since weldments generally do not fulfill this condition, modification of C* integral is necessary in order to recover its path independency. Such a modification is introduced in this paper, and an example is given to illustrate theoretical considerations. The procedure is based on Gurtin's approach, (1), leading to the integral expression analogous to the J integral for multi-phase body, defined in (2).

MODIFIED C* INTEGRAL FOR MULTI-MATERIAL BODIES

Starting point in the theoretical analysis is C* integral, with the integration contour, as given in Fig. 1. Taking into account the notations in Fig. 1, one can write:

* Faculty of Mechanical Engineering, University of Belgrade

$$C^* = \int_{G_1} \left(Wn_1 - \sigma_{ij} n_j \frac{\partial \dot{u}_i}{\partial x_1} \right) ds \quad (1)$$

$$0 = \int_{G_k} \left(Wn_1 - \sigma_{ij} n_j \frac{\partial \dot{u}_i}{\partial x_1} \right) ds \qquad G_k = G_2, G_3, G_4, G_5 \quad (2)$$

since only G_1 encompasses crack tip. Notations in eqns (1) and (2), not given in Fig. 1, are as follows:

$W = W(\dot{\varepsilon}_{ij}) = \int_0^{\dot{\varepsilon}_{ij}} \sigma_{ij} d\dot{\varepsilon}_{ij}$ is the strain energy density, σ_{ij} and ε_{ij} are stress and strain tensors, respectively, u_i is displacement vector and $\dot{}$ denotes time derivative. By simple adding of eqs (1) and (2) the following expression can be obtained:

$$C^* = \int_{G} \left(Wn_1 - \sigma_{ij} n_j \frac{\partial \dot{u}_i}{\partial x_1} \right) ds - \sum_{1}^{4} \int_{l_k} \left([W] n_1 - \left[\sigma_{ij} n_j \frac{\partial \dot{u}_i}{\partial x_1} \right] \right) ds \quad (3)$$

where G denotes the outer contour, and l_k, k=1,2,3,4, contours along material boundaries (Fig. 1). Squared brackets in the second integral term denote so-called jump function, defined as follows: $[F] = F^+ - F^-$ where F^+ and F^- denote function value at the positive and negative contour side, respectively.

It should be noted that only problems which are symmetrical to the crack planeare analyzed here. Considerations about "y component of crack driving force" (see e.g. (3)) are not in the scope of this paper. It should be also noted that the modified C^* integral was introduced for the three-material body, chosen as the representative for weldment with three different material regions: base metal (BM), weld metal (WM) and heat-affected-zone (HAZ). Modifications for any other number of different materials are straightforward.

FINITE ELEMENT CALCULATIONS AND RESULTS

Calculation of integral expression (3) was performed using post-processor, specially written for the finite element programme, published in (4), andmodified here to take into account complex material behaviour under creep conditions ("overlay concept"). All calculations are done on PC 486.

In order to illustrate theoretical considerations CT specimen (Fig. 2) was chosen for calculations. Since no experimental evidence was available, it was assumed that the specimen had been taken from "X" shaped welded joint, with the properties of BM, WM and HAZ as given in Tab. 1. These properties are fictitious, although chosen in accordance with literature data for Cr-Mo pipeline steel. Anyhow, having in mind the purpose of this investigation - material heterogeneity effects - they are quite satisfactory.

Table 1 Material properties

	E (GPa)	ν	B	n
BM	150	0.3	1E-16	5.0
WM	150	0.3	5E-16	5.0
HAZ	150	0.3	8E-16	5.0

notation:
B and n - constants in Norton law
E - Young's modulus,
ν - Poisson's ratio

The finite element mesh, Fig. 3, was based on the mesh recommended by ESIS (5).Ten integration path were used, 1-4 through all three material regions, 5-6 through HAZ and WM, and 7-10 through WM only. The calculation was done also for BM only in order to verify the procedure.

Results for the BM are given in (6,7) showing strong path dependency in the primary creep region. Anyhow, after the steady state conditions are reached C^* integral becomes path independent and its value for plane strain problem is 130 N/m/h, (7). The results obtained here for BM only are in perfect agreement.

Results for the weldment calculations are given in Tab. 2, both for C^* integral and its modification, as defined by the expression (3). Only steady state values are given.

Table 2 Results of finite element calculations (given in N/m/h)

path	1	2	3	4	5	6	7	8	9	10
C^* integral	148	145	139	137	135	130	130	128	131	129
modified C^*	128	125	129	126	134	133	130	128	131	129

ACKNOWLEDGEMENTS

The work on this paper has been supported by the Scientific Fund and by the Fund for Technological Developmentof Republic of Serbia, as well as by the Electricity Board of Serbia.

REFERENCES

(1) M.E.Gurtin, *J.Elasticity*, Vol.9, 1979, pp. 186-195.

(2) S.Sedmak, A.Sedmak, N.Vukomanović, Proc. ECF8, Torino, EMAS, 1990, pp.1596-1599.

(3) H.Miyamaoto, M.Kikuchi, Proc. II International Conference on Numerical Methods in Fracture Mechanics, Swansea, Pineridge press, 1982, pp. 359-371.

(4) D.R.J.Owen, E.Hinton, *Finite Elements in Plasticity - Theory and Practice*, Pinridge Press, Swansea, 1980.

(5) ESIS TC8, "Recommendation for use of finite element method in fracture mechanics, reported by W.Schmitt, ESIS Newsletter No 15, 1991

(6) T.Hollstein, R.Kienzler, Proc. of 5th International Fracture Mechanics Summer School 5, Dubrovnik, 1989, EMAS, pp. 175- 188

(7) M.Brunet, J.C.Boyer, Proc. III International Conference on Numerical Methods in Fracture Mechanics, Swansea, Pineridge press, 1984, pp. 519-532.

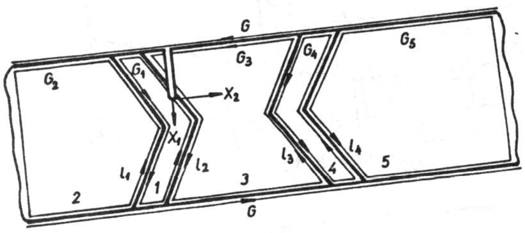

Fig. 1 Integration contours for C^* integral

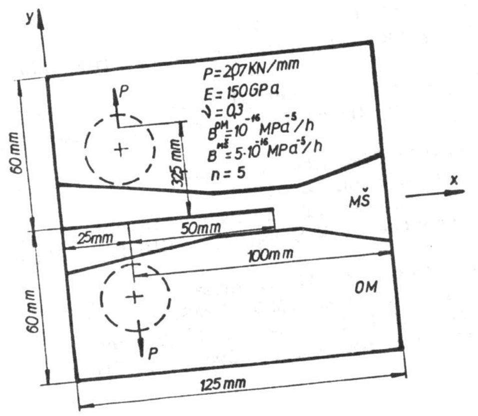

Fig. 2 CT specimen

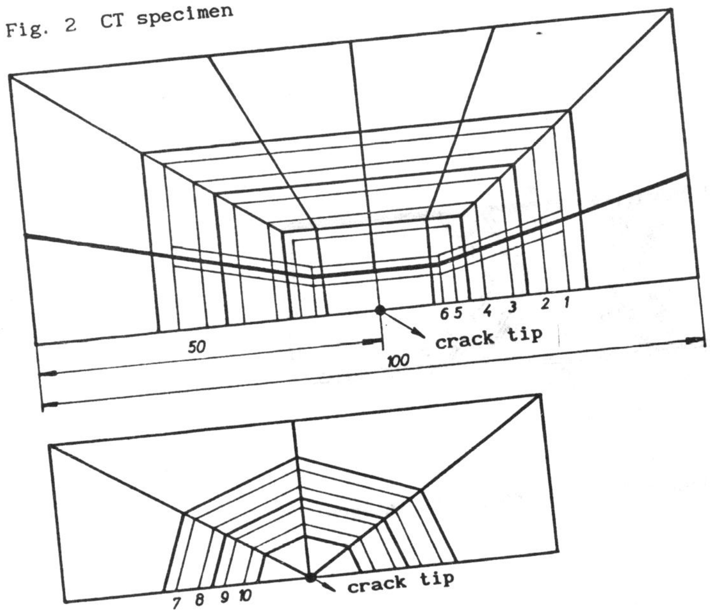

Fig. 3 Finite element mesh