MECHANICS OF MICRODEFECTED MATERIALS

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A new general method has been developed in composites mechanics.This is a multiparticle effective field method(EFM),which based on replacement of local external fields,containing separate inclusion by the effective field depending on the properties of the inclusion under study and those surrounding it. EFM of averaging the random structures based on the using of Green's function for the respective problem.

GENERAL RELATION

The paper discusses a macro domain w with a characteristic function W containing a set $X=(V_k,x_k,\omega_k)$, ($k=1,\ldots,N$) of ellipsoids v_k with characteristic functions V_k, centers x_k (that forms a Poisson set), semi-axes a_k^i ($i=1,2,3$) and aggregat of Euler angles ω_k. The local equation for the material state, that connects stress tenzors $\sigma(x)$ and strain tensors $\varepsilon(x)$ is given in the form

$$\sigma(x)=C(x)[\varepsilon(x)-\varepsilon^T(x)] \, , \qquad (1)$$

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where $C(x)$ is a fourth-order tensor for elastisity moduli, $\varepsilon^T(x)$ is a second-order tensor of stress-free strains (also called mismatch strains) . In the matrix $w\backslash v$ ($v=\cup v_k; k=1,2,\ldots$) the tensors $C(x)=C^{(o)}$, $\varepsilon^T(x)=\varepsilon^{T(o)}$ are assumed to be constant in each $v_k (k=1,2,.)$ $C(x)=C^{(o)}+C_1(x)=C^{(o)}+C_1^{(k)}$, $\varepsilon^T(x)=\varepsilon^{T(o)}+\varepsilon^T(x)=\varepsilon_1^{T(o)}+\varepsilon^{T(k)}(x)$.Substituting (1) in the equilibrium equation $\nabla\sigma=0$, we obtain an integral equation (Buryachenko and Lipanov (1),(2))

$$\sigma(x)=\sigma^o+\int\Gamma(x-y)\{M_1(y)\sigma(y)+\varepsilon^T(y)-[\langle M_1\sigma\rangle+\langle\varepsilon_1^T\rangle]\}dy \ , \quad (2)$$

where $M_1(y)=(C^{(o)}+C_1^{(k)})^{-1}-M^{(o)}, M^{(o)}=(C^{(o)})^{-1}$,at $x\in v_k$, $\Gamma(x-y)=-C^{(o)}*[I\delta(x-y)+\nabla\nabla G(x-y)C^{(o)}]$, G -the Green tensor of the Lam's equation of a homogeneous medium with an elastic modulus tensor $C^{(o)}$; δ-the delta function , I-the unit tensor, $\sigma^o\equiv\langle\sigma\rangle$.In (2) and below $\langle.\rangle,\langle.|x_1,\ldots,x_n;x_{n+1},\ldots x_m\rangle$ stand for the average and the conditional average taken from the ansemble of a statistically homogeneous ergodic field X ,on condition that there are inclusions at the points $x_1,\ldots,x_n$ and $x_1,\ldots,x_n\neq x_{n+1},\ldots,x_m$; $\langle.\rangle_k$ is the volume average involvingthe ellipsoid v_k.

The effective parameters of M^*,ε^*in the macromedium state equation $M^*\langle\sigma\rangle=\langle\varepsilon\rangle-\varepsilon^*$are defined by relations

$$M^*=M^{(o)}+B_\sigma^* \ , \ \varepsilon^*=\langle\varepsilon^T\rangle+B_\varepsilon^* \ ;\langle M_1\sigma\rangle\equiv B_\sigma^*\langle\sigma\rangle \ , \ \langle M_1\sigma\rangle\equiv B_\varepsilon^* \ , \quad (3)$$

where the tensor B_σ^* and B_ε^* are found from the solution of the elastic problem at $\varepsilon^T(x)=0$ and of the thermoelastic problem at $\langle\sigma\rangle=0$, respectively.

Let us introduce $\phi(v_m|x_m;x_l)$, which is a conditional distribution of the m-th inclusion in the domane v_m at fixed inclusion in the domane v_l ; $\phi(v_m|x_m;x_1,\ldots,x_n)=0$ at value of x_m lying inside the domain

$v_m^o \supset v_m$ with the characteristic functions V_m^o.

By way of conditional statistical averaging (with the help of various distribution functions $\phi(v_m|x_m;x_1,\dots,x_n)$), the problem of evaluation of the effective parameters of the medium is reduced to an infinite system of integral equations (n=1,2,...)

$$\langle\sigma|x_1,\dots,x_n\rangle - \sum_{i=1}^{N}\int\Gamma(x-y)V_i(y)\langle M_1(y)\sigma(y)+\varepsilon_1^T(y)|x_1,\dots,x_n\rangle dy = \sigma^o + \int\Gamma(x-y)\{\langle M_1(y)\sigma(y)+\varepsilon_1^T(y)|y;x_1,\dots,x_n\rangle-[\langle M_1\sigma\rangle+\langle\varepsilon_1^T\rangle]\}dy \quad (4)$$

Let us denote the right-hand member of the n-th line of the system by the field $\hat{\sigma}(x)_{1,\dots,n}$, then each inclusion $v_i (i=1,\dots,n)$ of the chosen fixed inclusions is in nonhomogeneous field $(x\in v_i)$

$$\bar{\sigma}_i(x)=\hat{\sigma}(x)_{1,\dots,n}+\sum_{j\neq i}\int\Gamma(x-y)V_j(y)[M_1(y)\sigma(y)+\varepsilon_1^T(y)]dy \quad (5)$$

THE EFFECTIVE FIELD

Let us apply the EFM (Buryachenko (3)) hypotheses:

H1) Each inclusion v_i has an ellipsoidal form, located in the homogeneous field $\bar{\sigma}(x)$ and $(x\notin v_i, \bar{v}_i=\text{mes}\,v_i)$

$$\int\Gamma(x-y)V_i(y)[M_1(y)\sigma(y)+\varepsilon_1^T(y)]dy= \quad (6)$$
$$= \langle\Gamma(x-y)\rangle_i\langle M_1(y)\sigma(y)+\varepsilon_1(y)\rangle_i\bar{v}_i$$

H2) At some sufficiently big n there occurs a closure $\langle\hat{\sigma}(x)_{1,\dots,j,n+1}\rangle=\langle\hat{\sigma}(x)_{1,\dots,n}\rangle_i$, where the right-hand member of the equality does not contain the index $j\neq i$, $1<j<n$ and $x\in v_j$.

Due to linearity of problem, there axist constant fourth and second-rank tensors B_i and C_i, such that

$$\langle\sigma(x)\rangle_i=B_i\langle\bar{\sigma}(x)\rangle_i+C_i,\ \bar{v}_i\langle M_1(x)\sigma(x)+\varepsilon_1^T(x)\rangle=R_i\langle\bar{\sigma}(x)\rangle_i+F_i, \quad (7)$$

where $R_i = \bar{v}_i \langle \Gamma(x) \rangle_i^{-1} (B_i - I)$, $F_i = \bar{v}_i \langle \Gamma(x) \rangle_i^{-1} C_i$.

THE EVALUATION OF C^*, ε^*

System (5) can be solved by analytical methods, if

$$\langle \hat{\sigma}(x)_{12} \rangle_i = \langle \bar{\sigma}(x) \rangle_i = \text{const.} \quad (i=1,2) \tag{8}$$

Then from (4), taking account of (7), (8), we get the expression for effective parameters

$$M^* = M^{(o)} + \sum_{i=1}^{N} R_i D_i n_i, \qquad D_i = R_i^{-1} \sum_{i=1}^{N} Y_{ij} R_j \tag{9}$$

$$\varepsilon^* = \varepsilon^{T(o)} + \sum_{i,j=1}^{N} Y_{ij} F_j n_i, \tag{10}$$

where n_q is a calculated concentration of inhomogeneity, matrix Y^{-1} has elements $(Y^{-1})_{ij}$ $(i, j=1,\dots,N)$ in the form of submatrices (6*6)

$$(Y^{-1})_{ij} = \delta_{ij}\Big[I - R_i \sum_{i=1}^{N} T_{iq}(x_i - x_q) Z_{qi} \phi(v_q | x_q ; x_i) dx_q\Big] -$$

$$- R_i \int \big[T_{ij}(x_i - x_j) Z_{ji} \phi(v_j | x_j ; x_i) - T_i(x_i - x_j) n_j\big] dx_j$$

$$(Z^{-1})_{ji} = I\delta_{ji} - (1-\delta_{ji}) T_{ji}(x_j - x_i) R_i, \tag{11}$$

$$T_{ji}(x_j - x_i) = (\bar{v}_j \bar{v}_i)^{-1} \iint \Gamma(x-y) V_j(x) V_i(y) dx dy$$

$$T_i(x_i - x_j) = (\bar{v}_i)^{-1} \int \Gamma(x - x_j) V_i(x) dx$$

STRENGTH OF COMPOSITES

Estimated are the average-volume values of elastic stress-fields in components

$$\langle \sigma \rangle_i = B_i D_i \langle \sigma \rangle, \quad (i=1,2,\dots) \tag{12}$$

A study of more-compex phenomena ,such as strength of composites ,however,requires a calculation of the second moment (Parton and Buryachenko(4))

$$\langle\sigma \times \sigma\rangle_i=\partial M^*/\partial M^{(o)}(\langle\sigma\rangle \times \langle\sigma\rangle)/c_i\ ,\ c_i\equiv\langle V_i\rangle\ . \quad (13)$$

In the case of composites with the strength properties of their components described by strength criteria

$$\Pi(\sigma)=\Pi^{2(k)}_{ij}\sigma_{ij} + \Pi^{4(k)}_{ijmn}\sigma_{ij}\sigma_{mn}=1 \quad (k=1,\dots,n) \quad (14)$$

the estimates (12),(13) justified the assumption of macroostrength criterion

$$\max_k \{\Pi^{2(k)}B_k D_k\langle\sigma\rangle + c_k^{-1}\Pi^{4(k)}\partial M^*/\partial M^{(k)}[\langle\sigma\rangle x\langle\sigma\rangle]\}=1, \quad (15)$$

where Π^2 , Π^4 are the second-and fourth-rank tensors of strength . In particular,for a composite with plane spheroidal cracks$(a^1=a^2 \gg a^3)$ in incompresible matrix that satisfies strength criterion $s_{ij}s_{ij}=k_p^2$ from (9), (15) we shall obtain macrostrength criterion

$$I_2+b^*(I_1)^2=(k_p^*)^2 \quad , \quad (16)$$

where

$$b^*=\frac{2}{9}c(1-448c/375\pi^2)[(1-848c/375\pi^2)(1-16c/15\pi^2)]^{-1} \quad (17)$$

$$(k_p^*)^2=k_p^2(1-448c/375\pi^2)(1-848c/375\pi^2)^{-1}$$

$(c=4\pi(a^1)^3n/3,\ I_1=\langle\sigma_{ii}\rangle/3\ ,\ I_2=\langle s_{ij}\rangle\langle s_{ij}\rangle)$ .Relation (16) obtained for additional assumptions $Z_{qi}=I\delta_{qi}+(1-\delta_{qi})*$ $*T_{qi}(x_q-x_i)R_i$ and $T_{ij}(x_i-x_j)=-C^{(o)}\nabla\nabla G(x_i-x_j)C^{(o)}$, $\phi(v_j|x_j;x_i)=(1-V_i)n_j$.

To predict durable strength of materials with the accumulated damages π (concentration of ellipsoidal pores) allowed for ,it is suggested that use is made of integrodifferential criterion of material durable strength . To reduce the bulk of computations we shall analyze only a one-dimenional case of loading by a variable load. It is assumed that with the load unchanged the kinetics of accumulated damages are described by simple law

$$d\pi/dt=f_1(\sigma,T)f_2(\pi) \quad , \tag{18}$$

where T is the temperature. The criterion proposed for a changing load is

$$\Phi(\pi)=\int_0^t H(t-\xi)dp(\sigma,T,\pi) \quad , \tag{19}$$

where $\Phi(\xi)$, $H(\xi)$ are the increasing monotonous functions, σ^*-effective stress determined by condition $\Pi^{(o)}(\sigma^*)=$ $=\Pi^*(\langle\sigma\rangle)$. If it is futher assumed that $H(t)=t^{\alpha}$, $p(\sigma,T,\pi)=$ $=p_1(\sigma,T)p_2(\pi)$, ($\alpha=const$) with kinetic curves $\pi=\pi(t)$ co- inciding in mode (18),(19) (when $p_1^{1/\alpha}=f_2(\sigma,T)$) we receive

$$\Phi(\pi)=\int_0^{\pi}[g(\pi)-g(\xi)]^{\alpha}dp_2(\xi) \quad , \quad g(\pi)=\int_0^{\pi}f_2^{-1}(\xi)d\xi \tag{20}$$

It is from the criterion of (21) that well-known criterias follow :

Baily's ($\alpha=1, p(\sigma,T)=1/\tau(\sigma,T)$) $\pi(t)=\int_0^t\tau^{-1}(\sigma(\xi),T(\xi))dt$,

Ilyushin's ($p(\sigma)=\sigma$) $\pi(t)=\int_0^t S_r^{-1}(t-\xi)d\sigma^*(t)$,

Moskvitin's ($p(\sigma)=\sigma^{1+m}, m=1$) $\pi(t)=\int_0^t S_r^{-1-m}(t-\xi)d\sigma(\xi)^{1+m}$,

where τ - durability of a material , $S_r(t)\equiv\tau^{-1}(\sigma)$ - is an areverse function of a durability .

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