FRACTURE OF METALS AS A RANDOM PROCESS

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A possibility for an integration of the statistical description of metal behaviour under load and the theories of Zhurkov and Griffith is illustrated and a principal analytical history of metal under load is presented.

INTRODUCTION

Metal behaviour under load - life and fracture - is described by different approaches: statistical, including its reliability (metal is not especially specified); after Zhurkov, considering temporary strength or durability ("perfect metal"); after Griffith, defining critical strength (metal with a crack).

The present study describes, based on random process theory, life and fracture of metals under load, integrating the possibilities of Zhurkov's and Griffith's approaches, defining the bounds: "perfect metal - metal with a crack". The aim is a presentation of a complete history of each defined metal in the interval "load imposing - fracture".

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EXPERIMENTAL RESULTS. STATISTICAL DESCRIPTION.

Tensile strengths σ_{Ti} and yield strengths σ_{Yi}, $(i=1,2,\ldots,50)$ are studied by testing two series of aluminium specimens (50 test specimens per set) at 293K and 373K on a universal testing machine FPZ-100; the strain rate is $dl/dt = 0.83.10^{-3}$ m/s. The durability or the time for fracture τ_i for all specimens is experimentally obtained.

In the most of the cases, the failure rate, concerning data of above type, is admitted to be a constant (Angelova et al (1). Then the statistical results description by the reliability function $P(\sigma)$ turns out to be only approximately qualitative.

Two types of functions are studied here. First function is of the form

$$P(\sigma) = \begin{cases} 1, & 0 \le \sigma \le \sigma_0 \\ A\exp[-B(\sigma-C)^2], & \sigma_0 < \sigma . \end{cases} \qquad (1)$$

It is necessary to fix preliminary C value by replacing $C = \sigma_{T51} =$ 86.4 MPa; σ_{T51} is the lowest tensile strength at which fracture is observed. Parameters A and B are obtained after the methods of the least squares and of Cramer: $A = 1.16395643$, $B = 0.0699458659$. The stress σ_0 is determined from conditions: $P(\sigma) \le 1$, $(A\exp[-B(86.4-86.4)^2] = A \approx 1.16 > 1)$; $P(\sigma)$ from (1) - strictly decreasing at $\sigma > \sigma_0$; $\lim_{\sigma\to\infty} A\exp[-B(\sigma-86.4)^2] = 0$; $P(\sigma)$ from (1) - continuous for each σ. From the pointed properties of function $A\exp[-B(\sigma-C)^2]$ follows, that there exists an unique $\sigma_0 \in [86.4,\infty)$: $\sigma_0 = 86.4 + \sqrt{[(1/B).\ln A]} = 87.87329$ MPa. With calculated A, B and σ_0, Eq.(1) can be rewritten in the form:

$$P(\sigma) = \begin{cases} 1, & 0 \le \sigma \le 87.873 \text{ MPa} \\ 1.164\exp[-0.0699(\sigma-86.4)^2], & \sigma > 87.873 \text{ MPa} . \end{cases} \qquad (2)$$

The average square distance $\mathcal{J}$ for the model function $P(\sigma)$ from (2) and the experimental data σ_{Ti}, $i=1,2,\ldots,49$, is $\mathcal{J} = [\sum_{i=1}^{49} (P(\sigma_{Ti}) - P_i)^2]^{1/2} = 0.129600794$; $P_i = P(\sigma_{Ti})$.

The second function is defined as

$$P(\sigma) = \begin{cases} 1, & 0 \leq \sigma \leq C \\ \exp[-B(\sigma-C)^2], & C < \sigma \end{cases} \quad (3)$$

To find parameters B and C we use the method of the least squares, which requires a minimization of function $F(B,C) = \sum_{i=1}^{49} [B(\sigma_{TSi} - C)^2 - Y_i]^2$, $P_i = P(\sigma_{TSi})$, $(-\ln P_i) = Y_i$. For the purpose, we find the solution of the system:

$$\begin{cases} \partial F/\partial B = B \sum_{i=1}^{49} (\sigma_{TSi} - C)^4 - \sum_{i=1}^{49} Y_i (\sigma_{TSi} - C)^2 = 0 \\ \partial F/\partial C = B \sum_{i=1}^{49} (\sigma_{TSi} - C)^3 - \sum_{i=1}^{49} Y_i (\sigma_{TSi} - C) = 0 . \end{cases} \quad (4)$$

From (4) for C we obtain the equation:

$$\Phi(C) = \sum_{i=1}^{49} Y_i (\sigma_{TSi} - C) . \sum_{i=1}^{49} (\sigma_{TSi} - C)^4 - \sum_{i=1}^{49} Y_i (\sigma_{TSi} - C)^2 . \sum_{i=1}^{49} (\sigma_{TSi} - C)^3 = 0,$$

which is impossible to be solved by analytical methods. In that case C is obtained after the method of bisection: $C \in (87.0313473, 87.0313475)$ MPa; $C = 87.0313474$ MPa. Then from (4) $B = 0.0824572$ and function of reliability, (3), can be rewritten in the form:

$$P(\sigma) = \begin{cases} 1, & 0 \leq \sigma \leq 87.0313474 \text{ MPa} \\ \exp[-0.0825(\sigma - 87.0313474)^2], & \sigma > 87.0313474 \text{ MPa}. \end{cases} \quad (5)$$

Here the value of the average square distance is $\mathcal{J} = [\sum_{i=1}^{49} (P(\sigma_{TSi}) - P_i)^2]^{1/2} = 0.09563002291$. It is smaller than $\mathcal{J}$ for (2), due to which we shall use $P(\sigma)$ from (5) further. On Fig.1 is defined the area of inition of the crack $P(\sigma_{YSi}) \div P(\sigma_{TSi})$, and on Fig.2 - the area of temperature influence $P(\sigma_{TSi}, 373K) \div P(\sigma_{TSi}, 293K)$.

INTEGRATION OF ZHURKOV'S, GRIFFITH'S AND STATISTICAL APPROACHES ON METAL FRACTURE

The obtaining of an adequate function $P(\sigma)$ from (5) and the experimentally tested durabilities τ_i allow us to determine the "Reliability" surface $P_1(\sigma_{TS}, \tau)$, Fig.3, each point of which defines an actual reliability of the metal P_i, under an acting stress σ_{TSi} for a defined time τ_i.

After Zhurkov, the metal durability τ is related to the temporary strength σ, to the temperature T, to the activated fracture volume γ, constant for each metal, and to the fracture activation energy U_0: $\tau = \tau_0 . \exp(U_0 - \gamma\sigma)/KT$; τ_0 is the period of thermal fluctuation of the atoms and K - the Boltzmann constant. It was established, that there exist actual activated fracture volumes αi for each pair (σ_i, T_i): $\alpha = \gamma\tau / \tau_0$, (Angelova (2). Physically, according to Zhurkov theory, the criterion for the specimen fracture is that of α. It is a result of either a break down of all atoms links in α or of an inition of a crack $a_\alpha = 2(3\alpha/4\pi)^{1/3}$, energy equivalent to the process of links break down for a time τ. The surface of "Defects" $P_2(\sigma_{TS}, a_\alpha)$, Fig.4, demonstrates an actual probability P_i under acting stress σ_{TSi}, at which after a time τ_i there initiates a defined crack $a_{\alpha i}$.

As fracture of α (or the inition of a_α) causes a definite fracture of the specimen, a_α can be considered as a Griffith crack. Our earlier studies demonstrate that the crack a_α is subject both to the theories of Zhurkov and of Griffith, (2):

$$\begin{cases} \pi(1-\nu)a_\alpha^2 \sigma_\alpha^2 / 8G\alpha + \sigma_\alpha - (U_0 - 2\bar{\sigma}_\alpha a_\alpha)/\alpha = 0 \\ \sigma_\alpha^2 = 8G\bar{\sigma}_\alpha / \pi(1-\nu) a_\alpha \; ; \end{cases}$$

where σ_α is the Griffith stress for a_α, $\bar{\sigma}_\alpha$ - effective surface energy, ν - the Poisson ratio and G - the Young modulus for aluminium. The surface $P_3(\sigma_\alpha, a_\alpha)$, Fig.5, is the surface of "Fracture", each point of which shows an actual probability P_i for a transformation of already initiated crack of defined size a_α to Griffith crack, as a result from a stress intensification from σ_{TSi} to $\sigma_{\alpha i}$. Thus metal history is completely described from the surface of "Reliability" $P_1(\sigma_{TS}, \tau)$, through the surface of "Defects" $P_2(\sigma_{TS}, a_\alpha)$ (by the appearence of the cracks a_α) up to the surface of definite "Fracture" $P_3(\sigma_\alpha, a_\alpha)$. All three surfaces are at 293K. The results at 373K are analogues; this being testified by:the $P(\sigma)$ changing in the interval of (293 ÷ 373)K, Fig.1 and the surface $P_4(\sigma_{TS}, T)$, Fig.6.

DISCUSSION

Our study offers, through the introduced surfaces "Reliability"

$P_1(\sigma_{TS}, \tau)$, "Defects" $P_2(\sigma_{TS}, a_\alpha)$, "Fracture" $P_3(\sigma_\alpha, a_\alpha)$, a principal analytical history of metal under load for a defined temperature interval through the surface "Thermo -reliability" $P_4(\sigma_{TS}, T)$. A combined approach, including the theories of random processes, of Zhurkov and of Griffith is used.

The transition from surface "Defects" $P_2(\sigma_{TS}, a_\alpha)$ to surface "Fracture" $P_3(\sigma_\alpha, a_\alpha)$ demonstrates: local stresses σ_α at the crack tips of a_α cover Griffith criterion, when the cracks reach a defined level of their size a_α under acting macrostresses σ_{TS}.

SYMBOLS USED

α = actual activated volume(m^3) for durability τ (s)
$P(\sigma)$ = reliability function

REFERENCES

(1) Angelova, D.G., Dishliev A.B. and Vasilev I.G., Int. J. Fracture, forthcoming, 1992.
(2) Angelova, D.G., "New Metals Fracture Hypothesis", Doctor Thesis, Sofia, Bulgaria, 1989.

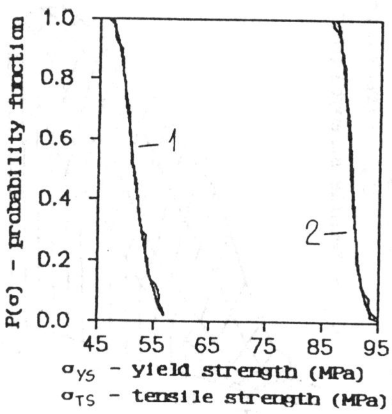

Figure 1 Probability function $P(\sigma_{YSi})$-1 and $P(\sigma_{TSi})$-2

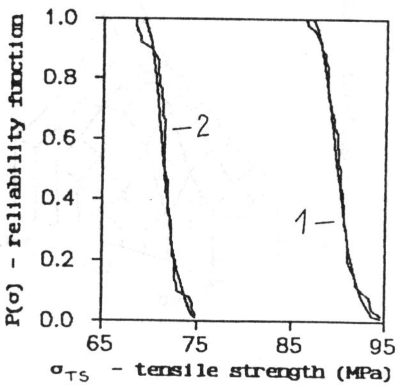

Figure 2 Reliability function at 293K-1 and 373K-2

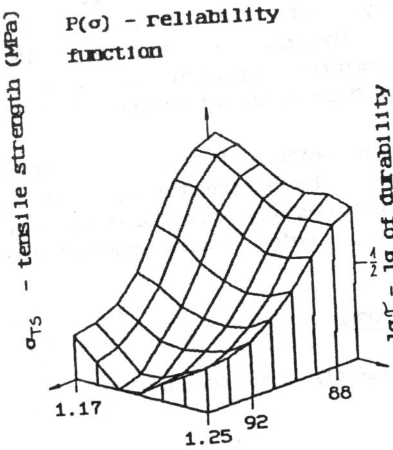

Figure 3 Surface "Reliability" $P_1(\sigma_{TS}, \tau)$

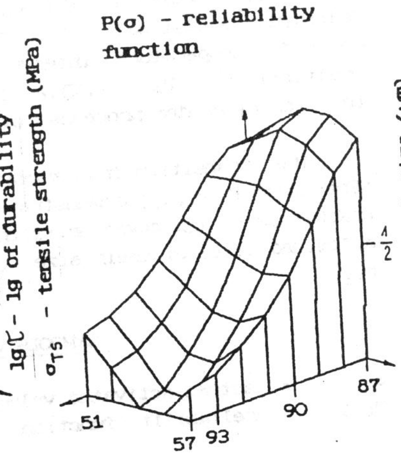

Figure 4 Surface "Defects" $P_2(\sigma_{TS}, a_\alpha)$

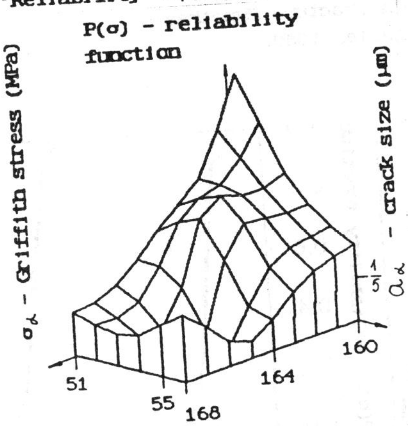

Figure 5 Surface "Fracture" $P_3(\sigma_\alpha, a_\alpha)$

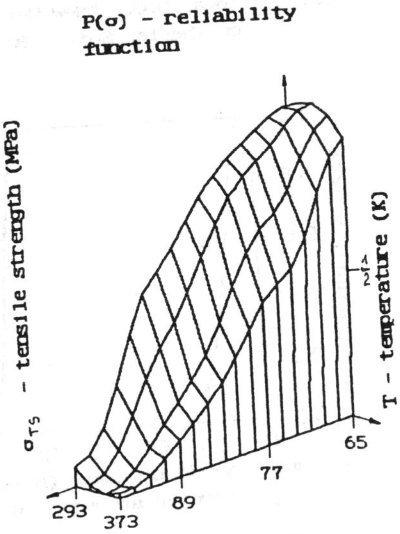

Figure 6 Surface "Thermo-reliability" $P_4(\sigma_{TS}, T)$