FATIGUE DAMAGE UNDER NON-STATIONARY STOCHASTIC STRAIN

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Various types of non-stationary Gaussian and Weibull strain processes were generated, analyzed and simulated using the 1-st and 2-nd order Markovian chains. Then they were treated by the Rain Flow Method and the fatigue lives according three fatigue damage accumulation hypotheses were assessed. The results show that (a) the best algorithm of simulation is the 2-nd order Markovian chain and that (b) the process non-stationarities have not a profound influence on the resulting fatigue life.

Measurements of operating processes reveal that most of them possess non-stationary properties (time-varying mean level, variance, etc.). As this phenomenon may be important for fatigue damage accumulation, in this paper we shall try to elucidate two partial problems, viz.
- simulation of a non-stationary process representing an input of a computer-controlled fatigue machine, and/or an input of the fatigue damage accumulation hypothesis,
- estimation of the influence of some non-stationary process properties on the resulting fatigue life.

PROBLEM FORMULATION

To run fatigue experiments or compute fatigue life under non-stationary stochastic loading it is indispensable to simulate (model) a non-stationary stochastic process by

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means of a suitable algorithm. The methods developed so far have one fundamental "drawback" from the practical point of view, viz. they require for the non-stationary statistical characteristic(s) to be described mathematically (Čačko et al (1)). It is almost impossible, however, to identify the corresponding equations and so we shall try to design an algorithm avoiding it. Dealing with fatigue, the criterion of successful simulation will be the fatigue endurance, in our case estimated through the two-parameter Rain Flow Method and three fatigue damage accumulation hypotheses, viz. *Palmgren-Miner (PM)*, *Serensen - Kogaev* (SK) and *Kliman* (K) hypotheses for strain processes, resp. (Bílý (2)), in order to obtain a possible "scatter" and "mean" of computational results.

SOLUTION OF FORMULATED PROBLEMS

From a variety of non-stationary processes two groups of typical representatives were selected:
(a) **Gaussian** processes described by a probability density function (PDF)

$$f_G(x,t) = \frac{1}{\sqrt{2\pi}\, s(t)} \exp\{-[x - \mu(t)]^2/s^2(t)\}; \qquad (1)$$

(b) three-parameter **Weibull** processes (covering also, e.g., **Rayleigh** and **exponential** processes), with a PDF

$$f_W(x,t) = A(t)[x - a(t)]^{m(t)} * \\ * \exp\{-A(t)[x - a(t)]^{m(t)+1}/[m(t) + 1]\}, \qquad (2)$$

where $\mu(t)$ and $s(t)$ are deterministic functions of time t (mean value and standard deviation, resp.); $A(t)$, $a(t)$, $m(t)$ are deterministic time parameters. These processes were simulated according to the algorithms developed in (1) and the variation of their parameters yielded a variety of non-stationary processes, some of them are characterized in Table 1. Gaussian G1 and Weibull W1 processes were used as stationary references.

Developing and verifying various simulation algorithms, a fairly accurate method (especially in the

Table 1- Some Variants of simulated Processes (G - Gaussian, W - Weibull, N - general non-stationary)

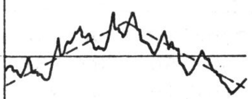
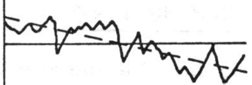
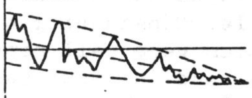
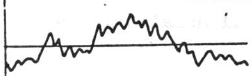
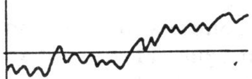
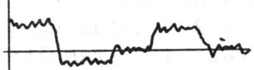
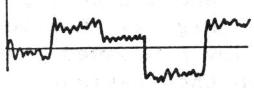
Variant	Type	Character	Non-stationary	Markovian ch. order
G1	stat.		----	first
G2	non-stat.		ramp $\mu(t)$	first & second
G3	non-stat.		ramp $\mu(t)$	first & second
G4	non-stat.		ramp $\mu(t)$	first & second
G5	non-stat.		sinusoidal s(t)	first
G6	non-stat.		sinusoidal s(t)	first
G7	non-stat.		ramp $\mu(t)$ & sinusoidal s(t)	first second
G8	non-stat.		ramp $\mu(t)$ & sinusoidal s(t)	first second
W2	non-stat. exp.		A(t) = = 1.2 t + 1 in Eq. (2)	first
W3	non-stat. Rayl.		A(t) = = t + 1 in Eq. (2)	first
W4	non-stat.		step-wise varying mean	first second
N	gen. non-stat.		stoch. mean & variance	first second

Table 2- Comparison of Numbers of Macroblock Repetitions to Fracture (in Thousands) for generated (g) and simulated (s) Processes from Table 1 and three Fatigue Damage Hypotheses H: s1 - 1-st order chain, s2 - 2-nd order chain,

Level $\times 10^{-4}$	H	G2 g/s1/s2	G3 g/s1/s2	G8 g/s1/s2	N g/s1/s2
Low	PM	635/209/480	733/335/510	130/105/148	3200/385/870
ε	SK	127/ 42/ 90	147/ 67/ 80	28/ 28/ 30	640/ 74/250
=3.6	K	185/125/235	368/215/220	98/ 96/ 94	1000/230/600
High	PM	55/ 29/ 65	74/ 54/ 58	1.5/7.0/3.5	103/9.6/ 45
ε	SK	11/5.8/ 13	14/ 10/ 11	0.3/1.5/0.6	21/2.0/7.9
=5.0	K	28/ 21/ 33	47/ 38/ 38	1.4/5.0/1.8	29/7.0/ 32

fatigue sense) appeared to be the **Markovian chain** of the 1-st order. Fig. 1 documents this by comparing the generated processes G1 (stationary Gaussian) and N and their simulated pairs. Nevertheless, when the generated processes had a time varying mean (processes G2, G3 and especially W4 and N), the simulation was not so successful. This discrepancy was more remarkable for low levels than for higher ones. In order to "improve" this result the 2-nd order Markovian chain approach was adopted taking into account three adjacent ordinate levels. The improvement was remarkable. Considering that also other statistical characteristics were well reproduced (differences do not exceed 5 % and in most cases are smaller than 1 %), this type of simulation seems to be very satisfactory.

A very interesting conclusion can be drawn from a general comparison of fatigue lives obtained for various strain levels and variants of simulated processes partially documented in Table 2. Excluding W4 and N, yielding evidently longer lives, all results fall into a relatively narrow scatter and can be approximated by one Manson-Coffin (MC) curve (Fig. 2). Consequently the PDF shape of the non-stationary processes examined above has no practical influence on the fatigue life. Only the step-wise varying mean (processes W4 and N) shifts the endurance to higher values. This may be, however, a matter of plotting because such processes have higher variances due to a jumping mean. These results are surprising but advantageous for practical analyses as most non-stationarities need not be taken into account when computing the fatigue life or simulating the operating strain.

CONCLUSIONS

Investigation of various non-stationary strain stochastic processes and the corresponding fatigue lives revealed:

- based on the fatigue life criterion the most universal and accurate method of stochastic process simulation seems to be the 1-st or even better, the 2-nd order Markovian chain; the corresponding algorithm is fast so it can be used in real time;

- whereas the 1-st order chain is capable of reproducing non-stationarities of the standard deviation type, reproduction of non-stationarities caused by time-dependent mean levels require the 2-nd order chain simulation algorithm;

- variation of the PDFs, time-dependent means, variances and other process parameters cause unessential endurance changes which can be attributed to the standard scatter of the MC curve, perhaps with the exception of processes with a suddenly changing mean level; these facts are very advantageous for the practical fatigue endurance estimation and process simulation.

REFERENCES

(1) Čačko, J., Bílý, M. and Bukoveczky, J. "Random Processes: Measurement, Analysis and Simulation", Amsterdam, Elsevier, 1988.

(2) Bílý, M. "Dependability of Mechanical Systems", Amsterdam, Elsevier, 1989.

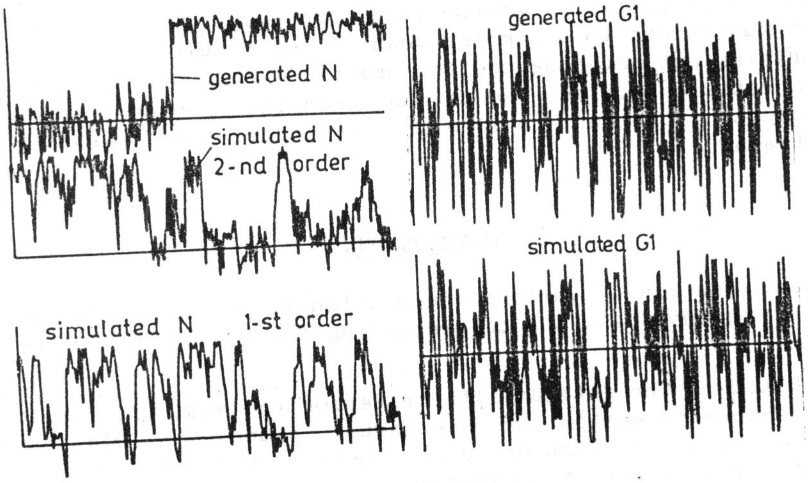


Figure 1 Comparison of generated and simulated processes

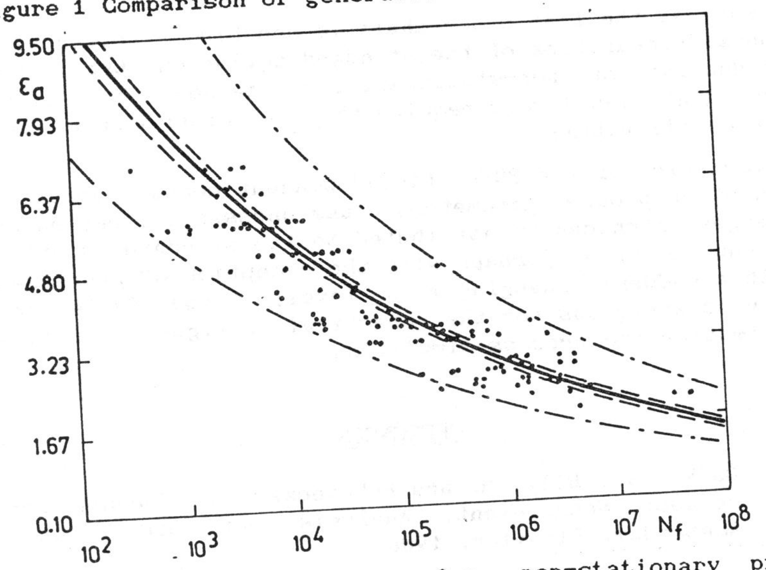


Figure 2 Manson-Coffin curve for non-stationary processes investigated with its confidence intervals