EXPERIMENTAL DETERMINATION OF THE FAILURE PARAMETERS $T_{D,0}$ AND $T_{V,0}$ IN MILD STEELS ACCORDING TO THE T-CRITERION

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The experimental determination of two elastic quantities is described in this paper in case of mild steels, by means of simple tension and torsion experiments. These quantities are considered as material properties, according to the T-criterion, controlling failure through general yielding or brittle fracture. An iterative method based on the Bridgman solution for necked specimens is employed to compute these quantities from the experimental data in case of tension. Finally, fracture limit diagrams (FLD) are plotted for the two materials used.

INTRODUCTION

The experimental behaviour of materials obeying to the Mises yield condition is, usually, described in terms of an equivalent stress-strain ($\bar{\sigma}$-$\bar{\varepsilon}$) curve. The area between this curve and the $\bar{\varepsilon}$- axis is considered as representing the strain energy density stored in the material. This is exactly true only in case of pure shear, where $\sigma_1=-\sigma_2$, $\sigma_3=0$. Any other combination of ($\sigma_1,\sigma_2,\sigma_3$) leads to a discrepancy between the area underneath $\bar{\sigma}$-$\bar{\varepsilon}$ curve and the strain energy density. The highest discrepancy is observed in case of hydrostatic tension/compression ($\sigma_1=\sigma_2=\sigma_3$) where $\bar{\sigma}=0$ and $\bar{\varepsilon}=0$, and consequently no energy transfer is indicated in the $\bar{\sigma}$-$\bar{\varepsilon}$ curve.

This unacceptable conclusion can be cured by considering an additional constitutive equation connecting hydrostatic pressure $\bar{p}$ and volume expansion Θ ($=\varepsilon_1+\varepsilon_2+\varepsilon_3$). Both curves ($\bar{\sigma}$-$\bar{\varepsilon}$) and (p-$\Theta$) are necessary for a complete description of the behaviour of mate-erials up to and including failure. This is the basic assumption of the T-criterion of failure (Andrianopoulos and Theocaris (1), Andrianopoulos and Boulougouris (2)).

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According to the T-criterion both the $\bar{\sigma}$-$\bar{\varepsilon}$ and p-Θ curves must have a terminal point corresponding to the maximum capacity of the material to store distortional elastic strain energy density T_D (measured in the $\bar{\sigma}$-$\bar{\varepsilon}$ curve) and dilatational elastic strain energy density T_V (measured in the p-Θ curve) respectively (Fig.1). When one of the terminal points is reached either in the $\bar{\sigma}$-$\bar{\varepsilon}$ or p-Θ curve the material fails by yielding or fracture, respectively. The existence of these terminal points is obvious because, otherwise, the material could store infinite amount of strain energy.

The two limiting capacities $T_{D,0}$ and $T_{V,0}$ are considered as material properties. Their evaluation necessitates two independent experiments: (i) pure shear, where by definition $T_V=0$, and the total elastic strain energy density at failure equals to $T_{D,0}$ and (ii) equal hydrostatic tension, where by definition $T_D=0$, and similarly the total elastic strain energy density equals to $T_{V,0}$. However, the second experiment is practically impossible and can be replaced by simple tension provided that, at failure, T_D is smaller than $T_{D,0}$. This experimental procedure is described here.

EXPERIMENTAL PROCEDURE

Tension Experiments. Rods of two mild steels (EN1A and EN24T) were machined to form cylindrical specimens of dimensions (gauge length x diameter)$(l_o x 2\alpha_o)=(0.075x0.0113)m^2$ and $(0.02x0.0036)m^2$ marked as "Big" and "Small" Directly measured quantities are the engineering stress $\sigma_{z,eng}$ and the neck geometry (α,R) from photographs, where α is the radius of the minimum cross section and R the neck radius of curvature. Pictures of the neck at the moment of failure cannot be obtained and, so, the function $\alpha/\alpha_o=f(\sigma_{z,eng})$ was plotted from data before failure. It was estimated through a least squares approximation that this experimental curve is:

$$\left.\begin{aligned} \text{EN1A: } & \alpha/\alpha_o=(79.6+1.84x\sigma_{z,eng})x10^{-3} \\ \text{EN24T: } & \alpha/\alpha_o=(160.0+0.77x\sigma_{z,eng})x10^{-3} \end{aligned}\right\} \quad (1)$$

Extrapolation of Eqs.(1) at $\sigma_{z,eng}$ equal to its failure value gives the minimum value of α. By a similar numerical procedure it was estimated that for both materials it is valid that:

$$\alpha/R=0.831x\ [\ln(A_o/A)]^{1.54} \quad (2)$$

with $A_o=\pi\alpha_o^2$ and $A=\pi\alpha^2$.

Neck geometry (α,R) and failure load P_f being known, the stress state at the central point of the minimun cross section, where failure initiates, can be evaluated by means of the following equations (Bridgman, (3)):

$$\sigma_z=\bar{\sigma}_f[1+\ln(1+a/(2R))], \quad \sigma_r=\sigma_\theta=\bar{\sigma}_f\ln[1+a/(2R)] \tag{3}$$

where: $\bar{\sigma}_f=P_f/[\pi a^2(1+2R/a)\ln(1+a/(2R))]$

Then the experimental points ($\bar{\sigma}_f$, $\bar{\varepsilon}_f=\ln(A_o/A)$) and ($\sigma_o,\varepsilon_o$) (first yield point) are put in Fig.2. Also, the respective values of hydrostatic pressure (p), dilatational (T_V), and distortional (T_D) strain energy densities and plastic work (w_p) are computed by means of:

$$\left.\begin{aligned} &p=(\sigma_z+\sigma_r+\sigma_\theta)/3, \quad T_V=(1-2\nu)(\sigma_z+\sigma_r+\sigma_\theta)^2/(6E) \\ &T_D=\frac{1+\nu}{3E}[(\sigma_z-\sigma_r)^2+(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)]^2_; \quad w_p=(\text{area under } \bar{\sigma}\text{-}\bar{\varepsilon})-T_D \end{aligned}\right\} \tag{4}$$

The values obtained are characterized as "EXPerimental" in TABLE 1. Note that in this table $\sigma_1=\sigma_z$, $\sigma_2=\sigma_\theta$ and $\varepsilon_1=\varepsilon_z$, $\varepsilon_2=\varepsilon_3=\varepsilon_\theta$.

By assuming bilinear shape of the curve $\bar{\sigma}$-$\bar{\varepsilon}$ of each material, an estimation of the slope $H=d\bar{\sigma}/d\bar{\varepsilon}$ can be made. Thus a unique load path $\bar{\sigma}$-$\bar{\varepsilon}$ is established for each material with mechanical properties given in the rows entitled as "MEAN TENSION EXPerimental" in TABLE 1. At this point an iterative procedure is applied with the following rules:

i) $\bar{\sigma}$ and $\bar{\varepsilon}$ must follow at each load step a given path (Fig.2).

ii) At each load step stresses are given by Eqs.(3).

iii) At each load step with given stresses, strains are obtained by means of the incremental flow theory of plasticity.

The results of this numerical procedure are given in the rows entitled as "MEAN TENSION THeoretical" of TABLE 1.

Torsion Experiments. The torsion specimens from both materials were thin walled tubes of identical dimensions with gauge length $l_o=1.2\times10^{-3}$m, outer radius $r_o=3.45\times10^{-3}$m and inner radius $r_i=3.0\times10^{-3}$m. A similar procedure as that described in the previous paragraph was employed. The flow theory of plasticity relations were used under plane stress conditions ($\sigma_1=-\sigma_2$, $\sigma_3=0$) in place of Eqs.(3). and the load path $\bar{\sigma}$-$\bar{\varepsilon}$ was the same as in tension up to the respective failure points. The results obtained are given, also, in TABLE 1 at the rows entitled as "TORSION".

CONCLUSIONS

By inspecting TABLE 1 the following conclusions can be made:

i) The critical value of the dilatational strain energy density T_V in tension is clearly greater than the (theoretically zero) T_V in torsion,

ii) T_V has an almost constant value for each material at the moment of failure, independently of specimen size,

iii) Critical distortional strain energy density T_D in torsion is clearly greater than T_D in tension. Consequently, both materials failed by "fracture" in tension according to the T-criterion because if they had failed by yielding then T_D had to have the same as in tension value, and

iv) Plastic work, w_p, cannot be considered as the critical quantity for failure because it increases by almost 100% from tension to torsion.

Hence, the two basic hypotheses of the T-criterion i.e. that:

i) Failure is caused by available (elastic) strain energy and not plastic work,

ii) The type of failure (fracture or yielding) is controlled by the available type of strain energy (T_V or T_D) seem to be considerably supported by experimental evidence.

APPLICATION

Plane stress fracture limit diagrams (FLD) are plotted in Fig.3 for the two materials studied. For that in Eqs(4) it was put:

$T_V=T_{V,o}=0.286$ MJoules/m^3, $T_D=T_{D,o}=0.970$ MJoules/m^3 for EN1A and

$T_V=T_{V,o}=1.655$ MJoules/m^3, $T_D=T_{D,o}=3.284$ MJoules/m^3 for EN24T.

The two steep lines correspond to failure by fracture of the respective material and the two elliptical ones (coinciding with the Mises ellipse) represent limiting strains for general yielding. Consequently, the working area for strains under plane stress conditions is enclosed by the solid lines in Fig.3. These lines pass through the torsion experimental points by definition. However, the respective tension experimental poins cannot be put in Fig. 3 since tension is not a plane stress state.

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	Spec→ Qnty↓	EN1A TENSION BIG	EN1A TENSION SMALL	EN1A TENSION MEAN	EN1A TORSION	EN24T TENSION BIG	EN24T TENSION SMALL	EN24T TENSION MEAN	EN24T TORSION
T	σ_o MPa	506.3	497.3	501.8	501.8	992.4	1067.	1029.7	1029.7
E		506.3	497.3	501.8	542.3	992.4	1067.	1029.7	1010.0
T	ε_o %	0.208	0.204	0.206	0.223	0.408	0.438	0.423	0.452
E		0.208	0.204	0.206	0.223	0.408	0.438	0.423	0.452
T	$\bar{\sigma}_f$ MPa	602.9	602.5	602.7	687.1	1146.	1219.	1183.	1264.
E		584.9	560.0	572.5	649.0	1180.	1220.	1200.	1154.
T	$\bar{\varepsilon}_f$ %	49.89	59.10	54.50	92.84	81.47	79.80	80.60	107.0
E		49.89	59.10	54.50	92.84	81.47	79.80	80.60	107.0
T	σ_1 MPa	683.1	704.7	693.9	396.7	1450.	1534.	1492.	729.9
E		662.9	640.2	651.6	374.7	1703.	1633.	1668.	666.0
T	σ_2 MPa	80.3	102.2	91.1	-396.7	303.4	314.4	308.7	-729.9
E		78.0	85.3	81.7	-374.7	394.6	404.7	399.7	-666.0
T	p MPa	281.2	303.0	292.0	0.0	685.5	720.8	703.0	0.0
E		273.0	270.3	271.7	0.0	830.7	814.0	822.4	0.0
T	ε_1 %	48.56	52.85	50.75	80.41	62.13	61.50	61.84	92.65
E		-	-	-	-	-	-	-	-
T	$-\varepsilon_2$ %	24.19	26.32	25.28	80.41	30.83	30.50	30.67	92.65
E		-	-	-	-	-	-	-	-
T	θ %	.1884	.2030	.1956	0.0	.4592	.4829	.4709	0.0
E		-	-	-	0.0	-	-	-	0.0
T	^TV MJ/m^3	0.265	0.308	0.286	0.0	1.574	1.740	1.655	0.0
E		0.228	0.224	0.226	0.0	2.112	2.027	2.070	0.0
T	^TD MJ/m^3	0.747	0.746	0.747	0.970	2.700	3.055	2.875	3.284
E		0.715	0.644	0.680	0.879	3.577	3.151	3.364	2.779
T	W_p MJ/m^3	267.6	288.9	278.6	550.2	657.7	695.4	676.7	1221.0
E		271.5	311.8	291.7	550.4	881.4	909.4	895.4	1197.1

TABLE 1. Experimental results. Values for E, ν for both materials are: $E=2.06\times10^5$ MPa, $\nu=0.27$. Slope, H of $\bar{\sigma}$-$\bar{\varepsilon}$ curve is H=200 MPa for EN1A and H=220 MPa for EN24T. "T" stands for theoretical and "E" for experimental values.

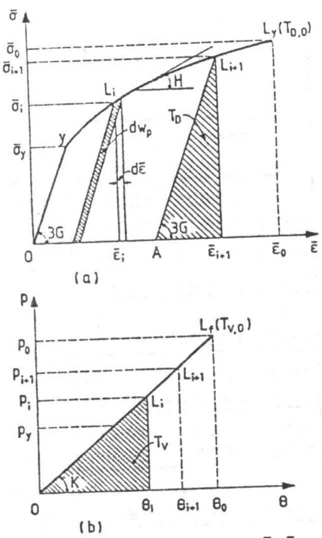

Figure 1 Typical curves $\bar{\sigma}$-$\bar{\varepsilon}$, p-Θ. Definition of symbols.

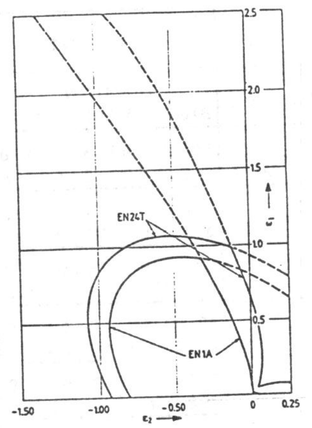

Figure 3 Plane stress fracture limit diagrams. T-criterion predictions.

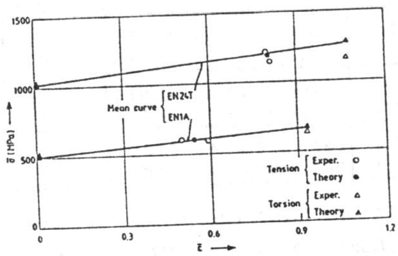

Figure 2 Experimental points and numerical simulation of curve $\bar{\sigma}$-$\bar{\varepsilon}$.