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A mechanical model of fatigue crack growth at low and intermediate ΔJ values is presented. The local stress and strain are calculated based on Rice solution. The required data for predicting fatigue crack growth rate can be found in standard material handbooks where cyclic and fatigue properties of the materials are presented.

INTRODUCTION

In low-cycle strain-controlled fatigue, cracks are initiated and propagated under general-yielding cyclic-loading. Dowling and Begley (1) and Dowling (2) correlated fatigue crack growth rate (FCGR) with, ΔJ . The value of J was evaluated as the rate of change of the strain energy density with respect to crack growth. Experimental data indicate that the (FCGR) behavior in metals follows the trend of the schematic (log-log) sigmoidal curve, wherein the FCGR curve da/dN is described as a function of delta J integral, ΔJ . The behavior of the FCGR is frequently described as a having three ranges. Range I is associated with the very slow FCGR behavior in the vicinity of the threshold value (ΔJ_{th}). Range II describes stable, subcritical FCGR behavior. Range III describes the behavior exhibited at very high FCGR where the mechanism of growth is influenced by onset of material fracture. An FCGR is frequently described analytically

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by the relation similar to the Paris-Erdogan (3) law in the form

$$da/dN = A (\Delta J)^m \quad (1)$$

where A and m are empirically determined constants. This equation has been observed to describe the behavior in the central range of the FCGR curve. A desirable feature of a crack propagation model under cyclic loading would be the incorporation of the fatigue properties in non-linear fracture mechanics. It is the subject of this paper to present such a model to determine FCGR.

A MODEL FOR FATIGUE CRACK GROWTH

In contrast to the monotonic loading, in the cyclic loading due to repeated loading and unloading of the plastic zone in the front of the crack, a crack-tip blunting occurs. Furthermore, microcracks are produced in the plastic zone, or more particularly in the front of the crack in generally termed damage process zone, δ . Thus, special care has to be taken in analyzing this region when continuum theories are used to determine the stress and strain distribution ahead of the crack. Since the fatigue damage is generally caused by the cyclic plastic strain, the plastic strain energy plays an important role in the damage process. Therefore, to describe the damage process in the front of the crack we should apply the fatigue criterion based on plastic strain or plastic strain energy density. In the present paper the criterion based on total strain energy density is adopted (Golos (4), Golos and Ellyin (5)). Since the process of damage in the process zone is controlled by plastic strain range the fatigue criterion can be expressed in the form:

$$\Delta W^P = \zeta (2N_f)^\alpha \quad (2)$$

where ζ and α are material parameters. In the cases where the values of ζ and α are not available, the approximated relationships to compute them can be presented as

$$\alpha \approx b + c \quad ; \quad \zeta \approx 4\sigma_f' \epsilon_f' \frac{1 - n'}{1 + n'} \quad (3)$$

where n' is the cyclic strain-hardening exponent, $(\sigma'_f/E$ and ϵ'_f are the strain amplitudes corresponding to the elastic and plastic intercept for one cycle, b is the fatigue strength exponent and c is the fatigue ductility exponent.

Assuming that the plastic strain components at each point within the plastic zone remain proportional to each other based on the Rice (6) solution the stress and strain distribution ahead of the crack tip, for plane strain small scale yielding can be expressed as follows:

$$\Delta\sigma(x) = \sigma'_y \left[\frac{E\Delta J}{(1+n')\pi(\sigma'_y)^2(x+r_c)} \right]^{n'/1+n'} \quad (4a)$$

$$\begin{aligned} \Delta\epsilon(x) = \frac{\sigma'_y}{E} \left[\frac{E\Delta J}{(1+n')\pi(\sigma'_y)^2(x+r_c)} \right]^{n'/1+n'} + \\ + \epsilon'_y \left[\frac{E\Delta J}{(1+n')\pi(\sigma'_y)^2(x+r_c)} \right]^{1/1+n'} \end{aligned} \quad (4b)$$

where x is the distance from the crack tip, σ'_y and ϵ'_y are the cyclic yield stress and strain respectively, ΔJ is the delta J integral, and r_c is the radius of the blunted crack tip.

The plastic strain energy density distribution ahead of the crack tip is given by

$$\Delta W^P = \frac{E(1-n')}{(1+n')\pi(x+r_c)} \Delta J \quad (5)$$

The plastic strain energy density within the process zone may be calculated from equation (5) setting by $x=\delta$.

The corresponding number of cycles, ΔN required for the crack to penetrate through δ can be determined from equation (2).

Substituting from equation (2) into (5) the crack growth rate per cycle, da/dN is therefore can be estimated as follows:

$$\frac{da}{dN} = \frac{\delta}{\Delta N} = \frac{\Delta J}{\zeta \frac{(1+n')^2}{(1-n')} \pi} \frac{2\Delta N^{-\alpha}}{\Delta N} - \frac{r_c}{\Delta N} \quad (6)$$

The r_c can be calculated, assuming that for $\Delta J = \Delta J_{th}$, $da/dN=0$. The experiments show unstable crack growth when the range of the stress intensity range approaches the critical value, i.e. $\Delta J = \Delta J_c$. Thus translates to the crack growth through the process zone at the instant loading. Then putting $2\Delta N=1$ in the equation (6) we can calculate the value of δ , as follows:

$$\delta = \frac{\Delta J_c - \Delta J_{th}}{\zeta \frac{(1+n')^2}{1-n'} \pi} \quad (7)$$

Rearranging equation (6), the FCGR can be described as:

$$\frac{da}{dN} = 2\delta \left[\frac{\Delta J - \Delta J_{th}}{\zeta \frac{(1+n')^2}{1-n'} \pi \delta} \right]^{-1/\alpha} \quad (8)$$

In the case when $\Delta J_{th} \ll \Delta J$, putting

$$m = -2/\alpha$$

$$A = 2\delta / \left[\zeta \frac{(1+n')^2}{(1-n')} \pi \delta \right]^{-1/\alpha} \quad (9)$$

we obtain the relationship (1).

COMPARISON WITH EXPERIMENT AND DISCUSSION

In the present study the experimental data for 1018 steel (7) and A 533 B steel (8) were used. In the analysis the values of ζ and α were calculated from equation (3). Therefore, substituting equation (3) into

equations (8) we obtain the following relationship for predicting the FCGR

$$\frac{da}{dN} = 2\delta \left[\frac{\Delta J}{4\sigma_f' \epsilon_f' (1+n') \pi \delta} \right]^{-1/b+c} \quad (10)$$

The experimental and theoretical results for these materials in full lines are shown in Fig.1. The model developed herein indicates that constants A and m in the empirical equation (1) are mutually dependent. Furthermore, the required data can be found in the material handbooks where fatigue properties of materials are listed.

Predictions of the proposed model are in good agreement with the FCGR, especially at low and intermediate values of ΔJ of those reported materials.

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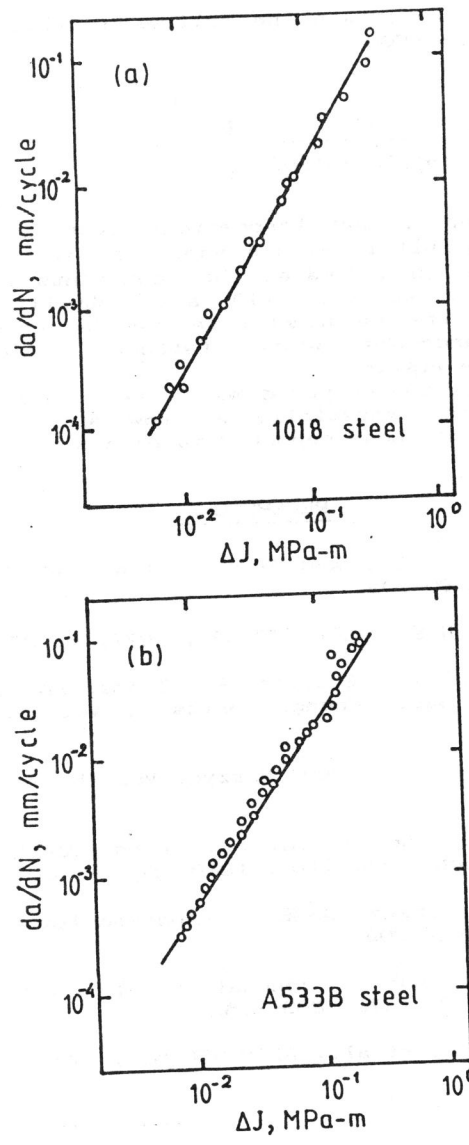


Figure 1 Correlation of da/dN versus ΔJ for 1018 steel (a) and A533B steel (b).