

# ESTIMATION OF INDUCED ANISOTROPY RESULTING FROM ORIENTED PLASTIC DEFORMATION AND DAMAGE

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A new method for separating the effects of plastic and damage induced anisotropy is proposed. Description model of the process is done. The model parameters are identified for copper.

## INTRODUCTION

It is well known that in the material of non-elastic metallic bodies formation of defects on two levels is observed: microdefects in the crystal lattice (vacancies, interstitial atoms, dislocations, stacking faults) and mesodefects along the boundaries of grains and phases (micropores and microcracks) (1). On macroscale it appears as if two opposite processes occur: plastic hardening and damage softening.

The orientation of the external stresses causes arrangement of defects which on macrolevel induces anisotropy in the non-elastic properties of the material. An important problem of material mechanics is to estimate separately the influence of anisotropic plastic hardening and anisotropic damage softening. Thus prediction of macrofracture of the material as a critical accumulation of mesodefects would be possible.

This paper offers a method for macroestimation of the both non-elastic processes, shows the possibility of their modelling and gives the way to

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determine the parameters of the presented model. The method is applied to copper, 99,91 % pure. Quasi-static and isothermal processes when damage begins in strongly developed plastic deformation conditions are examined.

### METHOD OF ESTIMATION

The method presented for estimation of oriented plastic deformation and damage is based on two main procedures:

#### **1. Heat treatment**

In strongly developed plastic deformation conditions, when initiation of defects on micro and mesolevels is possible, the sample body in unloaded and heat treated in the pre-crystallization range. For example, for 99,91 % pure copper it was established by metal physics examinations that the body should be isothermally heated for one hour at 290°C after unloading. This corresponds to the data reported for copper of different levels of purity (1). In these conditions processes of relaxation mainly run. Recrystallization is observed only in isolated microvolumes and its influence on the properties on macro level is negligible. Oriented microdefects in crystals are removed by heat treatment as much as to eliminate anisotropy in plastic hardening on macro level. Heat treatment does not eliminate mesodefects. That is why the remaining anisotropy is mainly due to oriented microdefects in the crystal grains and intercrystal areas (Fig.1). Thus separation of the anisotropic effects due to plastic deformation and damage is achieved.

#### **2. Method of deformation anisotropy determination**

Well known procedure of induced anisotropy determination in non-elastic properties of metals is applied (2). Metal sheets are subjected to uniaxial tension causing strong preliminary non-elastic strain (25 to 35 % for copper) without reaching localization of plastic macro-deformation. The plates are unloaded and small samples for one-dimensional tension are cut from them at various  $\alpha$ -angles (See Fig.2). By subsequent tension test the yield strength values for the various  $\alpha$ -directions are determined. Thus the induced anisotropy is determined from the joint action of plastic deformation and damage. To some of the plates heat treatment is applied according to 1. before the small samples were cut. By their testing the deformation anisotropy is determined only from the damage. The presence of initial anisotropy in the plastic properties of the metal is established with the help of not preliminary deformed plates.

By combining procedures 1. and 2. the estimation of induced anisotropy resulting from the development of the two types of defect producing micromechanisms is realized. The experimental data obtained can serve as a basis to determine the parameters of the mechanical-mathematical model

presented in Section 3 (See Section 4).

### 3. Mechanical-mathematical model of the twinned processes

Unlike the reported models (See 3, 4, 5, 6, 7) the present model reasonably describes a non-linear anisotropic behaviour of the metal. The model is an extension to that presented in (5) and is based on the following assumption:

(1) Up to date Lagrangian description which impedes the processing of the experimental data but naturally connected with convenient digital simulation of the processes is assumed (6). Cartesian coordinate system  $O x_1 x_2 x_3$  is introduced. The measures of the process:  $V_i(x_k, t)$  - velocity vector;  $\sigma_{ij}$  - stress tensor;  $\dot{\epsilon}_{ij}$  - strain rate tensor;  $x_k \in S_t$  (area occupied by the body for moment  $t$ );  $t \in [t_0, t_1]$ ,  $t_0$  - beginning,  $t_1$  - end of the quasistatic and isothermal process considered ( $i, j = 1, 2, 3$ ).

(2) The deformation of the body is rigid-plastic, i.e.  $\dot{\epsilon}_{ij} \approx \dot{\epsilon}_{ij}^a$  where  $\dot{\epsilon}_{ij}^a$  is the velocity tensor of non-elastic strains. It consists of a part  $\dot{\epsilon}_{ij}^p$  as a result of plastic deformation and part  $\dot{\epsilon}_{ij}^d$  as a result of the damage, i.e.  $\dot{\epsilon}_{ij}^a = \dot{\epsilon}_{ij}^p + \dot{\epsilon}_{ij}^d$ .

(3) The material is plastically incompressible ( $\dot{\epsilon}_{kk}^p = 0$ ) and has residual change of volume as a result of the micropores ( $\dot{\epsilon}_{ij}^d = 0$ ). It is determined by the relationship

$$\dot{\epsilon}_{ij}^d = f(\sigma_M) \dot{\sigma}_M, \quad \dot{\sigma}_M = 1/3 \delta_{ij} \dot{\sigma}_{kk}, \quad \sigma_M \geq \sigma_{D^0} \quad (1)$$

where  $\sigma_{D^0}$  is the mean stress limit  $\sigma_M$ , above which micropore formation starts;  $f(\sigma_M)$  is a constitutive function which is experimentally determined;  $\delta_{ij}$  is Kronecker's symbol;  $\dot{\sigma}_{ij}$  is the stress rate tensor.

(4) The variation of the deviatoric part of the non-elastic strain is defined by the deformation law:

$$\dot{\epsilon}_{ij}^a = \dot{\Lambda}_{(a)} \frac{\partial F(a)}{\partial \sigma_{ij}}, \quad \dot{\Lambda}_{(a)} = \begin{cases} 0, & \text{if } F(a) < 0, \text{ or } F(a) = 0, \dot{\lambda}_{(a)} < 0 \\ > 0, & \text{if } F(a) = 0, \dot{\lambda}_{(a)} > 0 \end{cases} \quad (2)$$

where  $\dot{\Lambda}_{(a)}$  is determined by the condition  $\dot{F}(a) = 0$ ;  $F(a) = 0$  is the condition of non-elastic deformation;  $\dot{\lambda}_{(a)}$  is the loading-unloading criterion.

(5) The variation of the deviatoric part of the plastic strain is determined by deformation law:

$$\dot{\epsilon}_{ij}^p = \dot{\Lambda}_{(p)} \frac{\partial F(p)}{\partial \sigma_{ij}}, \quad \dot{\Lambda}_{(p)} = \begin{cases} 0, & \text{if } F(p) < 0, \text{ or } F(p) = 0, \dot{\lambda}_{(p)} < 0 \\ > 0, & \text{if } F(p) = 0, \dot{\lambda}_{(p)} > 0 \end{cases} \quad (3)$$

where  $F_{(p)} = 0$  is the condition of plasticity,  $\mathcal{L}_{(p)}$  is determined by the condition  $F_{(p)} = 0$  and  $\mathcal{L}_{(p)}$  is the loading criterion in the process of plastic deformation.

(6) The condition  $F_{(l)} = 0, (l = a, p)$  is assumed in the form of:

$$F_{(l)} = \bar{\sigma}_{eq(l)}^2 - \bar{\sigma}_{Y(l)}^2 = 0 \quad (4)$$

where

$$\begin{aligned} \bar{\sigma}_{eq(l)} &= \sqrt{3/2 N_{ijkl}^{(l)} \bar{S}_{ij}^{(l)} \bar{S}_{kl}^{(l)}} \quad (i, j, k, l = 1, 2, 3) \\ \bar{S}_{ij}^{(l)} &= S_{ij} - S_{ij}^{(l)}: \quad S_{ij} = \sigma_{ij} - 1/3 \delta_{ij} \sigma_{kk} \\ N_{ijkl}^{(l)} &= I_{ijkl} + A_{(l)} S_{ij}^{(l)} S_{kl}^{(l)} \quad \dot{S}_{ij}^{(l)} = Q_{(l)} \dot{\epsilon}_{ij}^{(l)} \\ I_{ijkl} &= 1/2 (\delta_{ik} \delta_{jl} + \delta_{jk} \delta_{il}) \\ \gamma &= \int_{t_0}^t \dot{\gamma} dt, \quad \dot{\gamma} = \sqrt{1/2 e_{ij}^a l_{ij}^a}, \quad e_{ij}^a = \epsilon_{ij}^a - 1/3 \delta_{ij} \epsilon_{kk}^a \end{aligned} \quad (5)$$

The constitutive functions  $A_{(l)}, \bar{\sigma}_{Y(l)}, Q_{(l)}, (l = a, p)$  depend on  $\gamma$  and are defined according method (II) (See Section 4).  $F_{(a)} = 0$  is directly registered from the joint display of plastic deformation and damage (by  $\bar{\sigma}_{eq}^{(a)}$  and  $\bar{\sigma}_{Y(a)}^*$ ).  $\bar{\sigma}_{eq}^{(d)}$  and  $\bar{\sigma}_{Y(d)}^*$  are experimentally determined after the heat treatment. On this basis  $\bar{\sigma}_{eq}^{(p)} = \bar{\sigma}_{eq}^{(a)} + \Delta \bar{\sigma}_{eq}^{(d)}$ ;  $\Delta \bar{\sigma}_{eq}^{(d)} = \bar{\sigma}_{eq}^{(d)} - \bar{\sigma}_{Y(d)}$ ,  $\bar{\sigma}_{Y(d)} \neq \bar{\sigma}_{Y(d)}^*$  is calculated. The surface  $F_{(p)} = 0$  is fictitious: when along with plastic deformation a process of damage develops.  $S_{ij}^{(l)}$  are internal parameters of the metal state and  $S_{ij}^{(l)}$  are the active stresses of both processes (7).

#### 4. Application of the method. Identification of the material functions.

In order to illustrate the method presented in Section 2 and to indicate the reasonableness of the model given in Section 3 experiments with 99,91 % pure copper are carried out. The initial plates have dimensions 250x110 mm and the small samples have dimensions 4,20x9,0x25 mm cut at an angle of  $\alpha = 0^\circ, 22^\circ 30', 45^\circ, 67^\circ 30', 90^\circ$ . Preliminary residual strain in the direction of  $0 \times 1$  (See Fig.2) having values of 25 %, 30 % and 35 % was realized in the plates. Intensities of  $\gamma = 21,62 \%, 25,95 \%$  and  $30,27 \%$  correspond to it. The results obtained are presented in Fig.3 by polar diagram ( $\sigma_1, \alpha$ ).  $\sigma_1$  is determined by the yield strength at pure tension (at 0,2 % residual strain) for the small samples, heat treated and not heat treated. The experimental results in Fig.3 present graphically the developing deformation anisotropy in plastic deformation and damage. By the accuracy of the experiment it can be assumed that there is no initial anisotropy of the non-elastic properties.

For identification of the material functions of the model in Section 3:

$A_{(l)}(\gamma)$ ,  $Q_{(l)}(\gamma)$ ,  $\sigma^* \gamma_{(l)}(\gamma)$ , ( $l = a, p$ ) the approach announced in (7) is applied. The experimental points in the polar diagram ( $\sigma, \alpha$ ) are approximated to ellipses when they correspond to the conditions  $F_{(l)} = 0$ :

$$F_{(l)} = r_{(l)} [a_{(l)}^2 + a_{(l)} L_{1\alpha} \sigma_1 + 1/4 L_{1\alpha}^2 \sigma_1^2] + L_{2\alpha} \sigma_1^2 - \tau_{(l)}^2 = 0$$

where

$$r_{(l)} = 1/3 + 2/9 A_{(l)} \cdot a_{(l)}^2, \quad a_{(l)} = \sqrt{3} Q_{(l)} \cdot \gamma,$$

$$\tau_{(l)} = 1/\sqrt{3} \sigma \gamma_{(l)}^*, \quad L_{1\alpha} = 1 - 3\cos^2\alpha, \quad L_{2\alpha} = (\sin^2\alpha/4)(1 + 4\cos^2\alpha).$$

The approximation to ellipses indicates also the possibility of the theoretical model in Section 3 to describe reasonably the behaviour of the metals in which the two processes considered take place.

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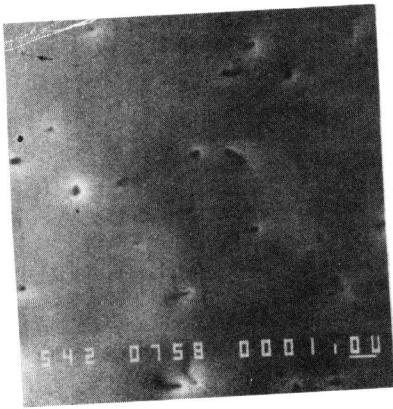


Fig.1. Micropores in heat treated sample, SEM , x5400

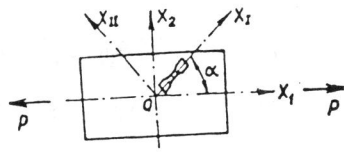


Fig.2. Experimental samples.

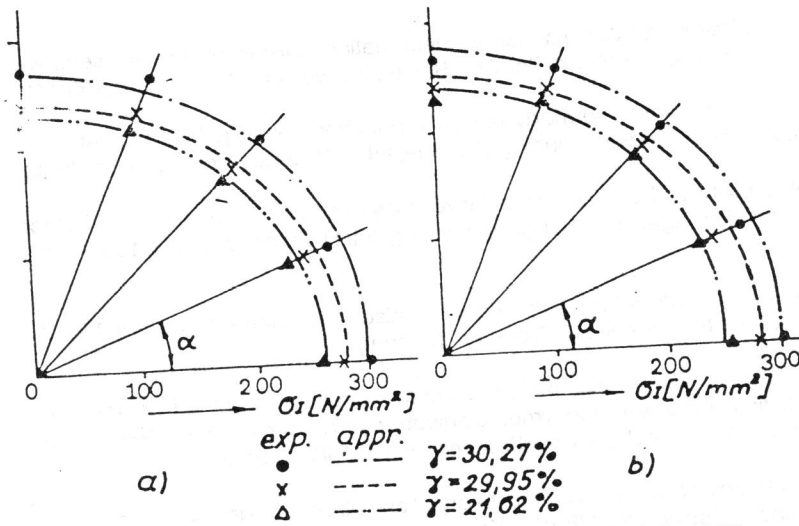


Fig.2. Experimental and theoretical approximation of yield stresses  
 a/ Yield conditions  $F(a) = 0$   
 $\sigma_{Y(a)} = 1.093 + 0.834\gamma - 0.000874\gamma^2$  [N/mm<sup>2</sup>]  
 b/ Yield conditions  $F(p) = 0$   
 $\sigma_{Y(p)} = -1.918 + 0.322\gamma - 0.00533\gamma^2$  [N/mm<sup>2</sup>]