DETERMINATION OF REMAIN LIFE OF SHORT AND LONG FATIGUE CRACK ON GEARS

B. Aberšek*, S. Sedmak** and J. Flašker*

During the initiation as well as during the further crack propagation the crack opening displacement and/or plastic deformation of the crack tip play a very important role. For this purpose we developed our own algorithm which we use for calculating the crack propagation up to destruction from the initial crack which we cannot detect. In the algorithm we used for short crack calculation the modified theoretic model of Bilby, Cottrell and Swinden (BSC). For long cracks calculation in this algorithm we use the model based on the probabilistic fracture mechanics.

INTRODUCTION

The problem of fatigue of various dynamically loaded components is a demanding engineering problem. If a component is also geometrically complex as, for instance, gears, the problem becomes more outstanding. In this case the stress/strain fields near the top were determined as follows :

- experimentally, by applying the photo elastic method and/or by means of strain gauge,
- by means of the FEM method and/or program package NISA II,
- with program STATFAG.

We calculated the service life of gears for the following cases:

- crack propagation up to destruction from the initial which we cannot detect. In the algorithm we used the modified theoretic model of Bilby, Cottrell and Swinden (BSC) and/or the Taylor's analysis for determination of the plastic zone at the tip of short cracks on the basis of which we determine the speed of the crack propagation and and/or its length (1). For long cracks calculation in this case we use the algorithm based on the probabilistic fracture mechanics (3).

* Faculty of Technical Science, University of Maribor
** Faculty of Technology and Metallurgy, University of Belgrade

- crack propagation from the size which we can detect by non-destructive testing methods, e.g. ultrasonic testing.

MATHEMATICAL MODEL OF CRACK PROPAGATION

To be able to determine the properties of very short cracks it must be kept in mind that in the analytical models describing the S-N curves it is necessary to take into account the dominant properties of the fields of the crack tip as well as the crack growth rate which is high in the beginning but then decreases (2). This can be reached by considering the separate regimes in the Kitagawa-Takahashi's diagram which is shown in Figure 1. A major difficulty however is defining of the boundaries of these regimes since in many materials the cyclic hardening and/or softening takes place.

Monte Carlo (MC) method

As a tool for describing the phenomena in the experimental mathematics, the MC method will be used. The MC simulation is in wide use in generation of the numerical results for the probabilistic fracture mechanic (PFM) models. The versatility and simple applicability are the reasons why the MC simulation is particularly suitable (4).

Basic principle of Monte Carlo Method. Any calculation made by the MC method, whose results are the quantitative data, can be considered to be the solving of the multiple integral. It is assumed that no calculations require more than N random numbers (e.g. 10^{10}). The results, i.e., the vector values will be the function:

$$R\ (\ \xi_1,\ \xi_2, \ldots \xi_N) \tag{1}$$

of the sequence of random numbers ξ_1, ξ_2, $\xi_3 \ldots$ so that the solution multiple integral can be simulated:

$$\int_0^1 .. \int_0^1 .. \int_0^1 R(x_1\ , \ldots\ldots,\ x_N) dx_1 \ldots\ldots dx_N \tag{2}$$

The method of solving such integral is not always the simplest, but it can always be used as a basis for further solving and/or for introduction of various MC techniques and/or other numerical methods as that generalization of the use is achieved. This integral is simplified in particular cases into the suitable form.

Model of propagation of short crack

For solving the problem of propagation of short cracks and/or determination of the size of the plastic zone connected with the lines of the slip we applied the theory of continuously distributed dislocations and/or the modified BSC model (1).

Under the assumption of continuously distributed infinitesimal dislocations, the equilibrium equation becomes a singular integral equation of the Cauchy's type with unknown distribution function. In general this integral equations have the form :

$$\int_D \frac{R(x)}{x - x_o} dx = \frac{P(x_o)\, 2\pi\, (1 - \nu)}{G\, b} \tag{3}$$

There are two different solutions of this equation, depending on whether a bounded or an unbounded solution function at the ends of the plastic zones is considered. The result is plastic displacement at the tip of the crack as a function of stress intensity factor given as follows:

$$\phi = \frac{b}{\pi^2 A} \frac{(1 - n^2)^{1/2}}{\tau} K^2 \tag{5}$$

Where n=a/c gives the dimensionless position of the crack tip, σ_f is the friction stress and τ is the applied stress.

By assuming the crack growth rate to be proportional to ϕ, the present theory described the model for its calculating:

$$\frac{da}{dN} = M(\Delta\phi)^m \tag{6}$$

where $\Delta\phi=\phi_{max}-\phi_{min}$; ϕ_{max} and ϕ_{min} are the crack tip displacements. Therefore, lifetime calculations are carried out with the following expression :

$$\Delta N_i = \int_{a_{j-1}}^{a_j} \frac{1}{M(\Delta\phi)} da_m \tag{7}$$

Model of long crack propagation

Deterministic models were extended with the statistical variability of the ratio of the crack growth X(t). If it is applied as a:

$$\frac{da(t)}{dt}=X(t)L(\Delta K,K_{max},R,S,a) \tag{8}$$

random variable it is possible to write , where X(t) is the positive stationary stohastic log normal random process as follows.

By integrating the equation (8) from 0 to t the crack size distribution a(t) is obtained. For solving this problem in our program package STATFAG (3), we used the simulation method Monte Carlo (4). The positive stationary stohastic log normal random process is simulated with the quick Fourier's transformation technique.

Loadings on gears are most frequently variable amplitudes. Therefore we incorporated in our program STATFAG (3) the possibility of generation of the loading by means of a mathematic model of a random gearing. In addition ,the loading due to residual stresses (5) are taken into account in the program.

EXPERIMENTAL ANALYSIS

For application in the probabilistic fracture mechanics the equation for the entire service life of a mechanical part, i.e., for the time of the initiation and subsequent crack propagation, can be written:

$$N = N_i(D,M,\Delta\phi) + N_p(B,\Delta\sigma,A,C) \tag{9}$$

For accurate simulation and for addition the MC method will be used.

For the experimental analysis we applied the mixed method, i.e., the photoelastic method of measuring the stresses by means of strain gauges (7). A specially made gear test piece was placed on a hydraulic testing machine INSTRON by means of a specially made clamping device as shown in (7). On one side the bearing tooth of the gear test piece was glued with special plastic material, while on the opposite side we placed a strain gauge HBM type DMS 1.5/120 LY11 in the critical cross section of the tooth and/or near the crack tip.

The test piece was continuously loaded and after each relieving the state of residual stresses, i.e. plastification was observed.

RESULTS - DETERMINATION OF (a - N) CURVES

Our algorithm was verified by using the FEM method. High-order singular elements were used in calculations. On the basis of the results the stress intensity factors and the direction of the crack propagation (6) were obtained and service life of gears was calculated accordingly. In Figure 2 a good accordance of the calculations made by the program STATFAG, the FEM method and experiment can be seen.

CONCLUSION

For the used analysis of the gear drive in mathematic modeling we tried to simulate as precisely as possible the actual conditions during operation. Researches have shown that in addition to the primary loading it is necessary to take into account also the secondary loading for heat treated gears. The most important are the residual stresses.

Of course, the results of the numerical analysis are not satisfactory, if they are not confirmed by the experimental results. Comparisons of the results of the numerical analysis and the experiments show good accordance and confirm that the proposed model is good and can be successfully used in practice for estimating the crack propagation rate and service life of gears.

REFERENCES

(1) BILBY B.A., COTTRELL A.H. in SWINDEN K.H.: The spread of plastic yield from a notch, Proc. Royal Soc. 272, 1963

(2) MILLER K. J. : The Behavior of Short Fatigue Crack and their Initiation, Part I - FFEMS, Vol. 10, No. 1, 1987 and Part II - FFEMS, Vol. 10, No. 2, 1987

(3) FLAŠKER J., ABERŠEK B. and JEZERNIK A. : Numerical Model for the Determination of Service Life of Gear, Proceedings of the Symposium on Finite Element Methods FEMSA 92, South Africa

4. HAMMERSLEY J.M. and D.C. HANDSCOMB: Monte Carlo Methods, Chapman and Hall, London, New York, 1983

5. FLAŠKER J., ABERŠEK B., JEZERNIK A.: Defining the residual stresses by applying the FEM, Comm. in Applied Numerical Methods, Vol.7, 589-594, 1991

6. ABERŠEK B., MIKLUŠ S., FLAŠKER J. and LAKOTA M.: Application of Probabilistic Fracture Mechanics Theory to Calculation of Life of Gears, Proceedings of the European Conference on Fracture - ECF-8, Torino, Italia, 1990

7. FLAŠKER, J. and ABERŠEK, B.: Calculation of Gears by Finite Element Method Considering the Gear Tooth Root Errors, Proceedings of Numeta - 90, Swansea, G.B., 1990.

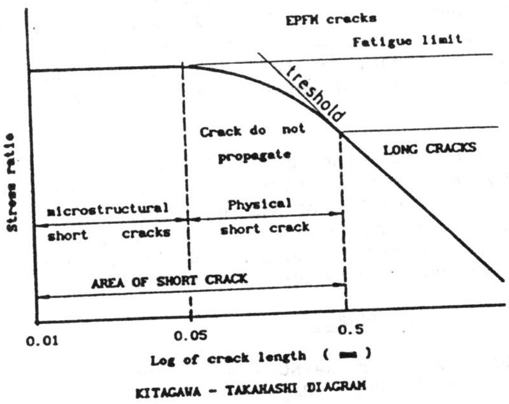

Figure 1 Three regimes of properties of short crack

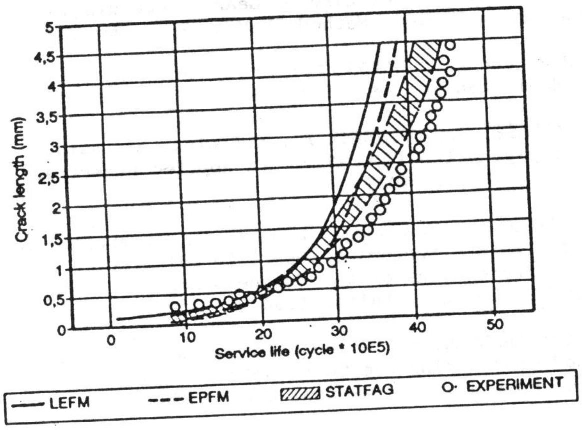

Figure 2 Crack growth curves