

M. FRANÇOIS*; M. KEYVANI*; D. BAPTISTE*; D. FRANÇOIS*.

A micromechanical damage model of a S.M.C.- type composite is developed and the results are compared with both classic and ultrasonic testings. Based on Mori and Tanaka's theory, the model predicts stiffness tensor of the material damaged or not, as well as the stress strain curve and the damage mechanisms, up to the point of failure.

INTRODUCTION

In order to predict the evolution of the elastic properties of a composite during damage, such as the S.M.C.(figure 1), we developed a micromechanical approach based on Mori and Tanaka's model. The only data needed are the geometrical parameters (volume fraction, orientation, aspect ratio of the reinforcement), the microstructural parameters (mechanical behavior of each phase, including interphase strength) and the kind of the external applied stress tensor.

Taking into account three local damage criteria, namely slipping, decohesion at the interface, and fiber fracture, we determined the applied stress tensor for several damage levels as well as the location and the nature of this damage. The damaged behavior is then modeled, assuming fictive properties for the debonded fibers.

The stiffness tensor of the damaged composite has been measured using classic and ultrasonic methods and compared to the theoretical ones.

* Laboratoire MSS Mat, Ecole Centrale de Paris. CNRS URA 850 Grande Voie des Vignes 92290 Chatenay-Malabry (France).

Description of the material

The Sheet Moulding Compound (figure 1) is a composite material with randomly oriented fibers in the plane.

Reinforcements are fiberglass, the fibers are cut with a length of 2,5 cm for a diameter of 10 µm; the aspect ratio L/d can then be considered as infinite.

Matrix is polyester added with 33% volume of chalk CaCO₃ (to add viscosity while the moulding).

Matrix Modulus Em	7 GPa	Fiber Volume Fraction f	0,32
Matrix Poisson's ratio νm	0,35	Fiber Massic Fraction	0,42
Fiber Modulus Ef	73 Gpa	Matrix strength Rm	55 MPa
Fiber Poisson's ratio νf	0,28	Fiber strength Rf	1500 MPa

Isotropic or sinusoidal plane fiber repartition.

The interphase strength is taken as the matrix one. Interphase shear strength τm is chosen equal to the half of the matrix strength Rm.

This material is essentially used for automotive engineering (body parts).

Modeling

The model is based on the Mori and Tanaka's theory (1). The stiffness tensor is given by:

$$C = C_m [I + f Q (I + N)^{-1}]^{-1}$$

where fQ and N are the average value of Qi and (Si-I)Qi on all the n families of reinforcement Ωi:

$$f Q = \frac{1}{V} \sum_{i=1}^n \int_{\Omega_i} Q_i dV$$

$$N = \frac{1}{V} \sum_{i=1}^n \int_{\Omega_i} (S_i - I) Q_i dv$$

V is the elementary representative volume (on which Σ, external stress tensor is constant and who contains a statistically representative quantity of different families of reinforcement)

$$Q_i = [(C_m - C_f) (S_i - I) - C_f]^{-1} (C_m - C_f)$$

Where C_m is the stiffness tensor of the matrix, C_f the stiffness tensor of the fibers and S_i the Eshelby's tensor who depends only on ν_m in our case of isotropic matrix. I is the identity tensor.

Using Eshelby's equations, we obtain stress tensor σ_i in each reinforcement Ω_i as a function of the arbitrary external stress tensor, Σ .

$$\sigma_i = C_m (\bar{E} + (S_i - I)) e_i^*$$

$$e_i^* = Q_i (E_o + \bar{E})$$

$$\bar{E} = - (I + N)^{-1} N E_o$$

$$E_o = C_m^{-1} \Sigma$$

Where e_i^* is the eigenstrain, E_o the strain tensor of pure matrix under Σ and $\bar{E} + E_o$ the strain tensor of matrix in the composite under Σ .

Otherwise, the continuity conditions at the interface (Mura (2)) (seen as a bidimensional adhesive) allow us to calculate σ : normal stress and τ : tangential stress around the equator of the reinforcement (figure 2)

$$\vec{T}(M, \vec{n}) = (\Sigma + \sigma_i) \cdot \vec{n}$$

Three local criteria are checked: decohesion and slipping at the interface, and fiber fracture (with the Lamé's criterion: maximum principal stress of σ_i : σ_{p1}). These are compared with their maximum values (figure 3) and, as we are in linear mechanics, we deduce the external tensor $\lambda_1 \cdot \Sigma$ for which we shall have the first damage. In the same time we get the nature of this damage.

For example, for a SMC under tensile load we predict a first damage mechanism by sliding of the fibers at 45 degrees to the load at a stress level nearly equal to that of the matrix.

The debonded fiber is then replaced by pure matrix. This approximation is loosely justified by the measured values of the slipping stress in pull-out testings. The damaged composite is then replaced by this "new" composite, where the debonded family of reinforcement does not exist anymore, and is modeled as previous. If the stress level $\lambda_2 \cdot \Sigma$ is under the previous level $\lambda_1 \cdot \Sigma$ we consider that we have a cascade of damage.

Tensile test simulations show some fiber decohesion cascades from 45 to 90 degrees to the direction of tension. The last stage of damage is associated with the rupture of the near to 0 degree oriented fibers. The entire stress - strain curves have been successfully simulated (figure.4). Simulations have been done using fibers non isotropic distribution (sinusoidal, with plus or minus 15% of fibers, as observed) showing the weak influence of this parameter.

Experimental evaluation

The elastic properties can be determined from the measurement of the velocities of ultrasonic wave propagation (Roux et al (3)). An analysis of the propagation of elastic waves through an anisotropic medium results in the Christoffel's equation:

$$\det (C_{ijkl}n_jn_k - \rho v^2\delta_{il}) = 0$$

In this equation C_{ijkl} are the elastic constants of the material. The direction cosines of the wave propagation direction are specified by n_j and n_k , ρ is the density of the material, v are the wave speeds of the three bulk wave propagation through the material and δ_{il} is the Kroneker symbol. With wave speed data, this equation can be inverted to determine the elastic constants of the material.

We have developed an ultrasonic technique (ultrasonic immersion method) for the determination of the stiffness tensor of anisotropic materials. We can compare them to the computed results.

	C33	C11	C13	C55	C44	
Ultrasonic	23,6	15,6	6,6	4,7	6,4	GPa
Computed	24,5	16,9	8,36	4,33	6,95	GPa

On the basis of theoretical studies, the damage may give rise to a reduction in ultrasonic velocities. In order to measure and characterize the level of anisotropic damage of our material under loading, using a test machine combined with the ultrasonic immersion method, we investigated the variation in wave velocities with damage evolution. Using these experiments we were able to evaluate the evolution of the different stiffness constants with the applied load. This ultrasonic device allowed us to characterize the stiffness tensor during uniaxial tensile test (figure 5).

CONCLUSION

The model, classic and ultra-sonic testing and are in a good agreement in the elastic domain. We can notice that the damaged material is stiffer than the modeled one. This certainly provides from the model used for the debonded fiber, assumed to be some matrix. We are currently developing another theory, using anisotropic reinforcements to model debonded fibers.

REFERENCES

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- (3) Roux, J; Hosten, B; Castagnède, B; Deschamps, M. Caractérisation mécanique des solides par spectro-interférométrie ultrasonore. *Revue Phys. Appl.*, **20**, 351-358.

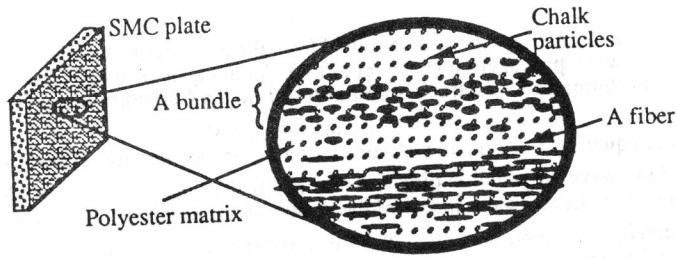


Figure 1: Structure of the SMC.

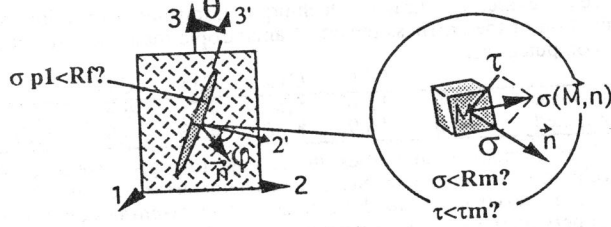


Figure 2: Three local damage criteria checked.

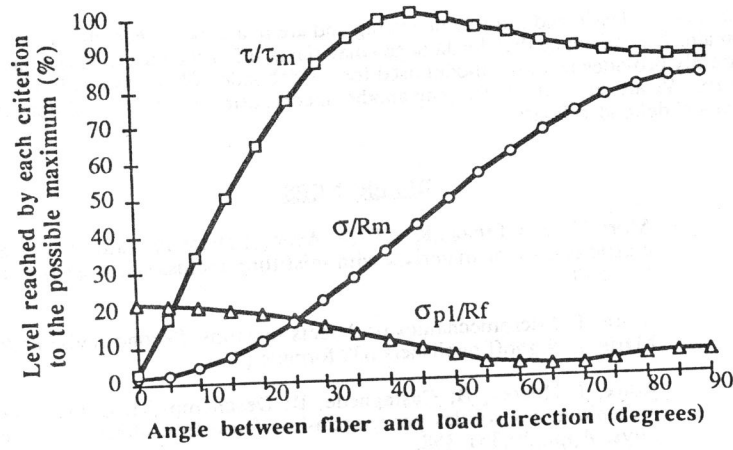


Figure 3: The evolution of the three criteria at first step of calculation (undamaged material) as a function of the angle between the load and the fiber.

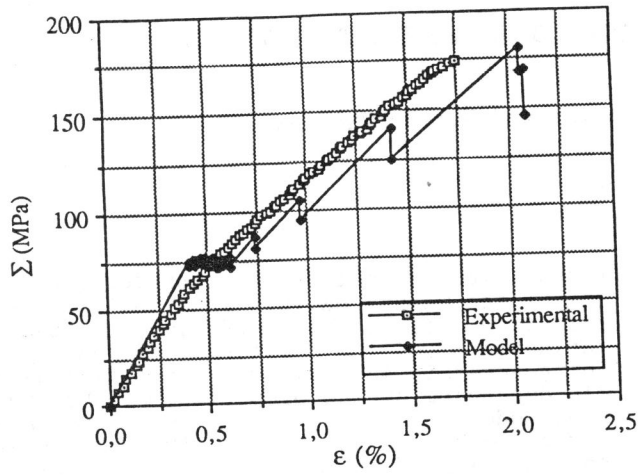


Figure 4: Simulated vs classic (strain gage) tensile testing.

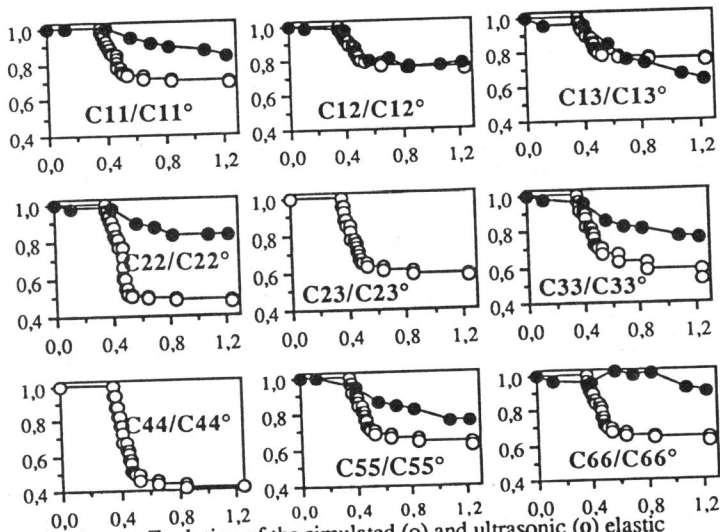


Figure 5: Evolution of the simulated (o) and ultrasonic (o) elastic stiffness tensor components as a function of the applied strain.