COMPUTATIONAL SIMULATION OF NON–COPLANAR CRACK GROWTH AND EXPERIMENTAL VERIFICATION FOR A SPECIMEN UNDER COMBINED BENDING AND SHEAR LOADING

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The Modified Virtual Crack Closure Integral (MVCCI)–method has proved to be a highly effective and versatile numerical procedure for the fracture analysis of various crack problems in linear elasticity. In this paper it is shown that the MVCCI–method also can readily be utilized for computer aided step by step crack growth simulations. By this method both, the direction and the width of the next crack extension step can be computed simultaneously, even under complex boundary conditions. For a non–symmetrical specimen, subjected to a combined bending and shear loading, the comparison of computationally predicted and experimentally obtained non–coplanar crack trajectories show excellent agreement in all cases considered.

INTRODUCTION

From failed structures and components it is known in engineering practice that cracks frequently originate and extend in regions characterized by complicated geometrical shapes and mixed–mode crack tip loading conditions. Hence the developing crack paths are found to be curved and standard solutions for plane cracks as those arranged in /1/ do not apply. The computer aided simulation and prediction of such non–coplanar crack trajectories still is a challenging problem of fracture analysis. Like in those approaches also in this investigation the crack growth simulation is based on a step by step finite element analysis, which results in a piecewise linear approximation of the curved crack path. In this case for each step of further crack extension the fracture analysis of the preceding crack tip, must provide both, the direction and the width of the next incremental crack extension. This analysis may be based on several fracture criteria /2/ which differ slightly in their basic assumptions and their numerical approach. Therefore only through the comparison with related experimental findings one can decide on the significance of the computer aided crack growth simulation, as it will be discussed in the following chapters.

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COMPUTATIONAL CRACK GROWTH SIMULATIONS

In this investigation the computational crack growth simulation is based on a step by step finite element– and fracture analysis (Fig. 1). Hereby both, the direction and width of the next incremental crack extension step is computed by the aid of the Modified Virtual Crack Closure Integral (MVCCI)–method. According to /3/ the direction φ of further crack extension is computed from

$$\varphi = -2\lambda \qquad (1)$$

with

$$\lambda = K_{II}/K_{I}. \qquad (2)$$

It is emphasized that for small SIF–ratios $0 \leq \lambda \leq 0.1$ practically the same values for φ are given by most criteria /2/ and that this orientation will result in conditions of local symmetry at the crack tip ($K_{II} \to 0$). Based on Eqs. (1) and (2) also the width Δa of the next incremental step of crack extension can be chosen according to the condition

$$\lambda(\Delta a) = K_{II}(\Delta a)/K_{I}(\Delta a) \leq 0.1. \qquad (3)$$

From these relations the need for an efficient numerical mode separation technique in conjunction with the step by step finite element– and fracture analysis can be seen. Particulary with respect to this requirement the MVCCI–method has proved to be highly advantageous, because it delivers the separated energy release rates G_i, $i=I,II$ simultaneously with good accuracy and without any additional effort.

According to BUCHHOLZ /4/ for a LSE–discretisation, as it is forming the actual crack tip in Fig. 3, the following FE–representation of IRWIN's crack closure integral relations can be given

$$G_{I}(a) = \frac{1}{t} \lim_{\Delta a \to 0} \frac{1}{\Delta a} \frac{1}{2} \left(F_{y,i}(a)\, \Delta u_{y,i-1}(a) + F_{y,i+1/2}(a)\, \Delta u_{y,i-1/2}(a)\right) \qquad (4)$$

$$G_{II}(a) = \frac{1}{t} \lim_{\Delta a \to 0} \frac{1}{\Delta a} \frac{1}{2} \left(F_{x,i}(a)\, \Delta u_{x,i-1}(a) + F_{x,i+1/2}(a)\, \Delta u_{x,i-1/2}(a)\right) \qquad (5)$$

In Eqs. (4), (5) the strain energy release rates G_i, $i=I,II$ are computed on the basis of the work to be done by the nodal point forces $F_{y,i}$, $F_{y,i+1/2}$ against the relative nodal point displacements $\Delta u_{y,i-1}$, $\Delta u_{y,i-1/2}$ in order to close the crack of length $a+\Delta a$ by an amount Δa. Finally, with the relation

$$\lambda = K_{II}/K_{I} = \sqrt{G_{II}/G_{I}} \qquad (6)$$

all parameters necessary for a computer aided crack growth simulation are provided.

The chosen FE–discretisation for the specimen is given in Fig. 4 together with a mesh detail, showing some of the simulated crack extension steps with $\Delta a = 1\text{mm}$ for crack initiation from the notch position at $\alpha = 40\text{deg}$.

EXPERIMENTAL CRACK GROWTH TESTS

In order to evaluate the significance of the computer aided crack growth simulations a reliable and detailed data base from directly related crack growth experiments is required. Here we can refer to a series of experiments carried out and monitored thoroughly by THEILIG /5/. Stimulated by crack growth observed in a structural component in service a special non-symmetrical specimen for lateral force bending has been designed (Fig. 2) in order to get an experimental basis for the assessment of the situation. Along the circular shaped transition region of the specimen notches have been attached at positions α=0, 20, 40deg. from which cracks of characteristic shape initiated and extended during the fatigue tests. In Fig. 5 some of these experimentally obtained crack trajectories are shown and it is interesting to see in which way the cracks are curved in this case (α=0deg.) and that the experimental scatterband is rather narrow. A different shape characteristic is found for cracks that have initiated from the roots of the notches at α=40deg. but about the same small scatterband applies.

These detailed experimental findings have formed the basis on which an early approach to computer aided crack path simulation has been evaluated. Here reference is given to a detailed report by THEILIG /5/. In that investigation constant strain triangular elements in combination with special singular elements forming the crack tip have been used. Although the simulation could only be performed for a rather coarse mesh the computational results were found to be in good agreement with the experimental findings for α=0 and α=20deg..

In the following chapter recent results by the MVCCI–approach to this problem will be discussed.

DISCUSSION OF RESULTS

In Fig. 6 the computationally simulated crack trajectories for the notch positions at α=0 and α=40deg. are shown together with the experimentally obtained scatterbands for the related crack growth experiments. For a given crack increment of Δa=1mm = const. an excellent agreement is found for both crack initiation positions, although distinctly different crack path characteristics are found. For α=0deg. the development of the related stress intensity factors versus crack length are given in Fig. 7. Clearly the implicit condition of local symmetry can be verified by $K_{II}(a) \approx 0$ and from $K_I(a)$ actual fatigue lifetimes may be calculated. After crack initiation the values of the SIF ratio K_{II}/K_I in Fig. 8 show very good convergence with further crack extension. Here the small numbers of $\lambda<<0.1$ would allow for increasing length $\Delta a_{n+1}>\Delta a_n$ of incremental crack extension steps on the basis of eq. (3), but here Δa=1mm=const. has been given.

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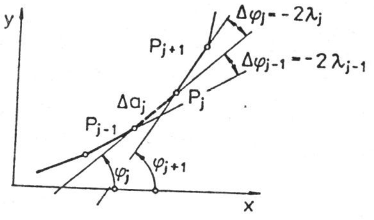

Fig. 1 Step by step approach for curved crack path simulation

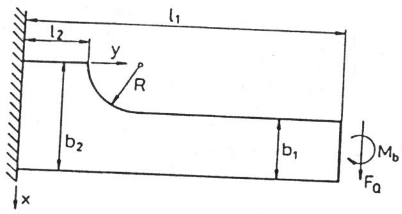

Fig. 2 Geometry of the non–symm. bending/shear specimen

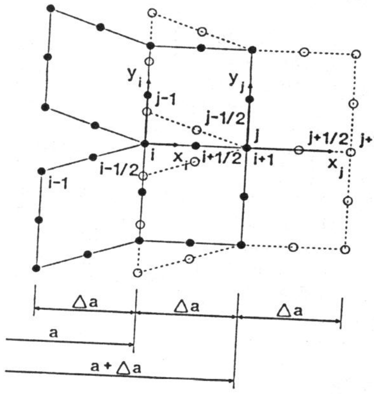

Fig. 3 Numerical VCCI– and MVCCI–methods, Δa>>0

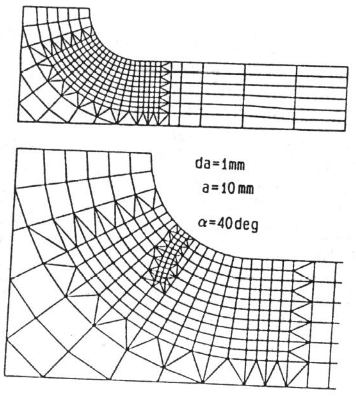

Fig. 4 FE–model of the specimen for crack path simulation

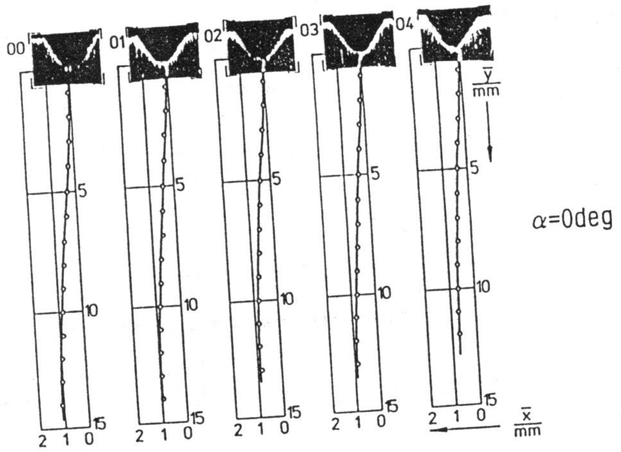

Fig. 5 Experimentally obtained crack trajectories originating from the root of the notch at position α=0deg

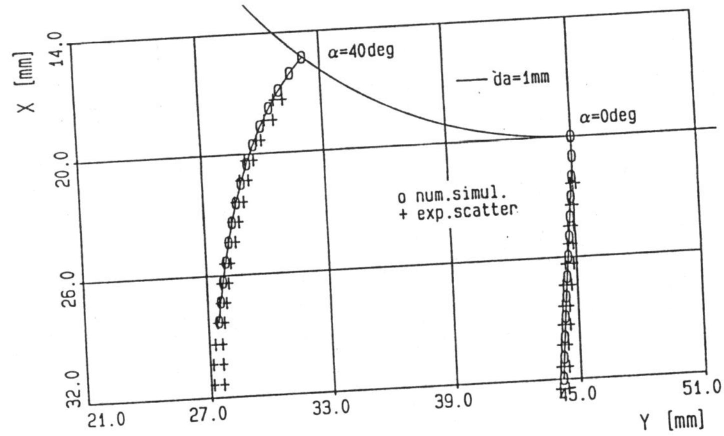

Fig. 6 Comparison of experimentally obtained and computationally simulated crack trajectories (α=0 and α=40deg, da=1mm)

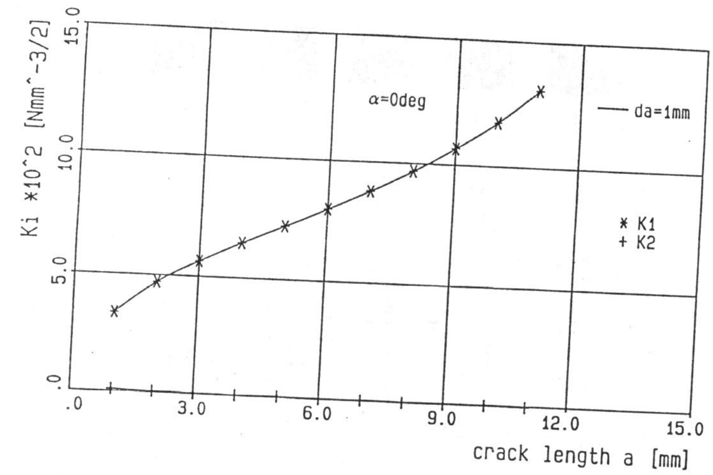

Fig. 7 Computed stress intensity factors K_i, i=I,II versus crack length (α=0deg, da=1mm)

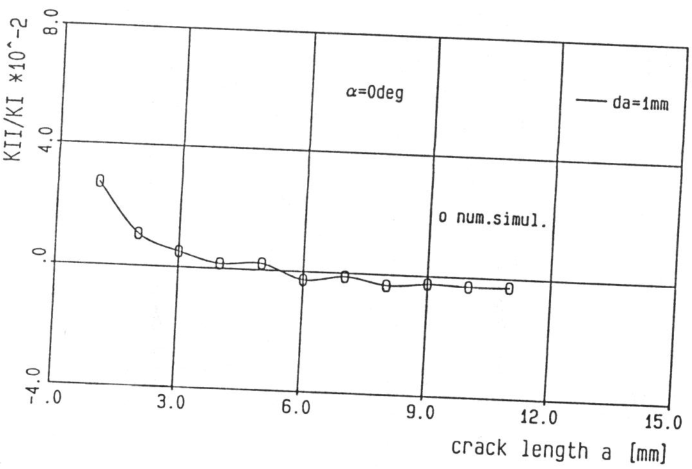

Fig. 8 Computed modulus of the SIF ratio K_{II}/K_I versus crack length (α=0deg, da=1mm)