CALCULATION OF CTOD OR J-Δa CURVE FOR A CRACK IN RESIDUAL STRESS ZONE

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Residual stresses are produced in weldments by local thermal expansion, plastic deformation, and subsequent shrinkage on cooling. Research reports (1), (2) indicated that in the region of the weld the resulting tension residual stress may reach a level close to the yield strength of the material, from which a problem arises as: when the distribution of residual stress in a weldment is known, how to calculate the crack driving force (CTOD or J-Δa relation) if a crack is detected there?

In documents from several countries it has been proposed by superimposing a definite stress or strain value to the applied stress or strain level to calculate the CTOD. For example, in the standard of JWES-2805 this strain is from $0.25\varepsilon_y$ to ε_y for different crack orientations(3), in PD-6493 from $0.6\varepsilon_y$ to ε_y (4).

Obviously the method of superimposition is based on experience and is convenient as well as conservative, sometimes, in application. But it can only give a rough estimate because the crack driving force depends not only on the maximum value of the residual stress but also on its distribution and the crack length. In Fig.1 an example is illustrated. So it is difficult to know the safety factor in some cases. In this paper an effort has been done to calculate the CTOD- or J-Δa in a more precise way.

The residual stresses are normal to or in the direction of welding, as shown in Fig.2a. Though the field of residual stress distribution is complicated, there are two keyfeatures of it; these are 1)Residual stress is acting only on the local area, which we name the "residual stress active area". In the whole structure it is always self-equilibrating. 2)In this local area the stress distributes always in a peak-like shape (Fig.4a). The exact solution of a crack problem with a cohesive zone under the peak-like shape stress distribution (Fig.4b) is solvable. The main idea of this paper is that we assume: the effects of different geometries and heterogeneity on mechanic analysis are only the variations of stress or strain distribution. If cracks are under the same stress or strain distribution, the behavior of them will be the same if there is no obvious influence of boundary condition. So if the rough information of residual stress distribution is in hand when there is no crack, we can simplify the complicated stress state in structures as a peak-like shape distribution of residual stress, then reproduce this stress state in a plate (Fig.4), and then put a through-thickness crack in this plate under the peak-shape stress distribution, finally we can solve the problem by modified the D-B model (Fig.3,4).

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Only the residual stress in one direction and perpendicular to crack is to be considered. The following binomial relation is suggested to describe the real residual stress distribution in structures (refer to Fig.3,4):

$$\sigma_{yy} = \sigma_{max} \frac{d^2}{d^2 + x^2} \left[1 - \frac{4}{\kappa + 3} \frac{x^2}{d^2 + x^2} \right] \qquad (1)$$

where *d*: *d*=1.43D; D is the reference width of peak-like stress distribution, and defined as the distance between the point of maximum stress and the point where the stress equals the half of maximum stress. In fact, the physical meaning of ***d*** or D is the dimension of residual stress active area.

From equ.1 we find: if the maximum stress, σ_{max} and the reference size D are at hand then the distribution of residual stress can be determined. There are already many references to give the value of σ_{max} (or ε_{max}) of residual stress in weldment. For D we suggest a conservative estimate that D= thickness/2 for the case in Fig.2a and D=the width of weldment for the case in Fig.2b in engineering application. The modified D-B model shown in Fig.4b was analyzed and the exact solution of it has been obtained as:

$$\delta = \frac{8\sigma_y a}{\pi E} \left\{ ln(s) + \frac{\pi \sigma_{max}}{2\sigma_y} \cdot \Phi \right\}; \quad s = sec\left(\frac{\pi \sigma_{max}}{2\sigma_y} \beta \right)$$

β , Φ are explained in the Appendix.

Fig.5 shows the relation between δ and peak strain (or stress) level for different ratios of *d*/a calculated by this relation as well as by JWES-2805 and PD-6493, from which one may find that under the most conditions the British Standard PD-6493 can offer conservative estimates but by JWES-2805 it is difficult to estimate the safety factor.when the peak strain is high or the size of residual stress active area is relatively large compared to crack length.

When the peak stress is not large or the ratio of *d*/a is relatively small, we may obtain the solution of "stress intensity factor" for the model in fig.4. It is also no difficulty to translate δ to J-integral by usual methods.

(1) Masubuchi, K., Welding Research Council Bulletin 174, 1974.
(2) Kelsey, R.A. and Nordmark, G.E., Proceedings of 5th Bolton Landing Conf., 1978, pp.117-128.
(3) British Standards Institution, PD-6493:1991.
(4) Japanese National Standard, JWES-2805, 1980.

NOMENCLATURE

ε_y : yield strain

δ : crack tip opening displacement

E : Young´s modulus

κ : plane strain:κ=3-4ν; plane stress:κ=(3-ν)/(1+ν)

σ_y : yield stress

σ_{max} : peak stress

a : half crack length

ν : Possion´s ratio

APPENDIX:

$$\beta = \frac{\sqrt{\omega_1}}{3 + \kappa} [1 + \kappa + 2\omega_1]; \quad \Phi = \frac{d/a}{3 + \kappa} \left[(1 + \kappa) ln\left(\frac{\omega_0 + \omega_2}{\omega_3} \right) + \frac{2\omega_4 \omega_0}{\omega_2} \right] - \omega_0$$

$$\omega_0 = tg\left(\frac{\pi \sigma_{max}}{2\sigma_0} \beta \right); \omega_1 = \frac{d^2}{d^2 + a^2 s^2}; \omega_2 = \sqrt{s^2 + \frac{d^2}{a^2}}; \omega_3 = \sqrt{1 + \frac{d^2}{a^2}}; \omega_4 = \frac{d^2}{d^2 + a^2}$$

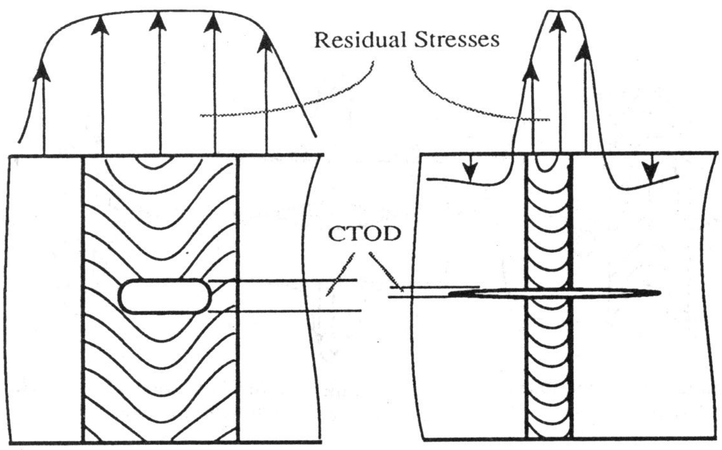

Fig.1 The effect of the width of residual stress peak on crack parameter

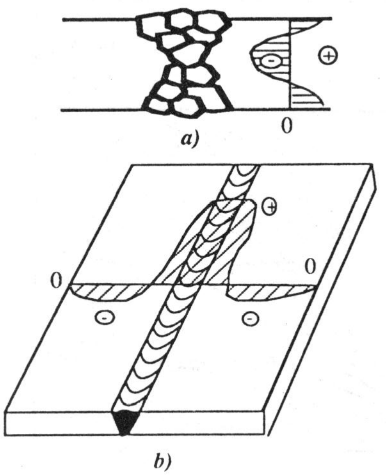

Fig.2 Origin of residual welding stresses

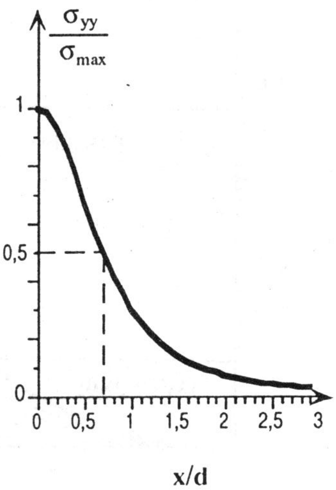

Fig.3 The peak-shape stress distribution described by (1)

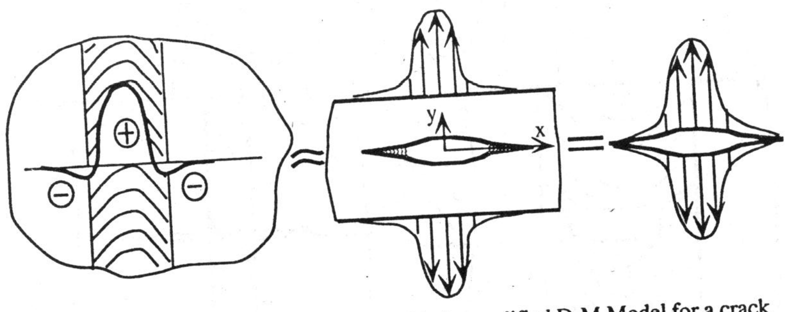

Fig.4 Strategy of analysis in this paper

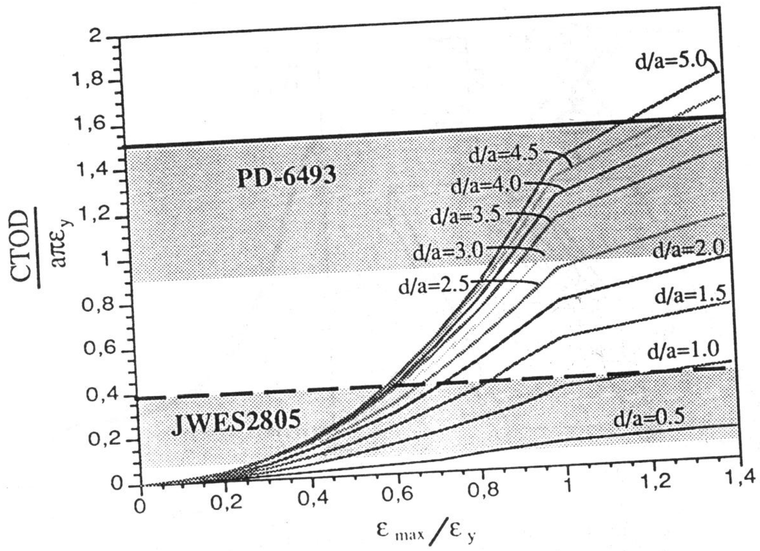

Fig.5 Comparasion of results calculated by equ.3 and other methods