AN EVALUATION CRITERION OF THE RESIDUAL LIFE OF STRUCTURAL ELEMENTS

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An effective engineering method of determination of residual life of structural elements containing plane defects is proposed. It consist in reduction of the initial equations of fatigue fracture kinetics to the relationships, that immediately describe the variation of the crack area during its growth. These dependencies are practically invariant in respect to the crack geometry, therefore allowing description of the prefracture growth of a complex configuration crack via the area of the initial defect

INTRODUCTION

Determination of the residual life of structural elements containing defects is based on investigation of their fatigue fracture kinetics, i.e. a slow subcritical crack growth under cyclic varying in time operation loading effect. In particular, if the crack propagates in one plane of a body (structural element),than the fatigue fracture kinetics is described by a differential equation:

$$\frac{\partial r}{\partial N}\left[1+\frac{1}{r^2}\left(\frac{\partial r}{\partial \varphi}\right)^2\right]^{-1/2}=V(K_I); \qquad r\Big|_{N=0}=r_0(\varphi), \qquad (1)$$

where $r(N,\varphi)$ - is the desired radius-vector of a moving crack contour in a polar coordinate system after N cycles of loading; $r_0(\varphi)$ is a radius-vector;

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$V(K_I)$ is a dependency of crack growth rate on maximum stress intensity factor for a cycle (SIF) at its tip, which is determined by a kinetic fatigue fracture curve for a given material and loading conditions.

The direct integration of equation (1) with step-by step determination of the position of the crack contour and variation of SIF along it is a rather complex mathematical task. Besides, practically it is rather difficult to establish the initial defect (i.e. a function of $r_0(\varphi)$), in the initial condition): the serial devices of non-destruction control used recently are oriented on determination of defects areas and do not give reliable information on their geometry. Therefore, from the practical point of view it is worth while to describe the fatigue crack growth kinetics(rates) via their area sizes.

Isolated internal and surface defects.

Let us consider a plane crack with a smooth contour L, propagating in a three-dimensional body under the influence of evenly distributed cyclic loadings σ, which are perpendicular to the crack plane (Fig.1).

Let's determine the dependency of crack area $S(N)$ on a number of loading cycles at its initial value of $S(0)=S_0$. Taking into account, that:

$$S(N) = \frac{1}{2}\int_0^{2\pi} r^2(N,\varphi)\,d\varphi \tag{2}$$

and differentiating this equality with respect to N, taking account of (1), we obtain:

$$\frac{\partial S}{\partial N} = \int_0^{2\pi} r(N,\varphi)\frac{\partial r}{\partial N}\,d\varphi = \int_L V(K_I)\,dt\,. \tag{3}$$

The equation (3) obtains the more simple shape, if instead of the crack area we shall consider a parameter which is a radius of the equivalent circle ($S = \pi a_{eqv}^2$) :

$$\frac{da_{eqv}}{dN} = V(K_{Ieqv}). \tag{4}$$

The values of K_{Ieqv} are the average value, which integrally allows for SIF variation along the crack contour and in case of the power function of $V=CK_I^n$ is determined by:

$$K_{Ieqv} = \left\{ \frac{1}{2\pi a_{eqv}} \int_L K_I^n \, dt \right\}^{1/n} . \qquad (5)$$

The advantages of the differential equation of fatigue fracture kinetics, presented by eq.(4) consist in the fact, that for a wide class of the convex contours the value of K_{Ieqv} depends insignificantly on cracks geometry and is determined, mainly, by its area. Thus, in case of elliptical cracks of the same area the value of K_{Ieqv} is changed not more than by 4% under variation of semiaxes b/a in the range from 1 to 0,2 (Andreykiv, Darchuk, 1988). Thus, the presented relationship describes the area variation of fatigue crack during its propagation in a closed, invariant in respect to the crack geometry form. For approximate determination of the K_{Ieqv} value we may use the solution that correspond to the circular crack of a_{eqv} radius:

$$K_{Ieqv} = \frac{2}{\sqrt{\pi}} \sigma \sqrt{a_{eqv}} . \qquad (6)$$

In that case it is not necessary to investigate the variation of the crack contour during its growth and the residual life is expressed in terms of the initial defect area:

$$N_* = \int_{a^\circ_{eqv}}^{a^*_{eqv}} \frac{da_{eqv}}{V(K_{Ieqv})} , \qquad (7)$$

where a°_{eqv} is an initial, and a^*_{eqv} is a critical (corresponding to the limiting state achievement) values of the a_{eqv} parameter.

Subsurface defects

From analysis of general solution for subsurface plane cracks follows, that for a crack of a given area and at a given distance of its geometrical centre H from the semispace surface (Fig. 2), the boundary effect will be maximum in case of a circular shape of a contour. Thus, if the circular crack sabstitution for a plane crack of an arbitrary shape is legitimate, then this substitution is even more legitimate for bodies of finite sizes: thus appearing additional values of SIF, contribute to the safely factor.

The K_{Ieqv} values for a presented in Fig. 2 scheme can be determined, using approximation of a numerical solution for a circular crack in a semispace (Ishida and Noguchi, 1984).

Further, presenting on the basis of eq.(1) the increment of the crack area and a shift of the center of its weight during fatigue crack growth we obtain a system of differential equations:

$$\frac{d a_{eqv}}{dN} = \frac{1}{2\pi}\int_0^{2\pi} V(K_{Ieqv})\,d\varphi; \quad \frac{dH}{dN} = -\frac{1}{\pi}\int_0^{2\pi} V(K_{Ieqv})\cos\varphi\,d\varphi \qquad (8)$$

solution of which allows to determine the sizes and location of a crack in an arbitrary moment of time and calculate its subcritical growth period, i.e. residual life of a body with a mentioned defect.

Defects systems.

Let us consider such coplanar defects systems (Fig. 3), consisting of the cracks m , limited by a smooth contour L_K . The investigation of fatigue fracture kinetics is reduced to a common solution of a system m of the differential equations of the type I used for determination of the unknown functions: $r_K(N, \varphi_K)$ which describe the contour variation of each of the cracks. In this case, as well as in the above mentioned, the main complexity of the task consists in plotting of SIF solution curves taking into account defects interaction in a system as well as variation of their shape and sizes during fatigue development. This solution for a K -crack can be presented as:

$$K_I^{(K)} = K_{I0}^{(K)} F^{(K)}, \qquad (9)$$

where $K_{I0}^{(K)}$ corresponds to solution of a given isolated crack, in a space, and $F^{(K)}$ is a correction function reflecting the neighbour defects influence.

The increase of the SIF value as compared to that of the isolated crack achieves its maximum values when crack is circular. Therefore the circular contours substitution for those of the neighbour cracks of a systems only enhances their influence on the SIF values in the vicinity of a given defect, i.e. increases the correction function $F^{(K)}$ in a relationship (9). Thus a problem for an initial system, consisting of an arbitrary shaped crack is reduced to a system of circular cracks of the similar areas, centers of which coincide

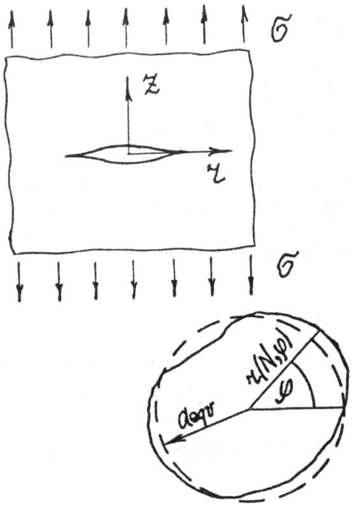

Fig. 1.

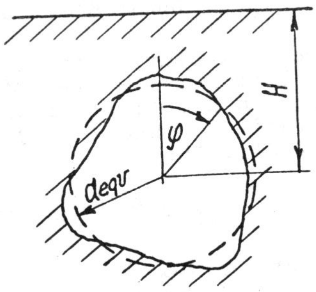

Fig. 2

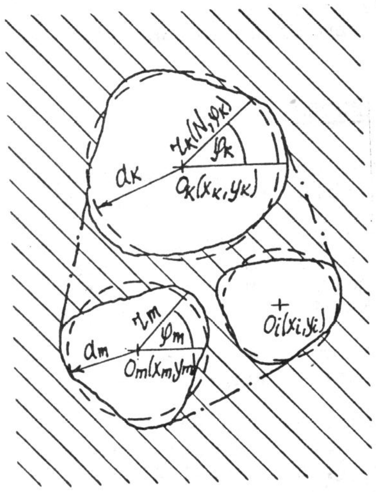

Fig. 3

with the geometrical centers of the considered cracks.
In the frame of this calculational model a state of defect system and its fatigue propagation is fully described by the parameters of $a_K(N)$, $x_K(N)$ and $y_K(N)$ which are "similar" radii and coordinates of the centers of each of the cracks. Basing on the kinetics equations, as it was presented above, a system of simple differential equations for determination of these values has been obtained:

$$\frac{da_K}{dN} = \frac{1}{2\pi}\int_0^{2\pi} V(K_{Ieqv}^{(K)})\, d\varphi_K ,$$

$$\frac{dx_K}{dN} = \frac{1}{\pi}\int_0^{2\pi} V(K_{Ieqv}^{(K)})\cos\varphi_K\, d\varphi_K ; \quad \frac{dy_K}{dN} = \frac{1}{\pi}\int_0^{2\pi} V(K_{Ieqv}^{(K)})\sin\varphi_K\, d\varphi_K . \quad (10)$$

For calculation it is also necessary to obtain the SIF values for a system of circular cracks. For these reason, a solution of the problem on two circular cracks (radii a_K and a_i) interaction can be used (Ishida et al., 1985)

$$K_I^{(K)} = \frac{2}{\sqrt{\pi}}\sigma\sqrt{a_K}\left\{F_3(\lambda,\varepsilon) + F_4(\lambda,\varepsilon)\cos^2\frac{\varphi_K-\theta_K}{2}\right\} , \quad (11)$$

where $\varepsilon = (a_K + a_i)/2H$; $\lambda = a_i/a_K$; $H = \sqrt{(x_K - x_i)^2 + (y_K - y_i)^2}$;
$\theta_K = arctg\{(x_K - x_i)/(y_K - y_i)\}$
and functions $F_3(\lambda,\varepsilon), F_4(\lambda,\varepsilon)$ - are approximated by polinomials.

Using the superposition of this solution, we can plot rather simple, convenient for practical calculations curves for the SIF values, if an arbitrary number of cracks occurs in a system. Using them we can numerically integrate a system of equations (10) without difficulty.

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