A CALCULATION OF THE EFFECTIVE ΔK UNDER VARIOUS FATIGUE LOADING CONDITIONS

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A crack closure model is described which can calculate the effective ΔK under different loading conditions. To test the model it has been applied on 8 constant amplitude fatigue tests with R ($=K_{min}/K_{max}$) values extending from 0.1 to 0.7. The material used is Al-2024 T351. The crack growth rate da/dN is calculated from the measured a (crack length) and N (number of cycles) data. The ΔK_{eff} value for each test is calculated, using the crack closure model, as a function of the crack length a. All tests at different R values coincide to a single line when da/dN is plotted against ΔK_{eff}. For comparison the test results have been plotted against the applied value of ΔK and also against the ΔK_{eff} values using the experimental crack closure functions of Elber (1) and Schijve (2).

INTRODUCTION

The fatigue crack growth rate, da/dN, is assumed to be a function of ΔK and R. In practice the combined effect of ΔK and R is often presented in a crack closure formula ΔK_{eff}. da/dN is assumed to be only a function of this ΔK_{eff}. For Al-2024 Elber (1) found:

$$\Delta K_{eff} = (0.5+0.4R)\Delta K \qquad -0.1 \leq R \leq 0.7 \qquad (1)$$

The ΔK_{eff}, used in this paper, is defined as:

$$\Delta K_{eff} = K_{max} - K_{cl} \qquad (2)$$

K_{max} is the maximum value of K, corrected for plasticity at the crack tip. K_{cl} is the calculated minimum value of the sum of two mode I stress systems:

$$K_{cl} = K_{min} + \int dK \qquad (3)$$

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where $K_{min} = RK_{max}$ and $\int dK$ is the contribution to K_{cl} of the compressed crack flanks. The integration sign is used because the crack flanks are divided in a number of compressed elements.

CALCULATION OF ΔK-EFFECTIVE

As shown in eq(2) the values of K_{max} and K_{cl} are needed in the calculation of ΔK_{eff}. At first K_{max} will be calculated. For the calculation of K the formula for a centre-cracked tension specimen (width w) is used:

$$K = \sigma\sqrt{\pi a}\sqrt{\sec\frac{\pi a}{w}} \tag{4}$$

In metals the fatigue crack growth is accompanied by plastically deformed material on the crack flanks due to the high stress concentration at the crack tip.Irwin (3) found that the plastic zone size at the crack tip could be given as:

$$2r_p = \frac{1}{\pi}\left(\frac{K_{max,applied}}{\sigma_{ys}}\right)^2 \qquad \text{(plane stress)} \tag{5}$$

$K_{max,applied}$ is the maximum value of the applied K, σ_{ys} is the yield stress of the metal and r_p is the radius of the (circular) plastic zone. Irwin pointed out that the crack behaves as if it were r_p longer due to the plasticity. The new crack length corrected for plasticity thus becomes $a + r_p$ instead of a. According to eq.(4) this will give a higher K (and thus a higher K_{max}). On its turn this higher K_{max} will give a higher r_p, etc. When the crack length and the plastic zone size are not too big this iteration of r_p and K_{max} will converge to stable end values. From now on the stable end value of the maximum K will be nominated K_{max} , while the applied maximum value of K is nominated $K_{max,applied}$. K_{max} is thus solved by an iteration procedure using eqs.(4) and (5). For constant ΔK and R tests $K_{max,applied}$ is simply constant; for a constant amplitude test $K_{max,applied}$ increases with crack length a as given in eq.(4). In Fig. 1 a calculation of K_{max} for a few discrete values of constant $K_{max,applied}$ is shown as a function of crack length a.
The calculation of K_{cl} is more complex. Consider the situation of a c.c.t. specimen (Fig.2) At maximum stress (i.e. at K_{max}) the crack is fully open. Suppose that the R-value is such that closing of a part of the crack flanks starts before $K_{min} = RK_{max}$ is reached. In order to calculate the value of K_{cl} the material along the crack growth direction is separated in elements with a length da (see Fig. 3).In Fig. 3 the elastic contour is the contour of the crack as it would be without plasticity induced closure. On both crack flanks the material elements have a length Δl in a stressless state. In a closure situation Δl will be compressed until half the crack opening v leading to a compressive stress in this element. This compression stress will give a contribution dK to the K

resulting from the external stress σ and will enhance the tensile stress in the remaining ligament. The value of K_{cl} can be found as the sum of all dK's of all compressed parts and the value of K_{min}:

$$K_{cl} = K_{\min} + \int dK \qquad (6)$$

As a first calculation step it is assumed that the width of the elastic contour at $K = K_{min}$ is given by K_{min}. The lengths Δl of the plastically deformed crack flank elements result from crack tip plasticity at $K = K_{max}$. A big compression of a number of elements will be the result at $K = K_{min}$. This cannot be a steady state value as the closure term, $\int dK$, together with K_{min}, influences the width of the elastic contour, i.e. the elastic contour becomes wider. This leads to lower compression stresses in the elements and thus a lower $\int dK$. So, the closure K, K_{cl}, decreases, resulting in a more narrow elastic contour of the crack. This, however, will again increase the compression stresses and thus $\int dK$ and K_{cl}, etc.
The calculation process is performed as just described. After a number of iterations a stable K_{cl} value can be reached. The iterations stop when the difference in $\int dK$ between two calculation steps is less than 10^{-4} times the applied ΔK. To avoid large numbers of iterations in some cases a maximum number of 20 has been chosen. In most cases the number of iterations that is needed to satisfy the criterion is much smaller.

CALCULATION DETAILS

Details about the unstressed element length Δl ,the formula used for the elastic contour , effects of thickness (plane strain/plane stress) and effects of shear lips will be discussed in a forthcoming paper (Ref. 4).Here only a few formulae are presented as used in the model. Δl is based on the crack opening value (ctod) at $K = K_{max}$:

$$\Delta l = \frac{1}{2} ctod - \frac{\sigma_{ys}}{E} r_p \qquad \text{(plane stress approximation)} \qquad (7)$$

The elastic crack contour is assumed to be elliptical (Fig.3). The long axis of the ellipse is assumed to be $2(a + r_p)$ with r_p resulting from eqs.(4) and (5). The short axis of the ellipse is taken from Tada et al (5).In the calculations the plane stress Δl of eq.(7) is corrected for plane stress/plane strain as:

$$\Delta l = \Delta l(\text{plane stress}) / \alpha \qquad (8)$$

α varies from 1 (plane stress) to 3 (plane strain).In the calculation the stresses in the elements are considered elastic due to the geometry of the closing part. The thickness Δl is very small compared with the other dimensions. A shear lip correction of the ΔK_{eff} found is applied using the results found in reference 6 .

VALIDATION OF THE MODEL

The ΔK_{eff} calculation model is tested using the results of constant amplitude tests performed on c.c.t. specimens of Al-2024 T351. Eight tests with different R-values were performed at 10 Hz. First the da/dN versus a data are obtained from the measured a-N data for all tests performed. Then the crack closure model is used to calculate ΔK_{eff} versus a for all tests performed. Hereby the calculation is performed inclusive the prefatigue zone ,as this zone of course has an influence on the amount of closure in the constant amplitude test (see Fig.4). It is assumed that there is no crack closure before the prefatigue is applied. By elimination of the crack length a, da/dN versus ΔK_{eff} is found. When the model works well it should be expected that at the same measured da/dN the same value of calculated ΔK_{eff} will be found. The combined test and calculation results are shown in Figure 8. For comparison the da/dN test results have also been plotted against the applied ΔK and ΔK_{eff} using Elber's and Schijve's experimentally found crack closure formulae in Figures 5,6 and 7.

CONCLUSION

The model works well for constant amplitude tests.

REFERENCES

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[3] G.R.Irwin . A.S.M.-Symposium "Fracturing of Metals", American Society of Metals ,Chicago (1947)
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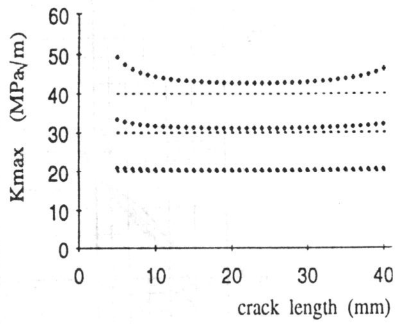

figure 1. K_{max} at $K_{max,applied}$ values of 20,30 and 40 MPa√m

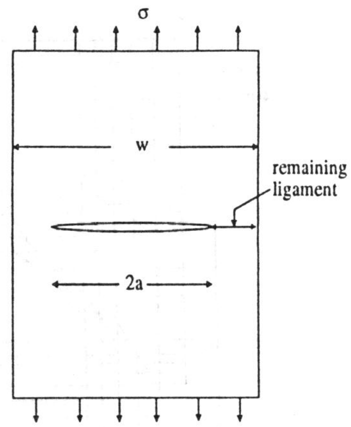

figure 2. The c.c.t. specimen

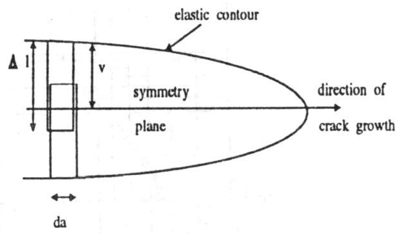

figure 3. The elastic contour, element length Δl and crack opening v.

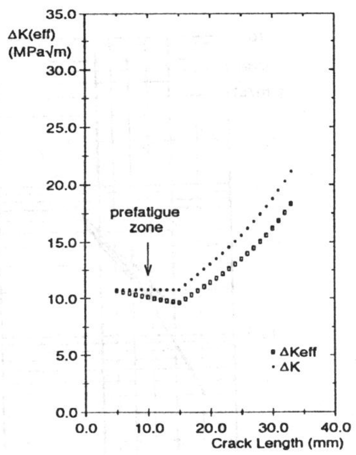

figure 4. ΔKeff calculation in a c.a.test with R=.25 and Smax=62.5 MPa.

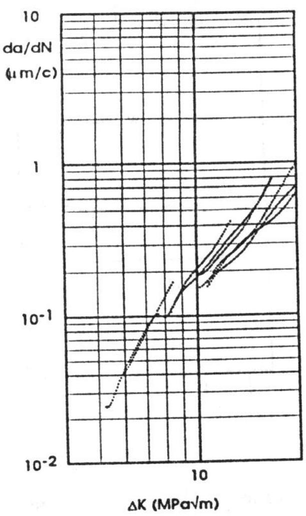

figure 5. da/dN versus ΔK as measured

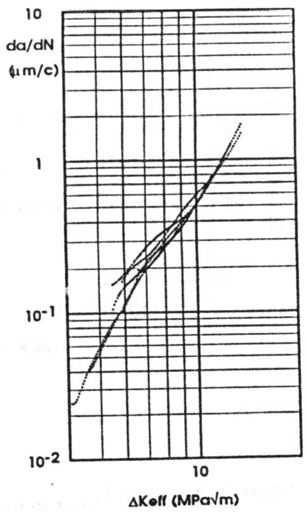

figure 6. da/dN versus ΔKeff using the crack closure relation of Elber.

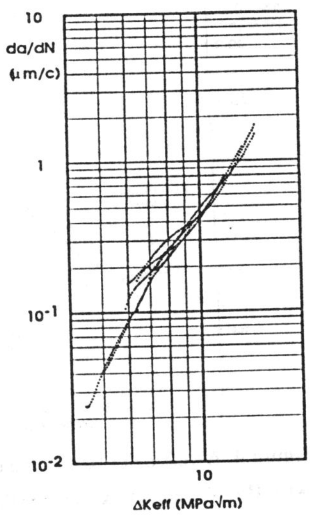

figure 7. da/dN versus ΔKeff using the crack closure relation of Schijve

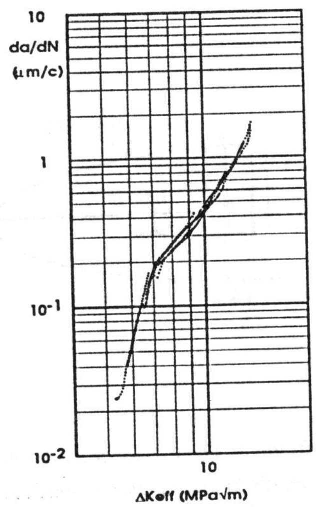

figure 8. da/dN versus ΔKeff using the crack closure calculation model