APPLICATION OF PROBABILISTIC FRACTURE MECHANICS THEORY TO CALCULATION OF LIFE OF GEARS

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The paper is concerned with comparison of the dynamic behaviour of gear with and without crack. Dynamic analysis is performed by means of the FEM method and programme package NISA II; the remain life of gears is determined on the basis of the probabilistic fracture mechanics by using the programme STATFAG.

INTRODUCTION

The problem of fatigue of components subjected to different dynamic loads is one of the most demanding engineering problems because of its sensitivity to the mechanical properties of materials, geometry of components, load history and the influence of environment. It is not possible to take into account all these parameters only through deterministic approach, therefore we applied the theory of probabilistic fracture mechanics to calculations.

The researches were performed on gears which are the most frequently used mechanical parts. The gear with and without defects were taken into account. The practice shows the defects occur most frequently in the tooth root. These defects are due to the fatigue, heat treatment or initial notch in case of clumsy or to rough machining. The results of the static and/or dynamic analysis were used as input data for probabilistic calculation of the remain life.

DEFINING OF PROBLEM

The subject of researches was the gear drive of the diffusor in the sugar factory Ormož. Figure 1 shows a detail of this gear

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during engagement.

The gear was divided into substructures representing the model for the analysis. The model was discretized into n = 310 finite elements with 993 nodes. High order 8-node elements were used. The crack was modelled with special singular elements.

Static analysis

The static analysis by using the NISA II programme was effected for the gear model according to Figure 2. It was loaded at the outer point of single engagement by the point force F = 1000 N/mm.

Analysis of eigenvalues

This analysis is used in order to obtain the dynamic characteristics of structures in the form of natural frequencies and mode shapes used herebelow for calculating the responses in case of dynamic loading and damping.

TABLE 1 - Natural frequencies for different crack length

crack length a [mm]	NATURAL FREQUENCIES in [Hz]		
	Mod 1	Mod 2	Mod 3
0.0	523.04	949.904	1352.20
1.5	512.74	938.730	1351.71
3.0	494.35	931.916	1348.06

Dynamic analysis

As on the diffusor drive the loading on the gear drive during operation are insignificant and nonstationary the problems can not be dealt with deterministically. Therefore the conditions during operation were simulated by random generation of the dynamic loading described by power spectral density (PSD) function.

The modal dynamic analysis is an efficient method for analysing the forced oscillations.

PROGRAMME PACKAGE STATFAG

It is a result of own development (1) and is used as a post processor for the conventional FEM methods. It is based on the probabilistic fracture mechanics. The deterministic functions of the crack growth, used in the programme, are the Paris', Forman's and Walker's functions. If it is taken into account as a random variable (3), it is possible to write as follows:

$$\frac{da(t)}{dt} = X(t)\, L\,(\Delta K,\ K_{max},\ R,\ S,\ a) \qquad (1)$$

For solving this equation in our programme package STATFAG we used the simulation method MONTE CARLO where X(t) is the positive stationary stohastic log normal random process defined with the mean value and autocorrelation function in the form of the Fourier's transformation function representing the (PSD):

$$\Phi_{zz}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{zz}(\tau)\, e^{-i\omega t}\, d\tau \qquad (2)$$

Analysis of remain life

With the method Monte Carlo we calculated the effective crack size depending on loading cycles. The results of calculations are indicated in Figure 3. For this purpose we used the Paris - Erdogan's law of crack propagation. The algorithm used was verified by the experiment results (2).

CONCLUSION

For the used analysis of the gear drive in mathematical modeling we tried to simulate as precisely as possible the actual conditions during operation.
The basic question to be answered after detecting cracks on the gear rim during the routine periodic inspection was the remain life. By applying the probabilistic approach, the programme NISA II and STATFAG we satisfactorily answered the question.

SYMBOLS USED

ΔK, K_{max} - stress intensity factor/range [$Nmm^{-3/2}$]

σ - stresses ... [N/mm^2]

a - crack length ..... [mm]

R - load ratio

ω - frequency ... [rad/sec]

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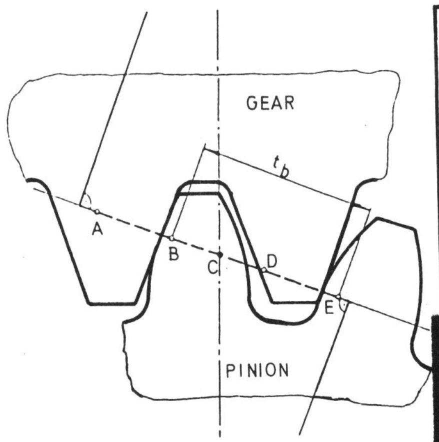


Number of teeth $Z_{2,1}$ = 306/21
Module m = 14 mm
Profile displacement $x_{1,2}$ = 0

Figure 1: Gears during engagement

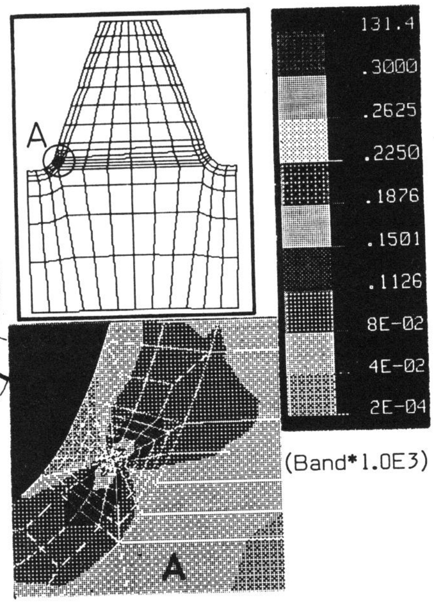
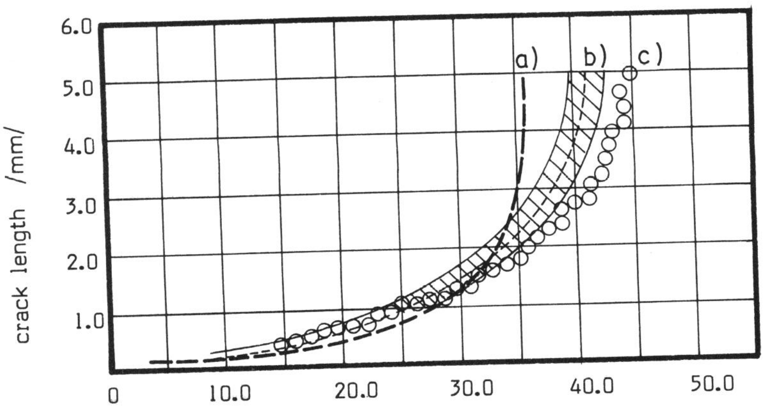


Figure 2: FE model of gear with Von-Misses distribution in the gear rooth.



a) with FEM b) with STATFAG c) Experimentaly
service life (cycle * 1.0E5)

Figure 3: Crack growth curve