THREE–CRITERIA–CONCEPT FOR THE DESIGN OF STRUCTURES WITH CRACKS AND SHARP NOTCHES

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A three–criteria–concept for the design of structures and components with narrow notches is to be presented. The applicability of this simple model will be demonstrated by the results of extensive experimental investigations. In order to determine the local load parameter which triggers off fracture, fracture tests are numerically simulated with the help of elastic–plastic finite element analyses. It will be shown that it is a critical stress at the notch root that causes fracture in narrow–notched components made from brittle materials (e.g. PMMA) where material behaviour is moderately nonlinear, and that it is a critical strain at the notch root in the case of more ductile materials (e.g. aluminium alloy 2017–T4) with elastic–plastic material behaviour.

INTRODUCTION

In order to be able to design structures taking into account the strength of materials, it is necessary to have an exact knowledge of the loads on the components, on the one hand, and the corresponding characteristic material values, on the other. Certainly, as far as the analysis of stress and the determination of the resistance parameters in components with sharp notches, slits, V–notches, and re–entrant sharp corners is concerned, there remains much to be explained.

THREE–CRITERIA–CONCEPT

The rounded narrow notch ($\rho << a$) represents an important exception. Where there is normal loading and linear–elastic material behaviour, the maximum stress at the root of a rounded narrow notch may be calculated with the help of the Creager/Paris equations (1) as follows (Kullmer et al (2)):

$$\sigma_{\max} = \frac{2K_I}{\sqrt{\pi\rho}} \qquad (1).$$

Here, ρ is the radius of the notch root and K_I, the stress intensity factor for the corresponding crack equivalent (i.e. crack of the same orientation and

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length as the notch in question). The stress intensity factor is defined with the following equation:

$$K_I = \sigma \sqrt{\pi a}\, Y_I \qquad (2).$$

The stress intensity factor is, therefore, dependent on introduced stress σ, notch depth a, and the dimensionless geometry factor Y_I.

A simple fracture criterion is to be found if it is assumed that fracture occurs when the maximum notch stress reaches a critical value σ_S. Which load leads to the fracture of the component can be illustrated in a notch fracture limit curve. A formulation of a notch fracture limit curve which is, in most cases, independent of both notch depth a and specimen geometry is obtainable by introducing notch toughness K_A. Notch toughness K_A is to be defined in the same manner as crack toughness K_{Ic}. If, for limit load, K_I is replaced by K_A, and σ_{max} by σ_S, in equation (1), the following is obtained:

$$K_A = \frac{\sigma_S \sqrt{\pi\rho}}{2} \geq K_{Ic} \qquad (3).$$

Notch toughness K_A can not go smaller than crack toughness K_{Ic}; for this reason, notch toughness K_A for notches with radii smaller than a limiting root radius ρ_g does not decrease further. Moreover, where there is greater ductility in a material, and where the notch radii are larger than the plastic limit radius ρ_p, the maximum load–bearing capacity, and therefore notch toughness, is limited by the plastic limit load. The plastic limit load σ_p is equal to the introduced stress σ from the point where the total load–bearing section is plasticized and further increase of load is impossible. By replacing K_I by K_A, and σ by σ_p, in equation (2), it is possible to compute formally the notch toughness at plastic limit load. An approximate calculation of the plastic limit load is obtained either by using characteristic material values from the uniaxial tension test in conjunction with equilibrium analyses on idealized stress curves in the ligament, or with the help of elastic–plastic finite element analyses (Kullmer and Richard (3)). Thus, the following is obtained for the notch fracture limit curve with the three–criteria–concept (Fig. 2 and Fig. 3):

$$K_A = \begin{cases} K_{Ic} & , \text{for } \rho \leq \rho_g = \frac{4}{\pi}\left[\frac{K_{Ic}}{\sigma_S}\right]^2 \quad \text{criterion I} \\[2ex] \frac{\sigma_S \sqrt{\pi\rho}}{2} & , \text{for } \rho_g \leq \rho \leq \rho_p \quad \text{criterion II} \\[2ex] \sigma_p \sqrt{\pi a}\, Y_I & , \text{for } \rho \geq \rho_p = 4a\left[\frac{\sigma_p Y_I}{\sigma_S}\right]^2 \quad \text{criterion III} \end{cases} \qquad (4).$$

Limiting root radius ρ_g is a material constant. The plastic limit radius ρ_p is also dependent on the geometry of the specimen.

EXPERIMENTAL INVESTIGATION OF THE THREE–CRITERIA–CONCEPT

Fracture tests were conducted on CTSN–specimens that were 10mm thick, sharp–notched, and made from either PMMA or the aluminium alloy 2017–T4 (orientation T–L). These fracture tests have shown that notch toughness K_A for notches with small enough notch radii is dependent not on notch depth a, but only on notch radius ρ (Kullmer and Richard (3,4)).

Criteria I and II are sufficient for the description of the notch fracture limit curve for the brittle material PMMA since a total plasticization of the ligament is not to be expected. Notch toughness increases linearly versus $\sqrt{\rho}$ and intersects the ordinate at origin clearly confirming the three–criteria-concept (Fig. 1). Critical stress is calculated from the gradient of the fitting curve at $\sigma_s = 168$ MPa, almost double the tensile strength $\sigma_{TS} = 89$ MPa.

Notch toughness in 2017–T4 also increases linearly versus $\sqrt{\rho}$; intersects, however, the ordinate not at the origin as assumed by the three-criteria–concept, but slightly lower as crack toughness (see also Begley et al (5)); and remains constant after the plastic limit load has been reached (Fig. 3). Critical stress is calculated from the gradient of the fitting curve at $\sigma_s = 1225$ MPa, almost three times the tensile strength $\sigma_{TS} = 415$ MPa. It is interesting to note that the plate thickness is not sufficient to determine crack toughness K_{Ic} according to ASTM E399, and that considerable plastic deformation occurs. It is likely then, that plastic deformation and crack toughness decrease until K_{Ic} is reached as plate thickness is increased. In which case, it may also be possible that the three–criteria–concept is better confirmed.

NUMERICAL SIMULATION OF THE FRACTURE TESTS

Three characteristic material values need to be determined to plot with the help of the three–criteria–concept, a notch fracture limit curve that may be used in the design of components. Crack toughness K_{Ic} is obtained from fracture tests; characteristic material values from the uniaxial tension test are required for the calculation of the plastic limit load. The exact determination of the critical stress σ_s is extremely time–consuming due to the fact that numerous fracture tests with specimens with notches of varying radii have to be conducted for this purpose. A simplified estimation of the critical stress by replacing σ_s with tensile strength usually results in a very conservative prediction for the fracture load.

In order to determine the load parameters which trigger off fracture, the fracture tests were numerically simulated with the help of elastic–plastic finite elements; the stress–strain curve from the uniaxial tension test was employed for the stress–strain law. The introduced stress σ was increased in load steps until the fracture stress obtained by experiment was reached.

For the numerical simulation of fracture tests with PMMA, a state of plane strain was assumed. The results of these investigations show that, despite nonlinear material behaviour, it is a critical stress σ_s at the notch root that triggers off fracture. The shapes of the stress curves are similar in the vicinity of the notch for different notch radii. The stress curves coincide when the abscissa is normalized at $2x/\rho$ (Fig. 2). The same normalization is appropriate for showing the similarity between the stress curves for linear-elastic material behaviour. Maximum stress at the notch root for nonlinear material behaviour is calculated numerically at 120 MPa; this corresponds to the crazes stress for PMMA which, according to Williams (6), lies between 110 MPa and 130 MPa. Crazes are, therefore, the cause of fracture in sharp–notched components made from PMMA.

A state of plane stress was assumed for the numerical simulation of fracture tests with 2017–T4 – since plastic yielding was hardly hindered. The result was material behaviour that was almost ideally plastic in the vicinity of the notch. Therefore, stress curves can be estimated with the slip line theory from Hill (7). It is not a critical stress σ_S that triggers off fracture, but a critical strain ϵ_S. The strain curves are, moreover, similar for different notch radii – since they coincide when the scale of the abscissa is $2x/\rho$ (Fig. 4). This is a surprising result for the reason that the linear curve of the notch toughness is supposed to intersect the ordinate at origin – whether we follow the assumption of the three–criteria–concept, or employ a critical strain criterion based on results from Rice (8). Instead we find that the linear curve of notch toughness intersects the ordinate almost at crack toughness.

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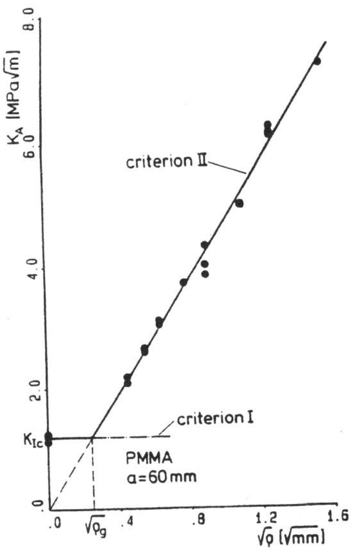

Fig. 1 Experimental notch fracture limit curve for PMMA

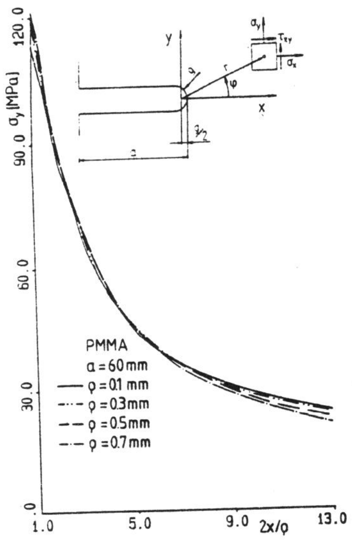

Fig. 2 Standardized stress curves under fracture loading for PMMA

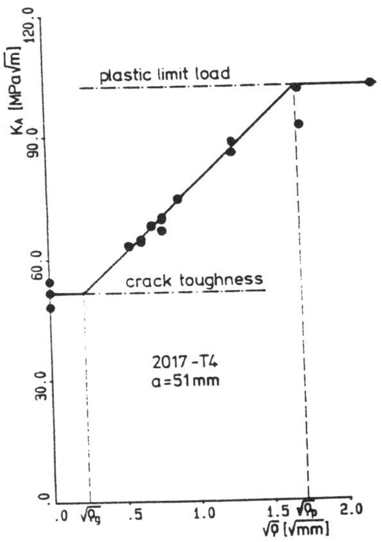

Fig. 3 Experimental notch fracture limit curve for 2017-T4

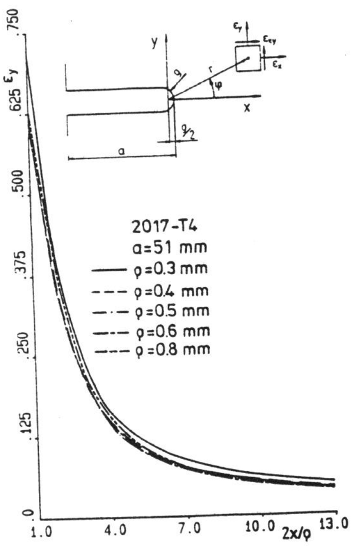

Fig. 4 Standardized strain curves under fracture loading for 2017-T4