CRACK GROWTH IN ROLLERS DUE TO MOVING HERTZIAN COMPRESSION

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In rollers of bridge supports straight radial cracks can occur although there are only small tensile stresses. This astonishing phenomenon could be explained by means of linear–elastic fracture mechanics. It is shown that the energy release rate of this system is very high, especially for relatively short cracks. The main contribution to it is made by the mode –II component of the crack–tip loading, which is for short cracks a nonlinear function of the support load. Both K_I and K_{II} are found to decrease rapidly with crack–length.

INTRODUCTION

In recent years several large cracks and even complete failures were detected in high–strength steel rollers of bridge supports. Typically the cracks started from the area of Hertzian contact and grew perpendicular to the surface (i.e. radially) towards the center of the roller, forming approximately plane radial fracture surfaces. An example is shown on Fig. 1. According to fractographical appeareance of the fracture surface, the crack growth mechanism was not cleavage but micro–ductile, forming a fine–dimpled structure, which is typical for forced rupture of the present fine–grained tool steel. According to the fractographical appearance of the crack surface the crack growth was stable and its rate decreasing with increasing crack length. This crack behavior is remarkable in many concerns:

– A crack usually does not grow through a compressive stress–field. How could the crack be initiated in the region of contact and driven through the Hertzian stress–field?

– In the general case of a slightly eccentric crack (with respect to the contact area) a considerable amount of mode–II loading was acting on the crack tip. Why did the crack still grow in a straight line?

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Under load controlled condition crack–growth rates generally increase with increasing crack–length in most cases. Why not in this case?

From a statics point of view there are no obvious reasons for any type of failure. The maximum compressive stresses are in general well below the elastic limit stress and fulfill the requirements of the corresponding standards. Considering the low stresses, the number of load cycles is too low to assume fatigue to be determinant for the failures.

Since the crack can not cross a compressive stress–field the crack obviously moved in vertical direction. The straight crack–path indicates that the crack followed exactly the load line, since any deviation from the axis of symmetry would give rise to an asymmetric Mode–II term in the crack stress–field, which would force the crack to turn aside and cause flaking. A leading action of a residual stress–field can also be excluded, since an axi–symmetrical stressfield in self–equilibrium is not able to drive a crack in a straight line to the center.

At present, the only plausible crack growth mechanism seems to be the following: Due to the daily temperature cycles the Hertzian stressfield moves slightly back and forth on the roller surface. When a crack is formed right between the two points of return, the contact stresses travel each half–cycle across the crack–mouth, changing each time the crack–tip stressfield significantly, especially the asymmetrical (Mode II) component, which is fully reversed each time. The resulting cyclic change of the stressfield in the vicinity of the crack might cause sort of low–cycle fatigue crack growth. However, this hypothesis is only plausible if the corresponding crack tip stressfields are of reasonable magnitudes. In the following the stress intensity factors are derived for the relevant load configurations. Thereby emphasis is layed on the qualitative behavior rather than on numerical accuracy.

STRESS INTENSITY FACTORS

Since the cracks are much wider (in axial direction) than deep, the cracked rollers can be considered as a two–dimensional plane strain system. The general crack configuration and its loading is shown in fig. 2. According to Roark (1) the maximum compressive stress σ_{cm} and the width e of the contact area due to a contact force P (force per unit length in axial direction) is given by

$$\sigma_{cm} = 0.591 \sqrt{(P\ E/2R)}, \qquad e = 2.15 \sqrt{(2\ P\ R/E)} \qquad (1)$$

where R denotes the radius of the roller and E its Young's modulus.

<u>Superposition.</u> The stress intensity factors K_I and K_{II} due to the Hertzian distribution of contact stresses $\sigma_c(x)$ strongly depend on the position of the contact area with respect to the crack mouth, given by the coordinate s. If the crack is "long", i.e a>> e/2 and a>>s, the sum of the compressive stresses can be considered as a point–load P acting on one side of the crack–mouth. This load configuration can be related to known solutions by the superposition shown in Fig. 3. The antisymmetrical system B gives rise to a pure antisymmetrical stressfield in the vicinity of the crack tip characterized by K_{II}, the symmetrical system C to a stressfield characterized by K_I.

System B. Although the outer shape of the bodies are quite different, the crack of system B in Fig. 3 behaves statically approximately like a surface crack in a strip of width 2R, which is loaded by two coplanar opposite forces acting on the crack–mouth. Thus the stress–intensity factor for the latter, which can be found in Tada et al. (Ref.2, p. 2.28)), also holds approximately for system B, i.e.:

– for a<<R: $K_{II} = 1.30\, P / \sqrt{(\pi\, a)}$

– for a<2R: $$K_{II} = \frac{P}{\sqrt{(\pi\, a)}} F_{II}(\xi), \quad \xi = a/2R \tag{2}$$

with $$F_{II} = \frac{1.30 - 0.65\,\xi + 0.37\xi^2 + 0.28\,\xi^3}{\sqrt{(1-\xi)}}$$

The stress intensity factor given by (2) is increasing with decreasing crack–length a , approaching infinity at a = 0. Of course this result does not hold from a physical point of view. For cracks comparable in size with e the effect of the distribution of P over the contact area, $\sigma_c(x)$, has to be taken into account. Doing this one obtains finite $K_{II}(a)$, which is 0 for a=0 and reaches its maximum at $a \simeq e$ (see Schindler, ref.(3))

System C. Due to the two point–loads the edges of the crack were elastically deformed in a way as indicated in Fig. 4. Thus the crack edges are pressed against each other. This gives rise to contact forces Q which produce a K_I–stressfield at the crack–tip. The magnitude of Q can be calculated from the displacement d which the crack edges would undergo if they were free to move across the symmetry–axis and the compliance of the crack–mouth with respect to the forces Q.

If the cracks are not too long, say $a \leq R$, a simple approximate calculation of the displacement d is possible by modeling the two triangular regions between the crack and the outer surface as clamped cantilever beams of linearly increasing cross–sections (Fig. 5). The horizontal displacement of these beams can be calculated roughly by using Castigliano's theorem on the basis of simple beam theory. One obtains

$$d = \frac{\partial}{\partial T} \left\{ \frac{2T^2}{3E} \int_{a0}^{a} (1/y)\, dy \right\} \tag{3}$$

where T denotes the transverse component of P/2, i.e. $T = P/2\sqrt{2}$. Because of the infinite stress at y=0 a suitable lower integration boundary a_0 has to be chosen. Integration of (3) with $a_0 = e/2$ gives a solution which is nonlinear in a and P:

$$d = \frac{P}{6E} \ln[0.465\, a \sqrt{(E/P\, R)} \tag{4}$$

In order to find the compliance of the system the crack–mouth opening 2Δ due to the horizontal forces Q has to be calculated. For relatively short cracks (a<<R) this can be done by using the basic solution (see Ref. (2)):

$$K_I = 2.60\ Q / \sqrt{(\pi\ a)} \tag{5}$$

for the system shown in Fig. 4. Applying Castigliano's theorem yields:

$$\Delta = \frac{\partial}{\partial Q} \frac{(1-\nu^2)}{E} \int_{a0}^{a} K_I^2(a)\ da \tag{6}$$

Inserting (5) in (6) and integration of (6) results in the same kind of a logarithmic function as (4), so it is reasonable to assume the same integration interval. One obtains

$$\Delta = \frac{13.52}{E\,\pi}\ Q\ \ln\ (0.465\ a\ \sqrt{(E/PR)}) \tag{7}$$

The condition of physical contact between the two edges of the crack requires $d=\Delta$. By using (4) and (7) one finds

$$Q = 0.0213\ \pi\ P \tag{8}$$

It is interesting to note that Q is a pure linear function of P, independent on the crack–length a. Eq. (8) inserted in (5) gives

$$K_I = 0.201\ P / \sqrt{(\pi\ a)} \tag{9}$$

This solution is valid for $a \leq R$. For larger crack–lengths the K_I according to (9) is too high. For short cracks, i.e. cracks comparable in size with e, (9) looses its validity for the same reason as (2).

DISCUSSION

By comparing (2) and (9) one finds, that a surface crack loaded by concentrated surface force acting on one side of the crack–mouth is predominantly loaded in Mode II (see Fig. 6). Inserting the typical numerical values of P and R (7800 N and 150 mm, resp.) one ends up with Mode–II stress–intensity factors which are very high, above or slightly below the K_{Ic} of the present high– strength steel. On the other hand the K_I, which is about 8% of K_{II}, stays well below it. Consider the case where P travels from a position $s>a$ across the crack–mouth to $s<-a$. In this case K_{II} rises from 0 to its maximum, changes the sign while passing the crack and goes back to 0 again. At the same time K_I goes from 0 to its maximum given by (9) and back to 0 again. Thus the material at the crack–tip is loaded by cyclic strains of high amplitudes, resulting in low–cycle fatigue crack–rowth. Thereby K_{II} probably causes the damage (void–growth) and K_I the subsequent material separation.

The question of crack–initiation is disregarded in this paper. It is not clear in detail how a crack can nucleate in the vicinity of the Hertzian contact. It is likely to assume that they are initiated either at pits on the surface or slightly below the surface according mechanisms as treated by Hahn and co–workers (Ref.4,5)

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FIGURES

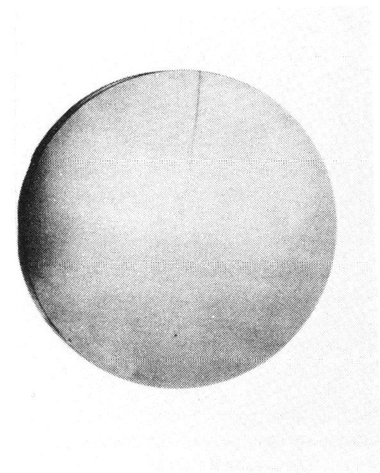

Fig. 1: Cross–section of a roller with a crack

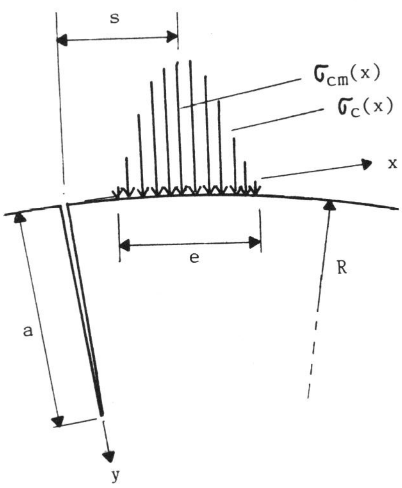

Fig. 2: Crack–model and loading a a system

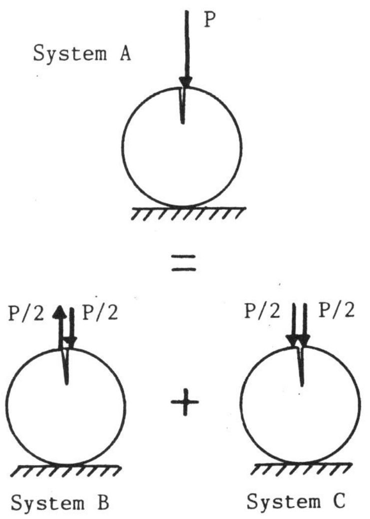

Fig.3: Superposition

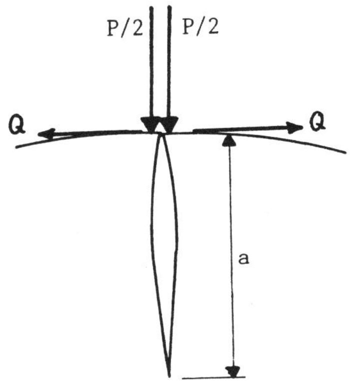

Fig.4: Schematic scetch of the crack load of system C in Fig.3

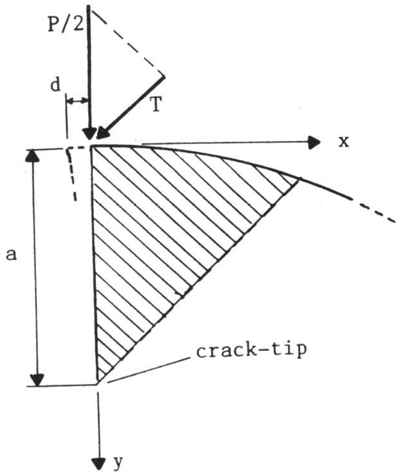

Fig.5: Statical model for displacement–calculation

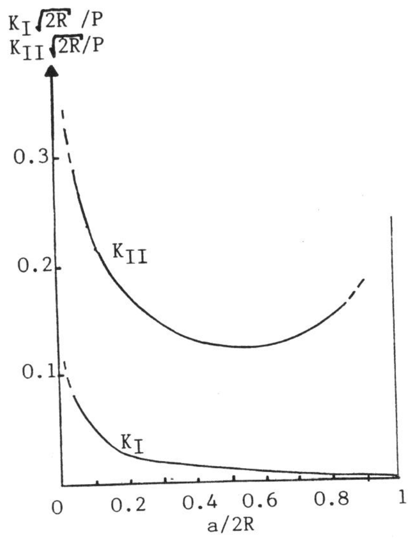

Fig.6: K_I and K_{II} in function of crack–length