FATIGUE OF WELDED STRUCTURES - A FRACTURE MECHANICS APPROACH

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This paper gives a brief description of the fracture mechanics approach for fatigue of welded structures. Crack growth laws and their constants are presented. General information is given about stress intensity factors and reference to analytical expressions for weld toe stress intensity factors is given. A calculation procedure suitable for a computer program is outlined. Multiple initiation of cracks at weld toes can be incorporated in the calculation procedure. Reference to comparisons of tests and calculations is given. The paper concludes with some general remarks and recommendations.

INTRODUCTION

The fatigue life of a welded joint in a steel structure can be determined either by the traditional S-N approach or with a more sophisticated fracture mechanics (FM) approach.

The S-N method is based on experiments, resulting in graphs with the stress range (S) on the vertical axis and the number of cycles to failure (N) on the horizontal axis. This method can be found in many codes and is widely used for fatigue design of welded steel structures.

The FM approach is based on a fatigue crack growth model. The material crack growth parameters in the model can be determined from standardized small specimens. The influence of the specific joint geometry is incorporated in the loading parameter (ΔK) (see section "Stress intensity factors").

With regard to the S-N method the FM approach has the advantage that the remaining fatigue life of defective welded joints can be assessed (Dijkstra et al[1]) and special effects

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such as weld toe grinding or scale effects can be studied without testing (Dijkstra et al[2]).

In this paper the fracture mechanics approach is outlined, with special attention to welded joints in steel structures.

CRACK GROWTH MODELS AND CONSTANTS.

The most simple and widely used fatigue crack growth model, based on linear elastic fracture mechanics, is the Paris-Erdogan relation. This model gives the relation between the crack growth rate (da/dN) and the fatigue loading parameter (stress intensity factor range (ΔK)) (see region II in figure 1) :

$$da/dN = C\ (\Delta K)^m \qquad \ldots\ldots (1)$$

where C and m are the material crack growth parameters.

In the near threshold range (region I in figure 1) the influence of the threshold value of ΔK (ΔK_{th}) can also be incorporated in the relation:

$$da/dN = C\ (\Delta K^m - \Delta K^m_{th}) \qquad \ldots\ldots (2)$$

In the upswing of the crack growth curve (region III in figure 1) the influence of the critical value of K (K_c), combined with the load ratio R ($=F_{min}/F_{max}$), can be taken into account.

$$da/dN = \frac{C\ (\Delta K)^m}{(1-R)K_c - \Delta K} \qquad \ldots\ldots (3)$$

The crack growth constants C and m, and ΔK_{th} have to be determined for the relevant material and conditions (environment, frequency, etc). Reference [1] gives recommended values when no specific data are available.

STRESS INTENSITY FACTORS

The stress intensity factor (SIF,K) range is the difference between the maximum SIF and the minimum SIF during a load cycle. The SIF is a measure for the magnitude of the stresses near the crack tip; eqn. (4).

$$K = Y\ \sigma\ \sqrt{(\pi a)} \qquad \ldots\ldots (4)$$

where: σ = remotely applied stress
Y = correction factor depending on geometry and loading conditions
a = crack depth

The stress variations for a fatigue crack growth calculation have to be determined from the complete load history during the (remaining part of the) service life.

The governing fatigue stresses for an as-welded structure are the nominal elastic stress ranges of the membrane stress (σ_m) and bending stress (σ_b) at the crack location for the uncracked geometry. The effect of the global geometry should be incorporated in the stress analysis, while the effect of the local geometry (weld shape, etc.) will be incorporated in the determination of the stress intensity factor by the stress intensity concentration factor (M_k).

The SIF of a constant depth edge crack in a welded 2D geometry (see figure 2) is generally given as follows:

$$K = [M_{k,m} M_m \sigma_m + M_{k,b} M_b \sigma_b] \sqrt{(\pi a)} \qquad \text{...............(5)}$$

where: M_k = stress intensity concentration factor for the influence of the weld geometry
M = stress intensity correction factor for the strip without the weld geometry
m and b as index means for membrane stress and for bending stress respectively.

M is a function of the relative crack depth (a/T). Formulas can be found in literature (Rooke and Cartwright[3]).
M_k is a function of a/T , the weld dimensions and the weld type. Assuming no interaction between the influence of the relative weld width (L/T), the weld angle (θ) and the relative weld toe radius (ρ/T) the following formula for M_k can be written.

$$M_k = f_L(a/T, L/T) . f_\theta(a/T, \theta) . f_\rho(a/T, \rho/T) \qquad \text{............(6)}$$

where: f_L = a correction factor for L/T for a specific weld type with a certain weld angle and weld toe radius.
f_θ = a correction factor for θ.
f_ρ = a correction factor for ρ/T.

More information and analytical expressions about M_k, f_L, f_θ and f_ρ can be found in literature ([1], [2], Smith and Hurworth[4] and Maddox et al[5]).
Figure 3 gives the general curve of M and M.M_k as function of a/T.

The SIF of a semi-elliptical crack at a weld toe in a 3D geometry (see fig. 4) is generally given as follows:

$$K_a = [M_{k,m,a} M_{m,a} \sigma_m + M_{k,b,a} M_{b,a} \sigma_b] \sqrt{(\pi a)}/\Phi \qquad \text{......(7a)}$$

$$K_c = [M_{k,m,c} M_{m,c} \sigma_m + M_{k,b,c} M_{b,c} \sigma_b] \sqrt{(\pi a)}/\Phi \qquad \text{......(7b)}$$

where: a and c as index means for crack depth and for crack width direction respectively,
Φ = elliptical integral of the second kind,
for other symbols see equation (5).

The correction factors for the flat plate ($M_{m,a}$, $M_{b,a}$, $M_{m,c}$ and $M_{b,c}$) presented by Newman and Raju[6] can be used. Van Straalen et al.[7] determined SIFs for a T-plate with a weld discontinuity. They found that 2D M_k values are a conservative approximation of 3D M_k values.

CALCULATION PROCEDURE

The lifetime can be calculated by integrating the crack growth law from the initial defect size (a_i) to the final (allowable) defect size (a_f). The integration can be done either analytically or numerically. Due to the complex relation between a and ΔK a numerical (step by step) calculation procedure carried out by a computer is recommended.
TNO-IBBC has developed the program FAFRAM (FAtigue FRActure Mechanics) (Snijder and Dijkstra[8]). The numerical procedure, used in FAFRAM, for the semi-elliptical crack of figure 4 is as follows :

1. With the actual crack depth (a_i) and half crack width (c_i) and the other geometrical parameters the values for M and M_k can be calculated.
2. Using the stress range ($\Delta\sigma$), the SIFs for crack depth (ΔK_a) and crack width (ΔK_c) can be calculated with equation (7).
3. Assuming a crack extension Δa, the corresponding number of cycles can be calculated with the Paris relation.

$$\frac{\Delta a}{\Delta N} = C\,(\Delta K_a)^m \qquad \text{or:} \qquad \Delta N = \frac{\Delta a}{C\,(\Delta K_a)^m} \qquad .....(8)$$

4. The crack extension in the width direction can also be calculated with the Paris relation.

$$\frac{\Delta c}{\Delta N} = C\,(\Delta K_c)^m \qquad \text{or:} \qquad \Delta c = \Delta N\, C\,(\Delta K_c)^m = \Delta a\,\left(\frac{\Delta K_c}{\Delta K_a}\right)^m \qquad ..(9)$$

5. The number of cycles has to be increased with ΔN.

$$N_{i+1} = N_i + \Delta N \qquad ....(10)$$

6. The crack dimensions have to be increased with the crack extensions

$$a_{i+1} = a_i + \Delta a \qquad \text{and:} \qquad c_{i+1} = c_i + \Delta c \qquad ....(11)$$

7. With the new crack dimensions (a_{i+1}, c_{i+1}) the next step can be calculated, starting with point 1 above.
8. The calculation has to be continued until the allowable crack depth (a_f) or until the required number of cycles (N_{req}).
9. The calculated number of cycles or crack size has to be assessed at its acceptability.

For 2D geometries the calculation has to be carried out in the a-direction only.

An extension of the crack growth procedure to cover growth of multiple initiated cracks as often occur at weld toes is described by Snijder et al[9].

A comparison of test results on welded T-joints with results of FAFRAM-calculations is given in [2]. The calculations give a good estimation of the influence of grinding and of the thickness effect. Dijkstra et al[10] showed good agreement between tests and calculations for beams with internal defects.

CONCLUDING REMARKS

- With the information in the paper and the references given to open literature a fatigue crack growth calculation of a steel welded joint can be carried out.
- The Paris-Erdogan relation can be applied in most cases as the crack growth law.
- A fatigue crack growth calculation procedure suitable for a computer program is given.
- Experimental crack growth data for 3-D geometries is needed to validate the crack growth model for a 3-D situation.

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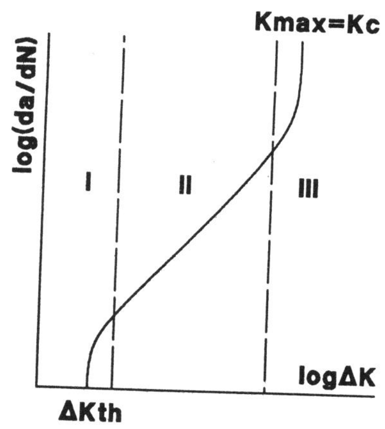

Figure 1 Crack growth curve

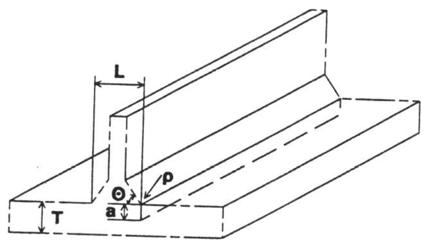

Figure 2 Constant depth crack

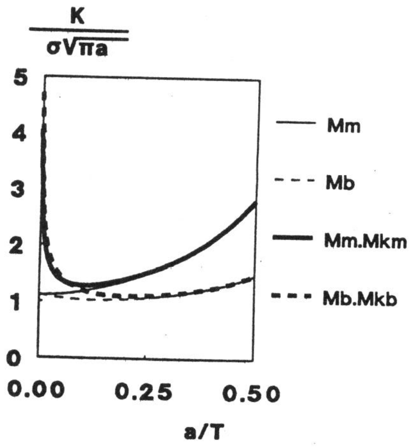

Figure 3 Geometry factors

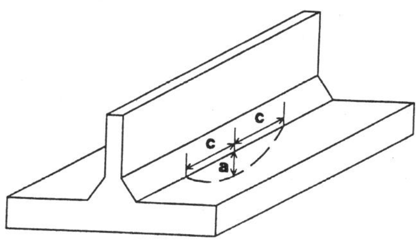

Figure 4 Semi-elliptical crack at weld toe