EFFECT OF THE LENGTH OF THE RANDOM LOAD HISTORY ON THE FATIGUE LIFE STATISTICS.

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To study fatigue crack growth under random loading, it is customary to consider finite length records as representative of the whole process. A few of the highest peaks determine the closure stress along the entire growth. This reduces the number of them which have a real influence in the life span.

This work studies the statistical distribution of fatigue lifetime as a function of the number of cycles reckoned to be representative of the load history. Both, narrow and wide band processes are investigated. It is shown that the number of cycles required to achieve a certain scatter in lifetime predictions depends on various parameters, band width and bulk stress level among the most important.

INTRODUCTION

The study of the fatigue resistance of mechanical systems may be formulated below the point of view of the reckoning of number of cycles needed for a crack to grow a determined length under certain load history. The estimates by means of the usual procedures can be quite different to experimental results. The reasons of theses differences can be divided in two groups: analytical or numerical errors of the model employed in the assessment; and statistical nature of the loads, environmental conditions and the material properties, that may produce different results, even between two experiences with, in theory, the same conditions. Among the error sources due to the statistical feature of the fatigue phenomenon, the followings can be remarked: random variation of the crack growth rate; variability of some environmental conditions; differences in critical stress intensity factors; random loads; etc.(Palmberg (1), Blom (2)).

When a system suffers random loads, part of the differences between analytic or empiric results and the service values are due to these loads. They are produced by modelling errors as well as differences in records which represent the same random process.

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Although belonging to the same random process, they will never have the same peaks and troughs, nor be distributed in the same manner.

Searching for the representative records of some process has been an important priority in the last years, specially in the aeronautic and, in minor intensity, in the automobile industries, Wetzel (3), Abelkis and Potter (4), Bryan and Potter (5), Potter and Watanabe (6). For instance, some load histories have been proposed for several sorts of aircraft, like TWIST, FALSTAFF (De Jonge et al. (7), Van Dijk and De Jonge (8)),and HELIX and FELIX for helicopter rotors, (Ten Have (9)).

In this searching of representative records, the effect of several variables on the crack growth has been analysed (Dill and Saff (10), Circle and Conley (12)). This will allow to decide whether their consideration or not is convenient when defining the record type for each case. Some of this factors are: the arrangement of loads inside a record; the set-up of load histories produced by the different working regimes that system must perform along its life; the elimination of the low amplitude cycles; the record length; etc. The record with the defined number of cycles is shorter than the total life of any component to be tested or analysed. The complete load process is simulated by repeating the unique record (block of load) until the component failure, instead of generating a record, with the same characteristics, as long as the expected life.

An essential requirement, so that a finite length record represents a random process, is that the statistical features of the selected record should represent the whole process. If the length is short, stocastic parameters belonging to two different records which represent the same process may be very different.

In fatigue crack propagation, the problem of the minimum record length selection becomes more difficult to solve, due to sequence effect. There might be situations in which two load records with very similar statistical features (peak distribution, number of them, ranges, etc.) supply very different results of crack growth parameters. This possible difference may be caused by the largest amplitude cycles of a record which affect the process more than others do.

In this work, fatigue life variability is studied relate to the number of cycles considered representative of the load process, in stationary ones. This distribution is analysed for different shapes of the spectral density (SDF) and for distinct load levels.

ANALYSIS PROCEDURE

To study the record length effect on the results, several groups of records have been used for the analysis, all records generated from the same SDF but with a very different length each group . The same analysis has been done using three different shapes of the SDF: one of them is a narrow band one, with constant value between 5 and 7.5 Hz, called type 1; another one, of wider bandwidth, with constant value from 5 to 40 Hz (type 2); and the third, with two frequency ranges in which the SDF is constant and non-zero (type 3). In this last one, the limits of the frequency range are: 5 and 40 Hz, for the first; 50 and 70 Hz for the second one, being the SDF value one-third the corresponding to the first range.

Several record lengths have been defined for each of these SDF shapes, ranging from 1.000 to 100.000 cycles. At the same time, for each defined length, several stationary stress histories, with the same SDF, have been generated. The simulation of the history have been performed by sine functions superposition and random phase of uniform distribution, Shinozuca (13). From the simulated process, a second one has been obtained which only has the peaks and valleys. Figure 1 shows an example of record and its SDF. Every generated record has been used for cycle by cycle simulation of crack growth, repeating the history several times until the final length is reached.

There are a lot of models to simulate crack growth, accounting for the retardation effect. Most of them can be included in one of the following groups: Those that use the crack closure phenomenon, Newman (14); and those based on the tip residual stress concept, Willemborg (15), Johnson (16). The model used in this work pertains to the second group. It is the proposed by Johnson (16) that has into account delay, produced by positive overloads, and acceleration, due to negative overloads.

The crack growth rate equation used is a modification of that proposed by Forman:

$$\frac{da}{dN} = C \frac{\Delta K^{m}}{(1 - R_{ef})\, K_{c} - \Delta K} \qquad (1)$$

Where Reff is the effective stress ratio defined by Johnson (16).

Table 1 shows data used in the model application. They correspond the Al-alloy 2219-T851, (16). af is the final crack length, and A, B, Y and Z are constants of this model.

RESULTS

Before analysing the effect of the record length on the dispersion of the results obtained with different histories from the same SDF, the variability of several parameters has been compared for two short record lengths. Five to ten histories with 1250 and 5000 cycles have been simulated from a SDF as that represented on the bottom of fig. 2. For each history, the following values have been

TABLE 1 - Data and Parameters of the Model.

C = 4.626 E-09
m = 3.171
Kc = 88.0 $\text{MPa}\sqrt{\text{m}}$
Kth = 3.3 $\text{MPa}\sqrt{\text{m}}$
σ_{ys} = 345.0 MPa
ao = 0.5 mm

af= 5.0 mm
A = 1.0
B = 2.3
Y = 0.0
Z = 0.5
(Plane Stress)

obtained: crack length increment produced during one repetition of each history (Δa); mean value of stress ranges (ΔS); extreme value (Ymax); and the mean effective stress range (ΔSef). All of them have been obtained during the repetition of the block when the crack length was 1mm. To normalise these values, they have been divided by the mean value of all histories.

Fig. 2 shows the results. The first and second columns represent the normalized values of Ymax and Δa per block for ten different records with 5000 cycles. The 3rd and 4th columns represent the values of S for records with 1250 and 5000 cycles, respectively. The fifth and sixth represent Sef for 1250 and 5000 cycles, respectively. Where ΔSef of each cycle is the stress range that using the simple Paris equation would produce the same fatigue crack growth rate than with this model.

It can be seen that ΔS almost does not change from one history to another, even with length of 1250 cycles. The same happens with the variance of ΔS, not represented. These two parameters could be considered representative of the histories in case of not considering sequence effects. Nevertheless, if sequence effects are present, as in this case, they represent very badly the history features. It can be seen a high dispersion of a and of Sef for 1250 and 5000 cycles. Thus, the mean and variance of ΔS are not valid to represent the histories, and their variability are not directly related to the dispersion in calculated lives.

Special interest has the observation of the dispersion of Ymax. It is similar to that of Δa. Veers et al. (17) found certain correlation between the values of Ymax in a history and the fatigue life obtained repeating that history record as a block until failure. Thus, the distribution of Ymax or any other parameter related to highest peaks in a history may be good candidates to estimate the dispersion in the fatigue life produced when different load histories of specific length are used.

In figure 3, a histogram of lifetimes, produced with 200 different records of the same length (10000 cycles), is shown. Histories are type 2, with 55.5 MPa^2 /Hz as SDF. Though the coefficient of variation (COV) is 0.063, there is a wide range of variation. The band of variation is about 45% of the mean value, being the maximum life 50% of the minimum of these 200 cases. The

TABLE 2 - **COV and Mean Values for each group of record.**

Type	Value	Number of Cycles			
		10000	20000	30000	40000
1	COV	0.054	0.045	0.036	0.023
	mean	508000	509700	508600	508800
2	COV	0.063	0.054	0.033	0.021
	mean	527200	527700	526200	525500
3	COV	0.048	0.042	0.024	0.014
	mean	520100	520900	518600	519400

distribution can be approximated by any of the extreme values ones. Figure 4 represents 104 values obtained from histories type 1 with 2000 cycles, drawn in weibull probability paper for maxima. Similar approximation is obtained using Frechet probability paper.

Table 2 represents the mean and COV obtained with different number of cycles and shapes of SDF. It can be seen that as the length of the record increases the COV decreases . The decrease rate being similar in all cases. When the number of cycles are enlarged from 10000 to 40000, the COV is divided by 3.

To know the stress level effect on the COV, two other cases have been considered. One with a load level 80% of the previous ones. Table 3 shows the result of 25 histories with 10000 cycles for each SDF shape. In order to obtain similar lifetimes with this new stress level, the final crack length in the simulation was reduced to 1mm. Compared with values in table 2, the COV decreases for the three shapes of SDF when the stresses are reduced. Opposite results are obtained when stresses are increased. Table 4 shows the mean and COV of the lives produced when the stress level is increased to produce lives five times lower than those with the original level. In this case only the SDF shape type 2 has been considered, and the number of cycles of each record were forced to change between 2000 and 25000. For each length, 104 cases were simulated. It can be seen that the COV for these cases are higher than for lower stresses.

CONCLUSIONS

It is known that the selection of the load history as representative of any random load process, is critical to obtain reliable results. The parameters to be accounted for depend on what kind of process is being considered. Some of them are: stress level; order and number of load excursions; stress levels that can be eliminated as non-damaging; number of cycles in the block, etc.

TABLE 3- **COV and Mean Value for lower stress level**

N	Value	Type		
		1	2	3
10000	COV	0.029	0.033	0.026
	mean	508400	510200	524900

TABLE 4 - **COV and Mean Values for higher stress level.**

Type	Value	Number of Cycles			
		2000	5000	10000	25000
1	COV	0.100	0.087	0.073	0.063
	mean	114900	115200	116100	115500

In the case of the length of the record to represent the history, it has been checked that its influence on the obtained life can be important. Depending on the length, the reliability of that simulation to represent any real condition and the possible difference of lives obtained in other cases will be higher or lower. These differences will depend on different factors, two of them are stress level and SDF shape. Other factors, not considered here but that may be important are: crack growth rate equation parameters C and m and yield stress (σy).

Finally, it has been seen that the distribution of life, produced with different load histories from a process with the same SDF, can be represented by an extreme value distribution. This may be important in order to know the uncertainty of results produced with any length of record. Future work will be done in order to estimate the parameters of these extreme value distributions from those of the histories. It will allow to know without doing many simulations the distribution of lives for different lengths and so, to increase the approximation of predictions.

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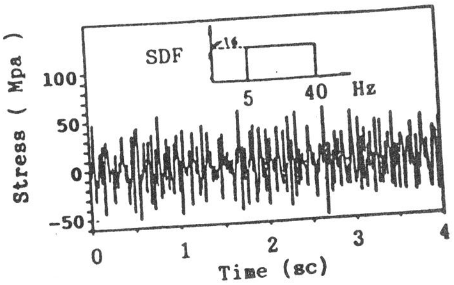

Figure 1 Stress Record for a constant SDF from 4 to 50 Hz

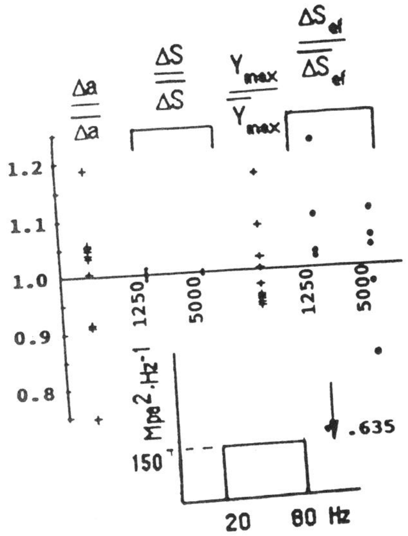

Figure 2 Dispers. of the Values. Records of 1250 and 5000 cycles.

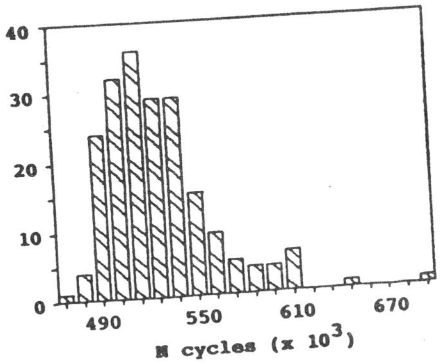

Figure 3 Histogram. 200 cases. Records: type 2; 10000 cycles.

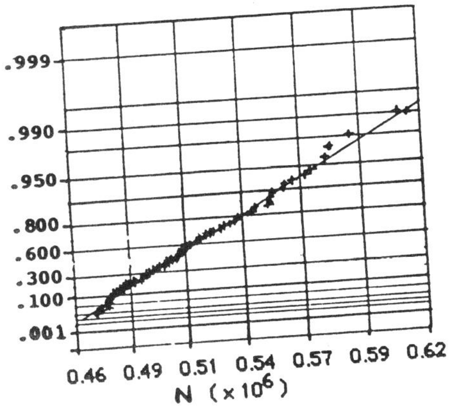

Figure 4 Weibull probability paper. Record: type 1; 2000 cycles