FATIGUE FRACTURE FRACTAL MECHANICS

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Some theoretical and experimental results are described those are directed in determination and explanation of relations between the fractal dimension of material damage and mechanical properties of material. It is shown that there are unique relations between micro- and macroparameters of fracture. Linear relation proposed between Paris's law exponent and fractal dimension of fatigue fractured surface enables to consider like phase transitions theory exponent.

INTRODUCTION

New physical concepts always give some insight into character of natural objects and processes. One of the- se concepts of last time is the fractal geometry displaying the selfsimilarity properties of physical structures. Recently the selfsimilarity of fractured surfaces was revealed by many authors and there is a lot of efforts to relate fractal characteristics with mechanical and physical values. Mandelbrot (I) defined a fractal as a set for which the Hausdorf-Bezikovich dimension D always exceedes the topological dimension D_T. The fractal dimension is a quantitative characteristic of fractal structures which are invariant under locai dilatations. In this work the concept of fractals has been used for the quantitative description of dissipative structures (2) controlling fatigue mode I fracture. This is a basis for establishing the relationship between the dynamic structure parameters and resistance to fatigue fracture.

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THE SELFSIMILARITY FUNCTION FOR FRACTAL DISSIPATIVE STRUCTURES

According to Mandelbrot (I) the selfsimilarity signifies that there is a function that copies a set into itself with the aid of scalar z being a selfsimilarity ratio. For nonstandart selfsimilar form an entity can be divided into N parts obtained through the selfsimilar ratio z which is related to the fractal dimention D, $0<D<3$, by the relation

$$z^D = 1/N \; . \qquad \text{(I)}$$

The use of the relation (I) in the analysis of dis sipative structures which control fracture of solids requires that the physical meaning of the parameters N and z be established. For this purpose let us consider the selfsimilar growth of a fatigue crack within the limits of which the crack rate da/dN depends only on the range of the stress intensity factor ΔK when

$$da/dN = B(\Delta K/A)^n \; , \qquad (2)$$

where A and B are dimensional constants which control the selfsimilarity boundaries (3). Under the conditions of selfsimilar growth of a fatigue crack the parameter n is a characteristic related to the dynamic structure which controls the fracture rate during the motion of the crack sides according to mode I fracture. Let us choose for analysis a bifurcation point which corresponds to the transition of a macrocrack to instability by the moment of attainment of the maximum size of a prefracture zone r_c^{max} and to transition of a microcrack to instability with the initial size r_0 by the moment of attainment of the microcrack size r_{0c} in the direction of the crack motion. With an increase of the crack by the value $\Delta a = r_c^{max}$ in one cycle the crack becomes unstable by the moment of attainment of $K_I = K_{Iq}^{max}$ which correspondes to the realization of the upper boundary of selfsimilar growth of the fatigue crack, Fig.I. The parameter K_{Iq} is related to n by the relation (3)

$$K_{Iq}^{max} = K_{IR}^{max} \, \Delta^{-1/2} \left[\frac{n_{max} - n}{n_{max} - 2} \right] = K_{IC}^{\phi} \qquad (3)$$

where K_{IR}^{max} is a dimensional constant which controls the maximum size of the selfsimilar prefracture zone, Δ is a tension fracture constant for alloys on the same basis (3,4), and n_{max} is the maximum value of n after realization of tension fracture. On the other hand, under the conditions of selfsimilarity the r_c^{max} depends only on the yield point σ_y of the material and is determined from the relation (3,4)

$$r_c^{max} = \frac{1}{2\pi}\left[K_{IR}^{max} / \sigma_y\right]^2 . \tag{4}$$

It allows to introduce the scale coefficient i_r^c :

$$i_r^c = r_c^{max} / r_{oc} \tag{5}$$

where r_{oc} and r_c^{max} are the minimum and maximum scales at $\Delta K = K_{I\varphi}^{max}$. Using strain energy density function proposed by G.Sih (5) the scale coefficient is expressed by the ratio

$$i_r^c = K_{IR}^{max} \cdot E \cdot W_{c*} / \left[(1+\nu)(1-2\nu)(K_{Ic} \cdot \sigma_y)^2\right] . \tag{6}$$

It combines the yield point of material which determines the maximum value of r_c^{max} , the resistance to nucleation of the microcracks W_{c*} , which determines the subsequent unstable behaviour of the fractal object, and $K_{Ic}^{*} = K_{I\varphi}^{max}$ which determines the energy for the unstable crack motion.

For the other hand, we use $\Delta^{1/m}$ function which helps to describe the stepwise growth of a microcrack. This function gives compact information on the kinetics of selfsimilar growth of the fractal object at different scale levels. The possibility of describing the stepwise growth of a crack using the function $\Delta^{1/m}$ was demonstrated experimentally (3). Therefore the selfsimilar growth of a fractal object can also be represented as intermediate asymptotics blocks

$$r_{oj}^{N-1} / r_{oj}^{N} = \Delta^{1/m} , \tag{7}$$

where r_{oj}^{N-1} and r_{oj}^{N} are the preceding and next sizes of the fractal object in the direction of crack motion and m is the coefficient that is equal to I,2,4... ∞ . This signifies that with each output from the intermediate asymptotics block the fractal cluster size is increased by a value of Δ at $m \to \infty$. This makes possible to use the selfsimilarity function $\Delta^{1/m}$ as $m \to \infty$ in order to represent the relation (I) as

$$\Delta^{D+M} = (K_{IC}\sigma_y)^2(1+\nu)(1-2\nu) / (K_{IR}^{max} \cdot E \cdot W_{c*}) . \tag{8}$$

Fig.2 demonstrates a good agreement of the above speculations with experimental data for greate number of different steels with M =0 (brittle steels) and M =I (ductile steels).

ON THE PHYSICAL MEANING OF EMPIRICAL PARIS'S LAW

TABLE I - Fatigue Fracture Testing Data for (I0Ni,I0Cr,0.98Ti)-Maraging Steel.

Heat treatments	R	ΔK_θ MPam$^{1/2}$	n'	n''	$\langle n \rangle$	D-2
Tempering	0.I0	none		bend	2.294	0.453
	0.60				2.709	0.397
Tempering and aging	0.I0	2I.9	3.659	2.254	3.235	0.297
	0.33	I5.3	3.872	2.200	3.383	0.280
	0.50	I4.4	3.420	2.I34	3.II3	0.282
	0.60	I2.2	3.6I2	2.I28	3.342	0.226
	0.70	7.9	3.944	2.04I	2.785	0.233

We try to sustain a well-known efforts to describe fatigue fracture stochastically (Krausz et al.(6)). Regarding the fatigue fracture process as a stepwise crack tip propagation after some degree of coalescing damage near the crack tip is achieved we suppose it's like a diffusion limited aggregation (DLA). From appropriate DLA model (described by Kang et al.(7)) and properties of its solution under critical condition on fractal patterns size of average "mass" it's easy to deduce linear relation

$$n = 2 + D\cdot 2(2\omega - 1), \qquad (9)$$

where $\omega > 1/2$ is a scaling exponent of the kernel of model equations. Equation (9) as additional to the so-called two-exponential scaling, enables to consider like phase transitions theory exponent. To protect this point the experimental results are presented in Table I. R is min/max load ratio, ΔK_θ is effective mode I stress intensity factor corresponding to the bend of fatigue $\log(\Delta K)$-$\log(da/dN)$-diagram; n', n'' and $\langle n \rangle$ are exponents of Paris's dependences before, afterwards and over all experimental points respectively. Determined correlations are

$$D-2 = 0.015\,\Delta K_\theta + 1.934, \quad r = 0.963$$
$$\Delta K_\theta = 23.71 - 20.98\cdot R, \quad r = 0.970$$
$$n'' = 1.85\cdot(D-2) + 1.66, \quad r = 0.793$$

D being measured for fatigue fractured surfaces profiles under ordinata/abcissa magnification ratio 20 demonstrates a positive correlation with n''. This tendency does agree with theoretical formula (9) because of D depends on magnification ratio monotonically.

SYMBOLS USED

$A, K_{Iq}, K_{Iq}^{max}, K_{IR}^{max}, \Delta K_{\theta}$ = effective ΔK -values (MPa$\sqrt{m}$)
B = dimensional constant (m/cycle)
Δ = tension fracture constant
D = fractal dimension
E = Young's modulus (MPa)
i_r^c = scale coefficient
m = index
ν = Poisson's ratio
$n, n_{max}, n', n'', \langle n \rangle$ = Paris's law (effective) exponents
N = selfsimilar part number of fractal
ω = scaling exponent
r = correlation coefficient
$r_c^{max}, r_o, r_{oc}, r_{oJ}^N$ = effective sizes (m)
W_{c*} = resistance to microcrack nucleation (MJ/m^3)
λ = selfsimilar ratio

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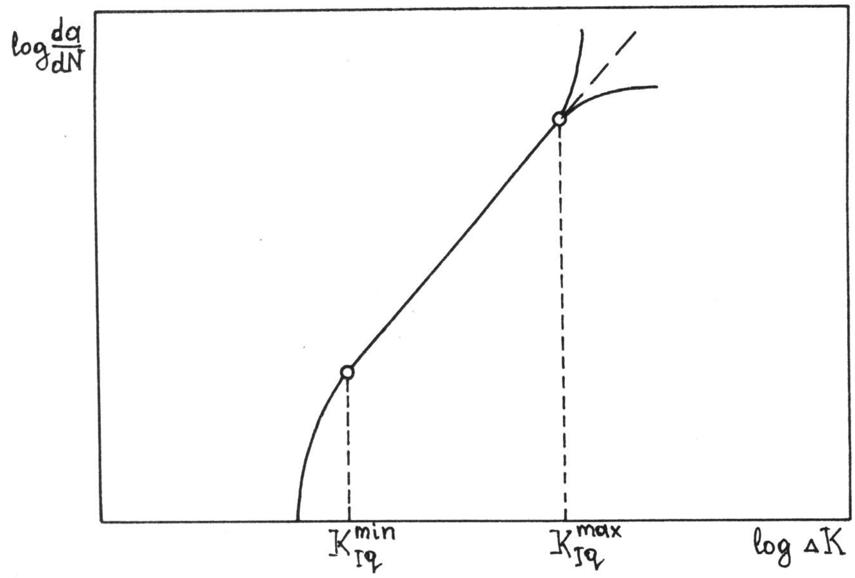

Figure I The fatigue diagram bifurcation point at $\Delta K = K_{Iq}^{max} = K_{Ic}^{\phi}$ controlling fatigue crack instability.

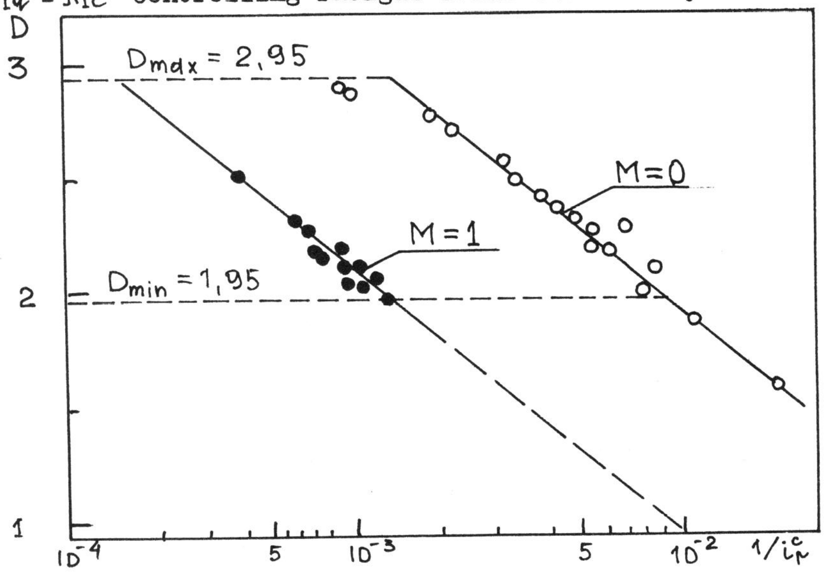

Figure 2 Dissipative structure fractal dimension D versus inverse value of scale coefficient i_r^c at $\Delta K = K_{Iq}^{max}$.